UNIVERSITY OF MUMBAI

No. UG/9 of 2018-19

CIRCULAR:-

Attention of the Principals of the affiliated Colleges and Directors of the recognized Institutions in Humanities, Sci. & Tech. Faculties is invited to this office Circular No.UG/122 of 2017-18 dated 28th July, 2017 relating to syllabus of the B.A./B.Sc. degree course.

They are hereby informed that the recommendations made by the Board of Studies in Mathematics at its meeting held on 3rd May, 2018 have been accepted by the Academic Council at its meeting held on 5th May, 2018 **vide** item No. 4.71 and that in accordance therewith, the revised syllabus as per the (CBCS) for the T.Y.B.A./T.Y.B.Sc. in Mathematics (Sem. –V) Paper-I Integral Calculas, Paper-III Topology of Metric Spaces and (Sem.VI) Paper-I Basic Complex Analysis, Paper-III Topology of Metric Spaces and Real Analysis, has been brought into force with effect from the academic year 2018-19, accordingly. (The same is available on the University's website www.mu.ac.in).

(Dr. Dinesh Kamble)
I/c REGISTRAR

MUMBAI – 400 032
12th June, 2018

To

The Principals of the affiliated Colleges & Directors of the recognized Institutions in Humanities, Sci. & Tech. Faculties. (Circular No. UG/334 of 2017-18 dated 9th January, 2018.)

A.C/4.71/05/05/2018

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No. UG/ 9 -A of 2018 MUMBAI-400 032 12th June, 2018

Copy forwarded with Compliments for information to:-

1) The I/c Dean, Faculties of Humanities, Science & Technology,
2) The Chairman, Board of Studies in Mathematics,
3) The Director, Board of Examinations and Evaluation,
4) The Director, Board of Students Development,
5) The Co-Ordinator, University Computerization Centre,
6) The Professor-cum-Director, Institute of Distance & Open Learning.

(Dr. Dinesh Kamble)
I/c REGISTRAR

UNIVERSITY OF MUMBAI

SYLLABUS for the F.Y.B.A/B.Sc.

Programme: B.A./B.Sc.

Subject: Mathematics

Choice Based Credit System (CBCS)
with effect from the academic year 2018-19

F.Y.B.Sc.(CBCS) Semester I				
CALCULUS I				
COURSE CODE	**UNIT**	**TOPICS**	**CREDITS**	**L/W**
USMT101	UNIT I	Real Number System	2	3
	UNIT II	Sequences		
	UNIT III	Limits and Continuity		
ALGEBRAI				
USMT102	UNIT I	Integers and Divisibility	2	3
	UNIT II	Functions and Equivalence Relation		
	UNIT III	Polynomials		
PRACTICALS				
USMTP01	UNIT I	Practicalsbasedon USMT101,USMT102	2	2
F.Y.B.A.(CBCS) SemesterI				
UAMT101	UNIT I	Real Number System	3	3
	UNIT II	Sequences		
	UNIT III	Limits and Continuity		

F.Y.B.Sc.(CBCS) Semester II				
CALCULUS II				
COURSE CODE	**UNIT**	**TOPICS**	**CREDITS**	**L/W**
USMT201	UNIT I	Infinite Series	2	3
	UNIT II	Continuous functions and Differentiation		
	UNIT III	Applications of Differentiability		
ALGEBRAII				
USMT202	UNIT I	System of Linear Equations and Matrices	2	3
	UNIT II	Vector Spaces		
	UNIT III	Basis & Linear Transformation		
PRACTICALS				
USMTP02	UNIT I	Practicalsbasedon USMT201,USMT202	2	2
F.Y.B.A.(CBCS) SemesterII				
UAMT201	UNIT I	Infinite Series	3	3
	UNIT II	Continuous functions and Differentiation		
	UNIT III	Applications of Differentiation		

- **TEACHING PATTERN FOR SEMESTER I**

  1. Three lectures per week per course.. Each lecture is of 48 minutes duration.
  2. One Practical (2L) per week per batch for courses USMT101, USMT 102 combined (the batches to be formed as pre scribed by the University). Each practical session is of 48 minutes duration.
  3. One Tutorial per week per batch for the course UAMT101(the batches to be formed as prescribed by the University). Each tutorial session is of 48 minutes duration.

- **TEACHINGPATTERN FOR SEMESTER II**

  1. Three lectures per week per course. Each lecture is of 48 minutes duration.
  2. One Practical (2L) per week per batch for courses USMT201, USMT202 combined (the batches to be formed as pre scribed by the University). Each practical session is of 48 minutes duration.
  3. One Tutorial per week per batch for the course UAMT201 (the batches to be formed as pre scribed by the University). Each tutorial session is of 48 minutes duration.

SYLLABUS FOR SEMESTER I

Note: All topics have to be covered with proof in details (unless mentioned otherwise) and with examples.

USMT101/UAMT101 CALCULUS I

Unit 1 : Real Number System (15 Lectures)

1. Real number system $\mathbb{R}$ and order properties of $\mathbb{R}$, absolute value $|\ \ |$ and its properties.

2. AM-GM inequality, Cauchy-Schwarz inequality, Intervals and neighbourhoods, Hausdorff property.

3. Bounded sets, statements of I.u.b. axiom and its consequences, Supremum and infimum, Maximum and minimum, Archimedean property and its applications, density of rationals.

Unit II: Sequences (15 Lectures)

1. Definition of a sequence and examples, Convergence of sequences, every convergent sequences is bounded. Limit of a convergent sequence and uniqueness of limit, Divergent sequences.

2. Convergence of standard sequences like $\left(\frac{1}{1+na}\right) \forall\, a > 0, (b^n)\ \forall\ 0 < b < 1$, $\left(c^{\frac{1}{n}}\right)\ \forall c > 0, \&\ \left(n^{\frac{1}{n}}\right)$,
3. Algebra of convergent sequences, sandwich theorem, monotone sequences, monotone convergence theorem and consequences as convergence of $\left(1 + \frac{1}{n}^n\right)$.
4. Definition of subsequence, subsequence of a convergent sequence is convergent and converges to the same limit, definition of a Cauchy sequences, every convergent sequences s a Cauchy sequence and converse.

Unit III: Limits and Continuity (15 Lectures)

Brief review: Domain and range of a function, injective function, surjectiove function, bijective function, composite of two functions, (when defined) Inverse of a bijective function.

1. Graphs of same standard functions such as
2. $|x|, e^x, \log x\ ax^2 + bx + c, \frac{1}{x}, x^n (n \geq 3), \sin x, \cos x, \tan x, \sin\left(\frac{1}{x}\right), x^2 \sin\left(\frac{1}{x}\right)$ over suitable intervals of $\mathbb{R}$.
3. Definition of Limit $\lim_{x\to a} f(x)$ of a function $f(x)$, evaluation of limit of simple functions using the $\epsilon - \delta$ definition, uniqueness of limit if it exists, algebra of limits, limits of composite function, sandwich theorem, left-hand-limit $\lim_{x\to a-} f(x)$, right-hand-limit $\lim_{x\to a+} f(x)$, non-existence of limits, $\lim_{x\to -\infty} f(x)$, $\lim_{x\to\infty} f(x)$ and $\lim_{x\to a} f(x) = \pm\infty$.
4. Continuous functions: Continuity of a real valued function on a set in terms of limits, examples, Continuity of a real valued function at end points of domain, Sequential continuity, Algebra of continuous functions, discontinuous functions, examples of removable and essential discontinuity.

Reference Books:

1. R.R. Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.
2. K.G. Binmore, Mathematical Analysis, Cambridge University Press, 1982.
3. R. G. Bartle- D. R. Sherbert, Introduction to Real Analysis, John Wiley & Sons, 1994.

Additional Reference Books

1. T.M. Apostol, Calculus Volume I, Wiley & Sons (Asia) Pte, Ltd.
2. Richard Courant-Fritz John, A Introduction to Calculus and Analysis, Volume I, Springer.
3. Ajit kumar- S. Kumaresan, A Basic Course in Real Analysis, CRC Press, 2014.
4. James Stewart, Calculus, Third Edition, Brooks/ cole Publishing Company, 1994.
5. Ghorpade, Sudhir R.-Limaye, Balmohan V., A Course and Real Analysis, Springer International Ltd.2000.

USMT102 ALGEBRA I

Prerequisites:

Set Theory : Set, subset, union and intersection of two sets, empty set, universal set, complement of a set, De Morgan's laws, Cartesian product of two sets, Relations, Permutations n_{P_r} and Combinations n_{C_r}.

Complex numbers: Addition and multiplication of complex numbers, modulus, amplitude and conjugate of a complex number.

Unit I : Integers & Divisibility (15 Lectures)

1. Statements of well-ordering property of non-negative integers, Principle of finite induction (first and second) as a consequence of well-ordering property, Binomial theorem for non-negative exponents, Pascal Triangle.
2. Divisibility in integers, division algorithm, greatest common divisor (g.c.d) and least common multiple (L.c.m) of two integers, basic properties of g.c.d such as existence and uniqueness of g.c.d of integers a & b and that the g.c.d. can be expressed as $ma + nb$ for some $m, n \in \mathbb{Z}$, Euclidean algorithm, Primes, Euclid's lemma, Fundamental Theorem of arithmetic, The set of primes is infinite.
3. Congruences, definition and elementary properties, Eulers φ function, statements of Eulers theorem, Fermats theorem and Wilson theorem, Applications.

Unit II : Functions and Equivalence relations (15 Lectures)

1. Definition of function, domain, co-domain and range of a function, composite functions, examples, Direct image $f(A)$ and inverse image $f^{-1}(B)$ for a function f, injective, surjective, bijective functions, Composite of injective, surjective, bijective functions when defined, invertible functions, bijective functions are invertible and conversely examples of functions including constant, identity, projection, inclusion, Binary operation as a function, properties, examples.
2. Equivalence relation, Equivalence classes, properties such as two equivalences classes are either identical or disjoint, Definition of partition, every partition gives an equivalence relation and vice versa.

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3. Congruence is an equivalence relation on $\mathbb{Z}$, Residue classes and partition of $\mathbb{Z}$, Addition modulo n, Multiplication modulo n, examples.

Unit III: Polynomials (15 Lectures)

1. Definition of a polynomial, polynomials over the field F where $F = \mathbb{Q}, \mathbb{R}$ or $\mathbb{C}$, Algebra of polynomials, degree of polynomial, basic properties.

2. Division algorithm in $F[X]$ (without proof), and g.c.d of two polynomials and its basic properties (without proof), Euclidean algorithm (without proof), applications, Roots of a polynomial, relation between roots and coefficients, multiplicity of a root, Remainder theorem, Factor theorem.

3. A polynomial of degree over n has at roots, Complex roots of a polynomial in $\mathbb{R}[X]$ occur in conjugate pairs, Statement of Fundamental Theorem of Algebra, A polynomial of degree in $\mathbb{C}[X]$ has exactly n complex roots counted with multiplicity, A non constant polynomial in $\mathbb{R}[X]$ can be expressed as a product of linear and quadratic factors in $\mathbb{R}[X]$, necessary condition for a rational number $\frac{p}{q}$ to be a root a polynomial with integer coefficients, simple consequences such as $\sqrt{p}$ is a irrational number where p is a prime number, roots of unity, sum of all the roots of unity.

Reference Books:

1. David M. Burton, Elementary Number Theory, Seventh Edition, McGraw Hill Education (India) Private Ltd.
2. Norman L.

PRACTICALS FOR F.Y.B.Sc

USMTP01 – Practicals

A. Practicals for USMT101:

1. Application based examples of Archimedean property, intervals, nieghbourhood. Consequences of l.u.b axiom, infimum and supremum of sets.
2. Calculating limts of sequences, Cauchy sequences, monotone sequences.
3. Limits of function and Sandwich theorem, continuous and discontinuous functions.
4. Miscellaneous Theoretical Questions based on full paper.

B. Practicals for USMT102:

1. Mathematical induction Division Algorithm and Euclidean algorithm in Z, primes and the Fundamental theorem of Arithmetic. Convergence and Eulers-function, Fermat's little theorem, Euler's theorem and Wilson's theorem,
2. Functions (direct image and inverse image), Injective, surjective, bijective functions, finding inverses of bijective functions. Equivalence relation.
3. Factor Theorem, relation between roots and coefficients of polynomials, factorization and reciprocal polynomials.

4. Miscellaneous Theoretical Questions based on full paper.

TUTORIALS FOR F.Y.B.A

A. Tutorials for UAMT101:

1. Application based examples of Archimedean property, intervals, neighbourhood.
2. Consequence of l.u.b axion, infimum and supremum of sets.
3. Calculating limits of sequences
4. Cauchy sequences, monotone sequences
5. Limit of a function and Sandwich theorem.
6. Continuous and discontinuous function.
7. Miscellaneous Theoretical Questions based on full paper.

Semester II

USMT 201 CALCULUS II

Unit-I : Series (15 Lectures)

1. Series $\sum_{n=1}^{\infty} a_n$ pf real numbers, simple examples of series, Sequence of partial sums of a series, convergent series, divergent series. Necessary condition : $\sum_{n=1}^{\infty} a_n$ converges $\Rightarrow a_n \rightarrow 0$, but converse is not true, algebra of convergent series,
2. Cauchy criterion, divergence of harmonic series, convergence of $\sum_{n=1}^{\infty} \frac{1}{n^p}$ (P>1), comparison test, limit comparison test, alternating series, Leibnitz's theorem (alternating series test) and convergence of $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$, absolute convergence, conditional convergence, absolute convergence implies convergence but not conversely, Ratio test (without proof), root test(without proof) and examples.

Unit –II : Limits and Continuity of functions(15 lectures)

1. Definition of Limit $\lim_{x\to a} f(x)$ of a function $f(x)$, evaluation of limit of simple functions using the $\in -\delta$ definition, uniqueness of limit if it exists, algebra of limits, limit of composite function, sandwich theorem, left-hand-limit $\lim_{x\to a^{-1}} f(x)$, right hand limit $\lim_{x\to a^{+1}} f(x)$ non existence of limits , $\lim_{x\to -\infty} f(x)$,$\lim_{x\to \infty} f(x)$, and $\lim_{x\to a} f(x) = \infty$.
2. Continuous functions: Continuity of real valued function on a set in terms of limits, examples, continuity of a real valued function at end points of domain, Sequential continuity, Algebra of continuous functions, Discontinuous functions, examples of removable and essential discontinuity. Intermediate value theorem and it's applications, Bolzano- Weierstrass theorem (statement only): A continuous function on a closed and bounded interval is bounded and attains its bounds.

3. Differentiation of real valued function of one variable: Definition of differentiation at a point of an open interval, examples of differentiable and non differentiable functions, differentiable functions are continuous but not conversely, chain rule , Higher order derivatives, Leibnitz rule, Derivative of inverse functions, Implicit differentiation (only examples)

Unit –III Applications of differentiation (15 lectures)

1. Definition of local maximum and local minimum, necessary condition, stationary points, second derivative test, examples, Graphing of functions using first and second derivatives, concave , convex , concave functions, points of inflection.
2. Rolle's theorem, Lagrange's and Cauchy's mean value theorems, applications and examples, Monotone increasing and decreasing function, examples,
3. L-Hospital rule without proof, examples of intermediate forms, Taylor's theorem with Lagrange's form of remainder with proof. Taylor's polynomial and applications.

Reference Books:

1. R.R.Goldberg, Methods of Real Analysis, Oxford and IBH, 1964.
2. James Stewart, Calculus, Third Edition, Brooks/ Cole Publishing company, 1994.
3. T.M.Apostol, Calculus, Vol I, Wiley And Sons (Asia) Pte. Ltd.

Additional Reference:

1. Richard Courant- Fritz John, A Introduction to Calculus and Analysis, Volume-I, Springer.
2. Ajit Kumar- S.Kumaresan, A Basic course in Real Analysis, CRC Press, 2014.
3. Ghorpade, Sudhir R, -Limaye, Balmohan V, A course in Calculus and Real Analysis, Springer International Ltd, 2000.
4. K.G. Binmore, Mathematical Analysis, Cambridge University Press, 1982.
5. G.B.Thomas, Calculus, 12 th Edition 2009

USMT202 ALGEBRA I

Unit I System of Equations and Matrices (15 Lectures)

1. Parametric Equation of Lines and Planes , System of homogeneous and non homogeneous linear Equations, The solution of m homogeneous linear equations in n unknowns by elimination and their geometrical interpretation for $(m, n) =$ $(1,2), (1,3), (2,2), (2,2), (3,3)$; Definition of n-tuple of real numbers, sum of n-tuples and scalar multiple of n-tuple.
   Deduce that the system of m homogeneous linear equations has a non trivial solution if m < n.

2. Matrices with real entries; addition, scalar multiplication of matrices and multiplication of matrices, transpose of a matrix, types of matrices: zero matrix,

identity matrix, scalar matrix, diagonal matrix, upper and lower triangular matrices, symmetric matrix, skew symmetric matrix, invertible matrix; Identities such as $(AB)^t = A^tB^t$, $(AB)^{-1} = A^{-1}B^{-1}$

3. System of linear equations in matrix form , Elemetary row operations , row echelon matrix, Gausian elimination method.

Unit II Vector Spaces (15 Lectures)

1. Definition of real vector space , Examplees such as $IR^n, IR[X], M_{mxn}(IR)$, space of real valued functions on a non empty set.
2. Subspace: definition, examples: lines , planes passing through origin as subspaces of respectively; upper triangular matrices, diagonal matrices, symmetric matrices, skew –symmetric matrix as subspaces of $M_n(IR)$ $(n = 2,3)$; $P_n(X) = a_o + a_1x + a_2x^2 + \cdots + a_nx^n,\ a_i \in IR, \forall 1 \le i \le n$ as subpace of $IR[X]$, the space of all solutions of the system of m homogeneous linear equations in n unknowns as a subspace of IR^n.
3. Properties of a subspace such as necessary and sufficient conditions for a non empty subst to be a subspace of a vector space, arbitrary intersection of subspaces of a vectorspace is a subspace, union of two subspaces is a subspace if and only if one is the subset of other.
4. Finite linear combination of vectors in a vector space; linear span L(S) of a non empty subet S of a vector space, S is generating set for L(S), L(S) is a vector subspace of V; Linearly independent/ Linearly Dependent subsets of a vector space, a subset $\{v_1, v_2, \ldots v_k\}$ is linearly dependent if and only $\exists\ i \in \{1,2, \ldots k\}$ such that v_i is a linear combination of other vectors v_j's .

Unit-III Basis of a Vector Space and Linear Transformation (15 Lectures)

1. Basis of a vector space, dimension of a vector space, maximal linearly independent subset of a vector space is a basis of a vector space, any two basis of a vector space have same number of elements, any set of n linearly independent vectors in an n-dimensional vector space is a basis, any collection of n+1 vectors in an n-dimensional vector space is linearly dependent.

2. Extending any basis of a subspace W of a vector space V to a basis of the vector space V.
   If W_1, W_2 are two subspaces of a vector space V then $w_1 + w_2$ is a subspace, $\dim(w_1 + w_2) = \dim(w_1) + \dim(w_2) - \dim\ (w_1 \cap w_2)$.

3. Linear Transformations; Kernel, Image of a Linear Transformation T ,Rank T, Nullity T, properties such as: kernel T is a subspace of domain space of T and Img T is a subspace of codomain subspace of T. If $V = \{v_1, v_2, \ldots v_n\}$ is a basis of V and $W = \{w_1, w_2, \ldots w_n\}$ any vectors in W then there exists a unique linear transformation

$T: V \to W$ such that $T(v_j) = w_j \ \forall\ 1 \leq i \leq n$, Rank nullity theorem (statement only) and examples.

Reference Books:

1. Serge Lang, Introduction to Linear Algebra, Second edition Springer.
2. S. Kumaresan , Linear Algebra , Prentice Hall of India Pvt limited .
3. K.Hoffmann and R. Kunze Linear Algebra, Tata MacGraw Hill, New Delhi, 1971
4. Gilbert Strang , Linear Algebra and it's Applications, International Student Edition.
5. L. Smith , Linear Algebra, Springer Verlang
6. A. Ramchandran Rao, P. Bhimashankaran; Linear Algebra Tata Mac Graw Hill.

PRACTICALS FOR F.Y.B.Sc

USMTP02-Practicals

A. Practicals for UAMT201:

1. Calculating limit of series, Convergence tests.
2. Properties of continuous and differentiable functions. Higher order derivatives, Leibnitz theorem. Mean value theorems and its applications.
3. Extreme values, increasing and decreasing functions. Applications of Taylor'stheoremand Taylor's polynomials.
4. Miscellaneous Theoretical Questions based on full paper

B. Practicals for UAMT202:

1. Solving homogeneous system of m equations in n unknowns byelimination for (m,n)=(1,2),(1,3),(2,2),2,3),(3,3),row echelon form. Solving system Ax = b by Gauss elimination, Solutions of system of linear Equations.
2. Examples of Vectorspaces , Subspaces
3. Linear span of an non-empty subset of a vectorspace, Basis and Dimension of Vector Space
4. Examples of Linear Transformation, Computing Kernel, Image of a linear map , Verifying Rank Nullity Theorem
5. Miscellaneous Theoretical Questions based on full paper

TUTORIALS FOR F.Y.B.A

A. Tutorials for UAMT201:

1. Calculating limit of series, Convergence tests.
2. Properties of continuous functions.
3. Differentiability, Higher order derivatives, Leibnitz theorem.
4. Mean value theorems and its applications.
5. Extreme values, increasing and decreasing functions.
6. Applications of Taylor's theorem and Taylor's polynomials.
7. Miscellaneous Theoretical Questions based on full paper

B. Tutorials for UAMT202:

1. Solving homogeneous system of m equations in n unknowns by elimination for (m,n) = (1,2),(1,3),(2,2),2,3),(3,3), row echelon form.
2. Solving system Ax = b by Gauss elimination, Solutions of system of linear Equations.
3. Examples of Vector spaces
4. Examples of Subspaces, Linear Span, Linear dependence/ independence of sets.
5. Basis and dimension of a vector space
6. Linear Transformations, Rank Nullity Theorem
7. Miscellaneous Theoretical Questions based on full paper

Scheme of Examination

I. Semester End Theory Examinations:

There will be a Semester-end external Theory examination of 100 marks for each of the coursesUSMT101/UAMT101,USMT102/UAMT102,of Semester I and USMT201/UAMT201, semester II to be conducted by the University.

Duration: The examinations shall be of 3 Hours duration.

Theory Question Paper Pattern:

a) There shall be **FIVE** questions. The first question Q1 shall be of objective type for 20mark s based on the entire syllabus. The next three questions Q2, Q3,Q4 shall be of 20marks, each based on the unitsI,II,III respectively. The fifth question Q5 shall be of 20 marks based on the entire syllabus.

b) All the questions shall be compulsory. The questionsQ2, Q3, Q4, Q5 shall have internal choices within the questions including the choices, the marks for each question shall be 30-32.

c) The questions Q2, Q3, Q4, Q5 may be subdivided in to sub-questions as a,b,c,d & e,etc and the allocation of marks depends on the weightage of the topic. (a) ThequestionQ1may be subdivided into10 sub questions of 2marks each.

II. Semester End Examinations Practicals:

At the end of the Semesters I &II Practical examinations of two hours duration and100 marks shall be conducted for the courses USMTP01,USMTP02.

In semester I, the Practical examinations for USMT101 and USMT102 are held together by the college.

In Semester II, the Practical examinations for USMT201 and USMT202

are held together by the college.

Paperpattern:

There will be Two parts Part A , Part B

USMTP01- Max Marks 80. Duration- 2hours

Part A: Questions from USMT101, Part B : Questions from USMT102

USMTP02- Max Marks 80. Duration-2 hours

Part A: Questions from USMT201, Part B : Questions from USMT202

Each part shall have two Sections

Section I

Objective in nature- Attempt any eight out of 12 multiple choice questions.

(8x3= 24)

Section II

Problems- Attempt any two out of Three

(8 x 2 =16)

Marks for Journals and Viva:

For each course USMT101, USMT102 and USMT201, USMT202

1. Journals: 5marks.
2. Viva:5marks.

Each Practical of every course of Semester I & II shall contain10 (ten) problems out of which minimum 05 (five) have to be written in the journal. A student must have a certified journal before appearing for the practical examination.