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M.A.

PART - I

PAPER - I

MICRO ECONOMICS

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CONSUMER CHOICE

Economic Problem originated from the nature of human beings and the environment that surrounds him. Human wants are unlimited whereas means or resources are scarce to satisfy these wants. If there has been no scarcity of resources and if the goods required by human beings have been available in plenty then perhaps no economic problem would have arisen. However, in reality the resources are limited and wants are unlimited. This is the root cause for the emergence of economic problem. In other words, it is the limitless wants of human beings, in combination with scarcity of resources that lead to the emergence of an economic problem. Thus, the economic problem is to satisfy the endless by competing human wants with the scarce resources. Since the basic economic problem is to allocate the available limited resources amongst alternative and competing ends, choice becomes inevitable. The choice process is facilitated by grading of various wants in order to their importance. The first priority is accorded to satisfy basic and urgent needs and the remaining resources are used for the satisfaction of the next important wants. What happens if a maximisation of satisfaction within his constraints is excluded by the tastes of the consumer? A rational consumer whose tastes preclude a maximum is an unhappy man indeed. Rationality drives him to seek a maximum that his tastes deny. Such a consumer is inherently restless, incessantly switching from one consumption pattern to another in the hunt for the elusive maximum. He is invariably in *disequilibrium.* It seems more plausible to assume that 'rational' consumers are 'at peace' with their consumption patterns. That is, they are in equilibrium. The characteristics of consumer equilibrium are studied by the utility theories. The utility theories use some concepts in common. This common conceptual tool kit relates to utility, its dependence on consumption, marginal utility, and the measurability of utility. Before studying the cardinal utility theory, it is useful to familiarize ourselves with this conceptual tool kit.

UNIT STRUCTURE

1.0 OBJECTIVES

After studying this lesson you will be able to specify what is utility? What are the different types of Utility? Cardinal utility and Ordinal utlility – consumer equilibrium through Indifference curve technique.

1.1 INTRODUCTION

The word utility denotes the want satisfying power of a commodity or service. A commodity may be harmful or injurious, but if it satisfies an economic want it possess utility. Wine and cigarettes are dangerous for health. Persons who are aware of it may not use them. These things do not possess utility for them. Those who consume them derive the utility. Utility is thus subjective and does not carry any ethical connotation.

The concept of utility was introduced to social thoughts by Bentham in 1789 and to economic thoughts by Jevons in 1871. The cardinalist school postulated that uiltiy can be measured. Various suggestions have been made for the measurement of utility. Under certainty, some economists have suggested that utility can be measured in monetary units, by the amount of money the consumer is willing to sacrifice for another unit of a commodity. Others suggested the measurement of utility in subjective units called utils. The ordinalist school postulated that utility is not measurable but is an ordinal magnitude. The consumer need not know in specific units that utility of various commodities to make his choice. It suffices for him to be able to rank the various basket of goods according to the satisfaction that each bundle gives him. He must be able to determine his order of preference among the different bundles of goods.

1.2 UTILITY

Utility or use value, originally referred to the *want satisfying power* of objects, but it now refers to the *satisfaction expected and obtained* from them. This transition from an objective to a subjective connotation was gradual, but is now complete. It may be explained as follows.

The want satisfying power of objects derives at least in part from their intrinsic qualities. However, with the same intrinsic qualities, objects give more satisfaction to some than to others. So it was argued early that the want satisfying power of objects, or utility, depends not on their intrinsic qualities, but on the *intensity of desire* for them.

The consumer's intensity of desire rests on the *satisfaction* he expects from consuming the object, if he is rational. And if the consumer is fully informed about the good and his own tastes, the satisfaction he expects will be the satisfaction he gets. Since the consumer is assumed to be rational and fully informed, his satisfaction equals the intensity of his desire for goods. Hence, it is now common to refer to utility and satisfaction as synonyms.

A subjective quantity

Satisfaction has some *quantitative* dimensions, in spite of qualitative differences. Thus, although the satisfaction we get from drinking water differs from the satisfaction got from watching a film, we can compare the two at any given time. We can say, for instance, that watching a film was more satisfying than drinking water. This comparison shows that satisfaction or utility, although subjective, is a quantity.

We are constantly quantifying utility in our heads, comparing the magnitude of satisfaction from one object with that of another. However, expressing this subjective exercise in objective terms may be difficult. This is the *problem of measuring utility,* which we will discuss later. For now, it is sufficient to note that utility is a subjective

Economists developed different theories to explain the consumer’s behaviour. The important among them are

1. **Marginal Utility Analysis (Cardinal approach)**
2. **Indifference Utility Analysis (Ordinal approach)**

The classical and neo-classical economist studied the consumer behaviour with the help of utility analysis. In the year 1876, Jevons, Karl Menger, Walras etc., have introduced this analysis. Alfred Marshall developed his theory of demand on this analysis only.

UTILITY ANALYSIS:

In the ordinary usage utility means –usefulness. But in economics more usefulness cannot be termed as utility. Utility may be defined as the want satisfying power or capacity of a commodity or service.

The concept of utility originated with an English philosopher, Jeremy Bentham (1748-1783) and his group of utilitarian. Utility got precise meaning in, only in 1870's with marginal utility analysis of William Stanley Jevons and others. But it was in the hands of professor Alfred Marshall that utility analysis took in its finer shape. Marshall assumed that utility is cardinally measurable like temperature or length. The most convenient measure is money.

1.3 TYPES OF UTILITY

Now let us examine different types of utility.

1. Form utility: Ex. Wood carved in to furniture
2. Place utlity:Ex:Sand from a river bank to the place of construction activity.
3. Time utility: Ex: Food grain stored during harvesting season, rain coats in rainy season etc., (Seasonal utility)
4. Service utility: Ex: Service of doctors, teachers, lawyers etc.,
5. Possession utility: Ex: A tool kit to a mechanic, a text book to a student etc.

Another set of classification is cardinal and ordinal utility approaches - This is related to measurement of utility also.

1.4 CARDINAL UTILITY

Some economists like Marshall assumed that utility is precisely measurable not only in principle but also in practice. Some other economists assumed that utility is measurable in principle, if not in practice.

1, 2, 3 are cardinal numbers. Each number has a specific value, just like weight, length, temperature etc utility was assumed to be having quantitatively measurable in cardinal magnitude. It means that we can sy how much satisfaction or utility we can derive from a commodity. Utility is assumed to be measurable in subjective units called “UTILS”.

In examining the above cardinal approach we will first stte the assumptions on which they are based.

1. **Rationality:** The consumer is rational. He aims at the maximization of his utility subject to his income limitation.

2. **Cardinal Utility:** The utility of each commodity is measurable in terms of money.

3. **Constant Marginal Utility:** This assumption is necessary if the monetary unit is used as the measure of utility. The essential feature of a standard unit of measurement is that, it should be constant. If the marginal utility of money changes as income increases that measuring rod for utility becomes an elastic ruler. In appropriate for measurement.

4. **Diminishing Marginal Utility:** The utility gained from successive units of a commodity diminishes.

5. **Total Utility** depends upon Quantities of individual commodities.

TOTAL UTILITY AND MARGINAL UTILITY:

According to the cardinal utility approach, it is possible to measure and express total and marginal utility in quantitative terms. Total utility from a single commodity, may be defined as the sum of the utility derived from all the units consumed of the commodity. For example, if a consumer consumes 5 units of a commodity and derives u1,u2,u3,u4 and u5 utiles from the successive units consumed then, If he consumes n units, then his total utility (TU)from n units may be expressed as

Marginal Utility:

Consumers, generally purchase those goods which given them the greatest amount of ssatisfaction for the expenditure made on them. This requires the consumers to measure and compare the amounts of satisfaction they derive from the various units of various commodities on one side and prices they pay for them on the other side. For this purpose two alternative techniques have been unsed by the economicsts. Viz., the marginal utility analysis and the indifference curve analysis.It is the additional units derived from consumption of an additional unit of a commodity**.** Simply speaking marginal means additional. It may be defined as the addition to total utility caused by consuming one more unit of a commodity.

$$\mathbf{M.U} = \frac{\Delta \mathbf{T.U}}{\Delta \mathbf{N}}$$

Δ**T.U :** Change in Total Utility

Δ**N:** Change in number of units

Total Utility:

Total Utility refers to the total satisfaction derived by the consumer from the consumption of a given quantity of commodity. Total utility is the summation or addition of the marginal utilities of different units of a commodity consumed.

$T.U = M.U_1 + MU_2 + \ldots\ldots\ MUn$

Relation ship between Marginal Utility and Total Utility:

1. When Marginal Uitlity decreses, Total Utility increases at decreasing rate.
2. When Marginal Utility becomes zero. Total utlity becomes maximum and constant.
3. When Marginal Utility is negative. Total Utility decreases.

Cardinal utility means that utility can be measured. It is usually measured and compared with the price which a person pays for one of the two commodities.

Utility is the degree of pleasure or satisfaction that arises from the consumption of specific goods. As we have already examined that Marshall assumed that utility is cardinally measurable like any other quantitative thing. He state that the money price one is willing to pay for a commodity rather than go without it is supposed to measure its utility.

Marshall assumed that utility is measurable not only in principle but also in practice. This concept he took from the writings of Jevons etc., with this assumption of measurable utility . Marshall provided an explanation for the demand and of the market demand curve.

1.5 ORDINAL UTILITY

Ordinal utility means that utilities can be ranked according to the preference of the individuals and they cannot be measured. Measurement of utility in cardinal units is not possible even where it possible to rank a magnitude “ordinally” ordinals are 1^{st},2^{nd}, 3^{rd}, 4^{th} etc.. It is not possible to measure just how much satisfaction an individual derives from the consumption of a good as we can measure the distance between two places or the weight of an object. Hence economists like Slutsky, Edgeworth, J.R.Hicks, R.G.D.Allen etc. developed an alternative method known as ordinal approach. Under this we assume that utility is comparable though not precisely measurable. Ordinal ranking merely indicates of whether the total utility obtained from a specific combination of goods is greater than, equal to, or smaller than the total utility of another combination of goods. Even when we say that the total utility is greater we can not say by how much it is greater.

In utility theory, we know whether a consumer is satisfied with the consumption of less or more of a commodity. In indifference curve analysis we know that, when faced with the consumption of two commodities how much more of one does the consumer give to compensate for reducing the consumption of the other commodity. By compensation, we mean, leaving the total utility of the consumer unchanged as a result of the changes in the goods and services consumed. In other words, the consumer is left indifferent as to a choice between the two alternative sets of goods and services.

1.6 THE UTILITY FUNCTION

The magnitude of utility depends upon the quantity of goods consumed by a rational consumer. This dependence is described by the *utility function,* which relates the consumer's utility to the quantity of goods consumed by him. Thus, generally:

$$U = F(Q_1,Q_2,\dots\dots\dots Q_N)$$

where q_i is the quantity consumed of the ith good; and 'f' is the general form of the utility function. The general form does not fit well with the cardinal utility theory. And so, many theorists of the cardinal utility school used a less general, 'additive' form of the utility function.

1.7 THE ADDITIVE UTILITY FUNCTION

The founders of the utility theory: Menger, Walras and Jevons worked with an additive utility function, and although the work of Edgeworth and Fisher showed the weaknesses of additive functions and the importance of the general form, the "additive utility function was only slowly and very reluctantly abandoned" (Blaug 1983, p. 346). This was because of the crucial role of additivety in the cardinal utility theory.

The 'strongly' additive utility function asserts that the *total utility of the consumer is simply the sum of the 'individual utilities' of the different goods consumed.* In this view, each good that is consumed yields an individual utility which depends on the quantity consumed of that good alone. Thus, we may say that the utility of apples *U* depends only on how many apples are consumed Ojj). Similarly, the utility of nuts *V* depends only on the quantity of nuts consumed *(q_2).* Marshall even suggests that the amount of money at one's disposal m also yields its own utility y. (Marshall 1920, p. 690). The sum of all these individual satisfactions or utilities is the total utility of the consumer (Ut). Algebraically,

$$U^T=U(_{qi}) + V(q_2)+fi(m)$$

***Properties*:** The strongly additive function has two restrictive properties.

(i) The individual utilities of each good are independent of other goods. That is to say, the satisfaction yielded by apples is not affected by how many nuts one consumes or how much money one has.

(ii) As a result, the partial derivatives of the total utility function with respect to any good *(q_v q_2, or m),* are independent of the quantities of other goods consumed. The partial derivatives of the utility function are called marginal utility.

Marginal Utility:

Marginal utility is a very important concept in utility theory. It has gained universal acceptance ever since it was first coined by Wieser in 1884. It is the additional Utility derived from consumption of an additional unit of a commodity.

Utility derived from the consuption of commodity X depends only on the amount of X consumed and not on the quantities of toerh commodities consumed ie.,

Ux = f(Qx);Uy = f(Qy),Uz = F(Qz). Therefore, the utlity function is U=Ux+Uy+Uz where U=Total utility derived from the consumption of various commoidities. This shows that the Utility function is additive. If X,Y,Z are interrelated, we cannot write the total utility function in this form. Thus independence of utility implies that ther are no related goods.

The total utility function in the case of two commodities x and y is

$$U = f(x,y)$$

The equation of an indifference curve is

$$U=f(x,y) =k$$

Where k is aconstant. The total differential of the utility function is

$$du = \frac{\partial u}{\partial y} =dy + \frac{\partial u}{\partial x} dx = (MUy)dy +(MUx)dx$$

It shows the total change in utility as the quantities of both commodities change. The total change in U caused by changes in y and x is (approximately) equal to the change in y multimplied by its marginal utility, plus the chnage in x multimplied by its marginal utility.

Along any particular indifference curve the total differential is by definition equal to zero.

Thus for any indifference curve

$$Du + (MUy)\ dy + (MUx)\ dx =0$$

Rearranging we obtain

Either $\frac{dy}{dx} = \frac{MUx}{MUy} = MRSx,y$ or $\frac{dx}{dy} = \frac{MUx}{MUy} = MRSy,x$

Law of Diminishing Marginal Utility: According to Marshall "The additional benefit which a person derives from a given increase in his stock of a thing diminished with every increase in the stock that he already has". It means that the Marginal Utility decreases as more and more units of a commodity consumed.

Law of Equi-Marginal Utility: The law is also known as the law of substitution or law of maximum satisfaction. It explains how a consumer allocates his limited income among various goods and services, whose prices are assumed to constant. The theory explains how a rational consumer acts his purchases.

Other things remaining the same, a consumer gets maximum total utility by spending his limited income when he allocates his expenditure on different goods in such a way that the marginal utility secured from the various goods become equal. When a consumer obtains equi-marginal utilities from all his purchases, he is said to maximum his satisfaction.

The consumer goes on substituting one good for another until the ratio of Marginal utilities to the prices of the goods that he purchases become equal. It is given by the following equation.

$$\frac{MUx}{Px} = \frac{MUy}{Py} = \frac{MUz}{Pz} = K$$

Price = P

K= Marginal Utility of Money assumed to be constant

1.8 INDIFFERENCE CURVE ANALYSIS

Indifference curve method was evolved to supersede the cardinal utlity analysis of demand. The indifference curve method seeks to derive all rules and laws about consumer's demand that are derivable from the cardinal utility analysis. At the same time the investors and supporters of new method contend that their analysis is based on fewer and more reasonable assumptions. The indifference curve analysis has , however, remained some of the assumptions of cardinal marginal utility analysis . Thus the indifference curve approach, like the cardinal utility approach, assumes that the consumer possesses complete information about all the relevant aspects of economic environment in which he finds himself. Further, it is assumed tht the consumer acts rationally in the sense that, given the prices of goods and the money income, he will choose the combinations that give him maximum satisfaction. More over, the assumption means that the consuemrs are capable of ordering or ranking all conceivable combinations of goods according to the satisfaction they yield.

1.9 ASSUMPTIONS

1. Rationality: The consumer is assumed to be rational. He aims aqt the Maximize his uitlity, given his income and market prices. It is assumed that he has full knowledge of all relevant information.
2. Unity is ordinal: It is taken as axiomatically true that the consumer can rank his preferences according to the satisfaction of each basket. He need not know precisely the amount of satisfaction. It suffices that he experesses his

preference for the various bundles of commodities. It is not necessary to assume that utility is cardinally measurable. Only ordinal measusrement is required.

3. Diminishing marginal rate of substitution: Preferences are ranked in terms of indifference curves, which are assumed to be convex to the origin and downwards sloping. The slope of the indifference curves explained the marginal rate of substitution of the commodities. The indifference curve theory is based, thus, on the axiom of diminishing marginal rate ofsubsitution.
4. Total utility of the consumer depends on the quantity of the commodities consumed

$$Q = f\,(Q_1, Q_2, \ldots\ldots\ldots\ldots\ldots\ldots\ldots Qx, Qy \ldots\ldots\ldots Qn)$$

An indifference curve is the locus ofpoints, indicating particular combinations of goods form which the consumer derives the same satisfaction and , as a result he is indifferent as to the particular combination he consumes.

Properties of Indifference Curve: (Draw the Diagrams)

1. Indifference Curves Slope Downwards from left to Right
2. Indifference Curves are Convex to the Origin
3. Indifference Curves Cannot Interesect Each Other.
4. Higher Indifference Curve yield Higher Satisfaction.

Modern demand theory uses an analytical approach called indifference curve analysis to explain how the household makes decision with regard to the economic choices and purchases.

The second root of the theory of demand is the indifference preference hypothesis. The theory of indifference preference developed as an alternative to the marginal utility theory of demand. It was meant to overcome the weakness of the marginal utility analysis arising out of some of the assumptions of that theory such as the rationality of the consumer the law of diminishing utility. Cardinal independent utilities and constant marginal utility of money. The indifference preference theory, however shared with the marginal utility theory two premises complete knowledge of prices and market conditions on the part of the consumer and rational choice of goods (marginal utility theory) and combinations of goods (indifference preference theory) of the consumer. In the earlier version of the indifference preference theory the assumption of continuity (that is that consumer ordered all possible combinations of goods) and in the latter version (Hicks : Revision of Demand Theory) the assumption of discontinuities (that is the consumer orders only combinations of goods within his range) was made.

While the marginal utility theory expressed consumer's tastes in terms of cardinal utility (that is the marginal utility curve the indifference preference curves). In the older theory quantitative utility schedules were adopted but in the new, scales of preferences. The theory of demand in terms of indifference schedules and curves was claimed to be more realistic than the marginal utility theory because econometric methods could be applied to the study of demand. Fisher and Frisch attempted to use these methods in the application of the marginal utility analysis, but econometrics was more effective as applied .to the indifference preference analysis in his value and capital by Hicks and his Foundations of Economic Analysis by Paul Samuelson. The Law of Diminishing Marginal Utility based on Psychological assumptions was found to be vague and the principle of diminishing marginal rate of substitution was adopted. Owing to numerous conditions to be fulfilled the marginal utility theory was of less value. In practice, the assumption of independent utilities and constant marginal utility of money were not fulfilled commonly. The theory of Marshall was limited in scope. Where essential commodities are involved such as food and house room which take up a major part of one's income and whose demand is inelastic, the law of demand has little significance. "Independent utilities" of Marshall leaves of account related goods, compliments and substitutes. The indifference preference theory covers such goods and the theory of demand is made more inclusive. Price effect is more fully analysed and broken up into income and substitution effects which was not done by Marshall. The indifference preference analysis was an attempt to make the theory of demand wider in its scope and operationally more effective Marshall assumed away the income effect on demand. On the other hand, the indifference preference theory of Hicks takes into account this. A change in demand may be attributed to changes in income, the price of the commodity and prices of other commodities. If the tastes of the consumer, that is the scales of preferences or utility schedules are given, in Marshall's analysis the demand for a good was a function of its price, he did not consider the other factors, the consumer's income and prices of other goods, that is, the income effect and the cross effects due to variations of their prices were overlooked. In Hick's theory, the change in demand is due to all the three factors when a consumer divides his income between X and Y if the price of one good changes as a result there may be an income effect (change in real income) due to the price change and a substitution effect (due to shift of consumption from the dearer to the cheaper good). While the substitution effect is positive the income effect might be positive or negative depending on whether the good was a normal good or an inferior good respectively. The net effect on demand of a price change is dependent on the strength of the income effect and of the substitution effect as well as the relative importance of the positive substitution effect and the negative income effect. Thus the demand curve might be either positively inclined or negatively inclined. Then, when the demand curve is positively inclined the normal law of demand as formulated by Marshall will be violated. Marshall further ignored the cross effects which Hicks and Allen fully considered. Both complementary and substitute goods are defined and discussed by Hicks and it is shown how they operate in the theory of demand. According to Marshall's assumption commodities were as already noted, unrelated or independent. Edgeworth and Pareto on the other hand regarded goods as non-independent and developed the technique of indifference curve to deal with non-independent goods. Goods to them were complementary independent or substitute. If X and Y are complementary an increase in Y causes in increase in the marginal utility of X given the quantity of X. If X and Y are substitutes an increase in Y reduces the marginal utility of X given the quantity of X. If X and Y are independent an increase in Y maintains the marginal utility of X at the same level given the quantity of X. In terms of price and demand complements substitutes and independent goods may be defined as follows, as Y increases and its price falls and its demand rises but at the same time if the demand for X also rises, X and Y are complements. As Y increases and its price falls and its demand rises but at the same time it the demand for X falls X and Y are substitutes. As Y increases and its price falls and its demand rises but if X is demand as much as before, neither more nor less X is independent of Y. Hicks and Allen have rejected these definitions of Edgeworth and Pareto as they involve introspective comparisons of utilities of X and Y. It is assumed in these cases that utility is uniquely measurable (that is the utility of X and Y are measurable). Instead, Hicks and Allen have defined such goods, substitutes and complements. In terms of marginal rate of substitution. The definitions given by Edgeworth and Pareto (in terms of Marginal utility) are in terms of individual rates. But the definitions of Hicks and Allen are in terms of price ratios and marginal rates of substitution and hence in terms of the market situation.

We have seen how in the case of inferior goods the income effect is negative. This may be true of an individual consumer, when there are many consumers, while a good may be inferior to some it may be normal to others. Then the negative income effect will be neutralised by the positive income effect of the different consumers. The negative income effect in the market will therefore be weakened, how much will depend on the distribution of income, the greater the variation in income and the wider the use of the commodity, the less important the negative income effect. Sir Robert Giffen in England had noticed the perverse law in the case of inferior goods leading to a positively inclined demand curve but since his time the distribution of income has changed and a commodity is used now more widely than at that time. The possibility of the perverse law of demand occuring is statistically less likely. That is the slope of the demand curve being positive is less probable. Marshall has regarded the negative income effect, therefore as exceptional in which case the core of Marshall's theory of demand has remained intact.

We have seen the relationship between Marshall's and Hick's theories of demand and the similarities and difference between them following the marginal utility analysis the indifference theory of demand marked an advance (which we shall consider later) but the theory suffers from certain short comings. These are due to the unrealistic assumptions made even by indifference preference theorists. Hicks assumes perfect competition. But according to the theory of monopolistic competition sellers enjoy a measure-of monopoly power and control over supply and price due to product differentiation which is closer to reality. Hicks on the other hand, assumes' that sellers cannot influence the market being so many and therefore, insignificant. The developments in the theory of price due to the contribution of Joan Robinson and Edward Chamberline have been ignored by perfect competition which is unreal 'Hick's theory of demand cannot reflect the true market as it operates on the basis of monopoly power. In his review article on Value and Capital Prof. Hailey has questioned the validity of perfect competition underlying the theory of demand in terms of indifference curves, Writing in the Economic journal on Value and Capital R.F. Harrod has similarly accused Hicks of being unrealistic in postulating perfect competition (with numerous sellers on the supply side and buyers on the demand side, none of the either growth being able to control prices. In this sense of the Hicks, term he points out accepts perfect competition but not perfect competition among goods for the consumer's income which might exist when he spends on each of the goods a very small part of his income. When a change in price occurs and therefore a change in the amount of expenditure on it, his real income will not be effected. There is in such a case no income effect or the marginal utility of money is constant. By rejecting the assumption of constant marginal utility of money Hicks as in effect rejected perfect competition among goods. But such perfect competition is truer and more realistic than perfect competition in the sense of a given price for each producer and consumer in the market. Hicks by so doing has rejected the more realistic concept of constant marginal utility of money imducit in perfect competition among goods and at the same time accepted the concept of perfect competition in the market. Hicks found it difficult to incorporate in his analysis the imperfect competition hypothesis knowing that the introduction of perfect competition would vitiate the analysis and he did not consider the error of importance. In view of the challenge to the perfect compeitition postulate the value was considerably modified and yet Hicks disregarded the amendments in his own theory of demand. On the other hand by denying perfect competition among goods which is not importable he was guilty of an anomaly. Out of this rejection has developed the income effect which he fully worked out in his indifference preference analysis.

Again the assumption of homogeneous good in the indifference theory is far from true because normally a consumer does not buy duplicate units of a good like hats or shoes. But on an indifference curve homogeneous goods alone-the axes are supposed to be consumed in various combinations implying duplication of commodities. In point of fact we use different kinds of a good like hats and shoes, such as a felt hat and a straw hat or a black pair and a brown pair of shoes. Then homogeneous goods of different amounts represented on any indifference curve are not true to life. On the other hand, if heterogeneous goods were portrayed on an indifference curve they could be shown in terms of value, that is money spent on hats and money spent on shoes might be shown and stand for different kinds of hats and shoes. However, this might pose difficulties of measurement when a price change occurs and the real income changes and there is a shift of the consumer's equilibrium. Normally, a price change leading to a change in the level of satisfaction may be shown by the price consumption curve. This assumes homogeneous goods and a given price for each good and a change in the price of each good. But if there are heterogeneous good consisting of various kinds of hats and shoes and their prices change it is more difficult to show the change in the real income or the income effect. Thus the indifference curve technique which is based upon the homogeneity of goods does not reflect reality be cause actually in practice goods purchased by consumers are various and not the same. Hicks could adopt homogeneous goods for his .theory because he assumed goods in the abstract that is X and Y. it he had taken actual goods like hats and shoes he would have realised the un-reality of his assumption and the difficulty of showing heterogeneous goods on an indifference curve.

In a modern economy, controls of various kinds in peace as well as war interfere in the free play of supply and demand. Public utility regulations governing railway rates, trade union control over wage fixation and controls under planning on the one hand and war time controls over supply and demand are relevant to theories of supply and demand. Hence, general theories of supply and demand which do not take into account such institutional economic controls are academic. To the extent that Hick's theory of demand falls to consider price controls it is removerd from reality and has limited applicability.

An indifference schedule is based on the assumption that people can choose from various combinations and stay on the same level of satisfaction. Logically such an assumption implies that a consumer might in the ultimate analysis be indifferent between two combinations (a) 10 pairs of shoes and zero hats and (b) zero pairs of shoes and 12 hats, in reality the consumer might want both shoes and hats and will not care for either A or B. Because he wants a minimum of each of the two goods say one hat, in which case no amount of shoes can compensate him for the loss of one hat. He will resist any reduction beyond this minimum. Such a resistance as shown L.L.Thurstone and Schultz work both at the upper lower limits. To the extent that Hicks assumes consistant choice from an indifference schedule his theory of indifference curves is removed from reality.

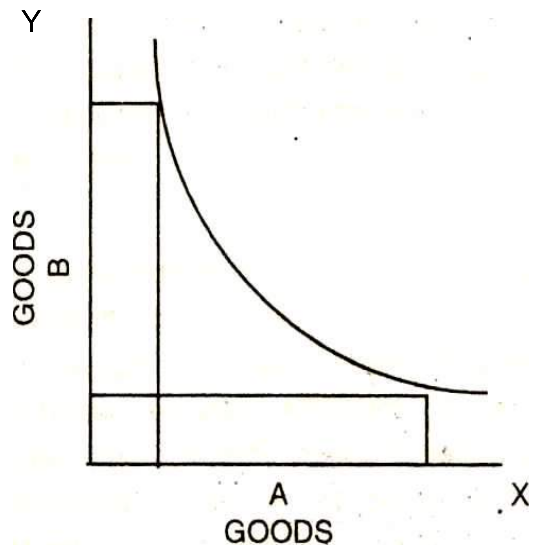


Figure 1.1

Some critics have said that an indifference curve isnot valid in real life for goods which are not close substitutes. If A and B are the two goods and A is increased ten fold, can B decreased tenfold, unless A and B were close substitutes? Such a substitution implies, a large movement along the indifference curve. Would this be valid ? If not be movement must be within a narrow range when A and B are not close substitutes. Further it has been pointed out that consumers are not aware of the effect of a change in price on their demand if the price on their demand if the price deviates much from the usual level they are familar with, that is when the price rises or falls sharply. Hence only within a narrow range the effect of a price change can tell how his demand is affected only from his expeience and sharp movements of price upwards or down wards may be beyond his experience. To his criticis Allen has replied that only very small changes in goods in a combination are meant to be taken in an indifference curve and not changes widely seperated combinations. Notris who shares Allen's.view on this question has said that in indifference man only the middle reaches of the curve and no! the extremeties are to be considered and the combinations will then be within the.limited range of a real consumer's choice. If on the other hand the ends of the middle reaches of the curve and not the extremeties are to be considered and the combinations will then be within the limited range of a rear consumer's choice. If on the other hand the ends of the indifference curve are taken the combinations will be necessarily widely separated and the criticism of Benthen and Schultz is justified. Further, only if an indifference curve is-extended to meet the axis combinations with only one good (like 12 hats and zero shoes and 10 pairs of shoes and zero hats) will arise, not if we keep the indifference curve clear of OX and Oy. Thus if only, small arcs of indifference curve not cutting the axes were taken, changes in combinations of goods will be marginal and not large and will involve small movements along the curve by a consumer. In such a case an indifference curve are taken it would mean that one commodity increases unduly at the expense of the other and this might be unrealistic. Thus if normally a consumer buys two pairs of shoes a large movement along the curve would imply that he would purchase 10 pairs of shoes which may be improbable. Again according to the indifference preference theory the point of tangency is where the consumer achieves equilibrium and this must be in the middle reaches of the curve since an indifference curves by definition is convex. The position elsewhere- on the indifference curves like either extremities would be possible only if he were on a higher income level. Then he will be on a different curve. Hence with some qualifications as just observed an indifference curve can be made more realistic.

Inspite of the shortcomings just discussed the theory of Hicks is an improvement on that of Marshall. He took over from Pareto" the concept of ordinal utility and developed it further. To know how much of a good a buyer buys, Marshall postulated a utility surface (marginal utility) while Pareto adopted a scale of preference (indifference curve). While the indifference map tells that a buyer prefers one set of goods to another it does not say by how much it is preferred. Cardinal utility (utility surface) on the other hand, implies that it is known how much satisfaction is given by a good. In terms of ordinal utility all that is said that one good is more important than another or they are of equal importance. One does not claim to measure the importance of either of the goods. The ordinal utility theory is then more realistic. However the shift from cardinal to the ordinal utility still involves introspectiorvWhen there are marginal utility schedules the utility of goods is measured by introspection. Even so in an indifference schedule the consumer when comparing the various combinations can express his preferences or otherwise only through introspections. But in ordinary terms the consumer compares different combinations. Whereas in the cardinal utility analysis measurement is assumed, in the ordinal utility theory it is not. But in either the element of introspection is present. This shift from cardinal to ordinal according to Hicks, involves a change in the exposition of the problem, When a scale of preferences replaces marginal utility'concepts arising from marginal utility will have to be abandoned and others substituted: Therefore, marginal rate of sustitution takes the place of marginal utility. Diminshing marginal rate of substitution replaces the principle of diminshing marginal utility. The identity between the slope of the price line and the indifference curve displaces the proportionality rule. The demand theory based on indifference preferences and ordinal utility may be better than the earlier Cardinal utility, theory in terms of marginal utility. But as Hicks has enuaciated it, it has failed to take note of the recent advances in the general theory of value, due to the recognition of imperfectmarkets, which have been significant. The assumption of rationalism is shared the two theories of demand. The hypothesis of perfect competition and the rule of reason in consumption are questionable. The theory of monopolisitic competition, there fore has left behind it the Hicksian indifference preference analysis and the findings of social scientists have exploded the theory of rational conduct in the realm of economics which again does not seem to have made any impact on Hick's analysis. It has been proved that man is a creature of habit, he may not change his patterns of consumption through experiments with new combinations or substitute one good for another but continue to buy the same goods as before, lacking the will and the energy to find anything new. Since man is not rational the theory of Hicks based upon the infallibility of reason is inadequate to explain irrational behaviour. Hicksian demand theory has not made much head way in regard to the psychology of the consumer beyond what the orthodox marginal utility theory had already done. Owing to these limitations a behaviouristic theory of demand was developed by Samuelson.The concept of indifference has been challenged by W.E. Armstrong. This may be explained as follows :

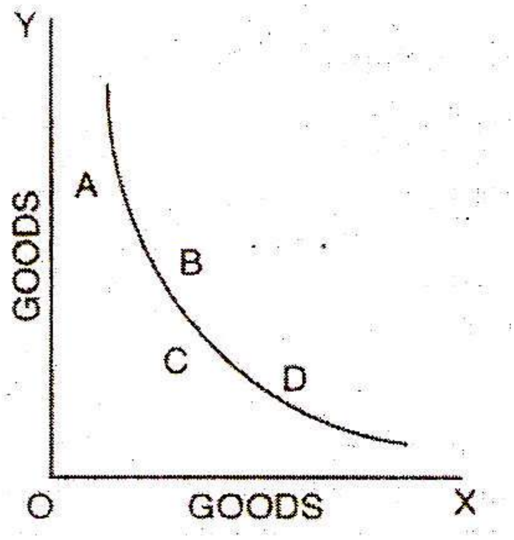


Figure 1.2

Let A,B,C and D lie close to each other (continuously) the indifference curve and represent combinations of X and Y goods take a pair of combinations A,B lying near each other. The consumer is indifferent between A and B. Take another pair of combinations B and C again he is indifferent between C and D. On the principle of transitivity if the consumer is indifferent between A and B and between B and C he must be indifferent between A and C similarly, if he is indifferent between B and C between C and D he must be indifferent between B and D. In the end he must be indifferent between A and D. Now if the consumer is indifferent between A and B the total utility of A equal the total utility of B accordingly to indifference preference theory but in fact this may not be so because the difference in the total utilities of A and B may be imperceptible and consequently the consumer is different between A and B. He consequently the consumer is different between A and B. He thinks that A and B are of equal significance. He cannot choose between the two combinations. However, if he takes A and C the difference in their total utilities may be perceptible. Then he will not be indifferent between A and C but prefer A to C or C to A. Thus even though A appears to be equal to B and B appears to be equal to C but as argued above A may be greater than C or C may be greater than A since A and C are not equal but different in significance. In this case it cannot be argued that if A equals B and B equals C so A equals C. The principle of transitivity is not valid. It was noted before that one of the fundamental characteristics' of indifference curves was that no two indifference curves could intersect.

The theory of indifference curves has been balmed for its lack of empirical data. Hicks assumed transitivity and related one combination to another on an indifference curve as being indifferent. As already noted the identify between A and B, B and C, and C and D ana therefore, A and D was taken for granted. But it was open to quetion (Armstrong). The weakness of the theory therefore, was in its lack of emperical evidence to prove the fact of indifference and preference. Moreover even the logic behind indifference as enunciated by Hicks was disputed. To Hicks and Allen the indifference map was a postulate adopted for their theory of demand. It was not derived from any utility function (or utility data) Edgeworth had derived his indifference curves from a unity surface. And yet the indifference curves were imaginary. He had first used the technique of difference curves to illustrate the diffulties of barter. He applied this theory to a Crusoe economy. Marshall, similarly used hypothetical examples of two persons exchanging apples and nuts for his explanation of Edgeworth's indifference curve (contract curve). The indifference theory is imaginary and based on assumptions about consumer's behaviour. It was a compromise between the marginal utility theory of demand and the behaviour theory of demand according to knights and Schumpeter. Some economists and psychologists have attempted a measurement of indifference curve but they used hypothetical combinations of goods and studied consumers under controlled conditions. As a result their findings were unrealistic and imaginary.

One limitation of the indifference preference analysis is that it might explain only choice between riskless alternatives but not risky ones. In real life there is uncertainity as illustrated by insurance and gambling. In the former the choice is between certainity and uncertainity of which certainty is preferred even though such a preference involves cost-premia to the insurer while in the later uncertainity is preferred to certainity in the hope of a large reward. Such risk due to uncertainity is involved in economic life in regard to expectation of income in occupations, movement of prices of securities and earning of profits in business. The problem attracted the attention of earlier thinkers Adam Smith (wealth of Nations) and Marshall (principles). They explained the choice of risky alternatives as againest riskless ones in terms of adventurousness, optimism and faith in success due to good luck. On the other hand, choice with regard to riskless alternatives was explained by them in terms of maximum utility. This could not be applied to the case of risky alternatives due to the law of diminishing marginal utility. Marshall thus rejected the maximum utility principle in considering gambling modern writers have questioned this rejection. They have attempted to show that the maximum utility principle may be extended to risky choices. In their theory of games. Neumann and Morgenstern have demonstrated the use of the maximum utility principle to risky choices. By this method the expected utilities of different alternatives bearing different amounts of risk may be derived and the choice of a person predicted. (See Guideline No.3)

1 He had a consistent set of preferences.
2 The preferences can be expressed in terms of utility.
3 When risk is absent, the alternative with 'the highest utility is preferred.
4 When risk is present, that alternative is preferred for which the expected utility is the highest and

1.10 CONSUMER EQUILIBRIUM : MAXIMUM SATISFACTION

The consumer is in equilibrium when he maximizes his utility, given his income and the market prices. Two conditions must be fulfilled for the consumer to be in equilibrium.

The first condition is he should be on highest indifference curve that the marginal rate of substitution be equal to the ratio of commodity prices

$$MRS_{XY} = MU_X / MU_Y = P_X/P_Y$$

The second condition is that it should be with budget line. The slope of the budget line Px/Py. The two conditions will be satisfied if one point in common for the indifference curve and the budget line. The point of tangency of the budget line with that of the indifference curve satisfied

$$MRS_{xy}=MU_x/MU_y=P_x/P_y$$

This is necessary but not sufficient condition for equilibrium. The sufficient condition is that the indifference curve should be convex to the origin. This condition is fulfilled by the axiom of diminishing MRS_{XY}, which states that the slope of the indifference curve decreases (in absolute terms) as we move along the curve from the left downwards to the right.

Graphical Presentation of the Equilibrium of the Consumer

Given the indifference map of the consumer and his budget line, the equilibrium is defined by the point of tangency of the budget line with the highest jm >ssib le i ndi f Tlrencc curve (point E in figure 6.12).

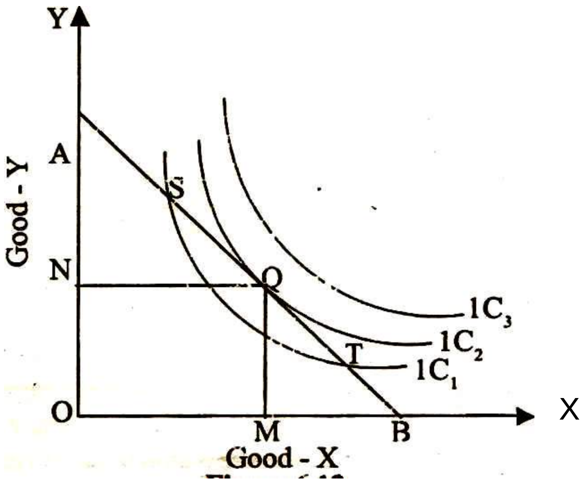


Figure 1.3

At the point of tangent the slope of the budget line (P_X/P_Y) and of the indifference curve ($MRS_{XY} = MU_X/MU_Y$) are equal:

$$MU_X/MU_Y = Px/Py$$

Thus the point of tengency of the two relevant curves denotes the first order condition graphically. The convex shape of the indifference curves implies the second order condition. The consumer maximizes his utility by consuming x* and y* of the two commodities.

We observer that the equilibrium conditions are identical in the cardinalist approach and indifference curve approach. In both theories we have

MUi/Pi =MU2/P2..,...MUn/Pn = ie.,MUx/Px = MU_Y/P_Y =... =MU,/Pn« Thus, although in the indifference-curve approach cardinality of utility is not required, the MRS requires knowledge of the ratio of the marginal utilities, given that the first order condition for any two

SLUTSKY SUBSTITUTION EFFECT:

Slutsky has given a slightly different version of substitution effect. In this version when the price of a good changes and consumer's real income or purchasing power increases, the income of the consumer is changed by the amount equal to the change in its purchasing power which occurs as a result of the price change. In other wards, in Slutsky's approach, income is reduced or increased by the amount which leaves the consumer to be just able to purchase the same combination of goods, if he so desires, which he was having at the old price, that is, the income is changed by the difference between the cost of the amount of good x purchased at the old price and the cost of the same quantity of X at the new price. Income is then said to be changed by the cost difference. Thus, in slutsky substitution effect, income is reduced or increased not by the compensating variation but buy the cost difference. Slutsky substitution effect is illustrated in Fig6.16 With given money income and the given prices of two goods as represented by budget line PL, the consumer is in equilibrium at Q on the indifference curve ICi buying OM of X and ON ofY. Now suppose that price of X falls price of Y and money income of the consumer remaining unchanged. As result of this fall in price of X the price line will shift to PL* and real income or purchasing power of the consumer will increase.

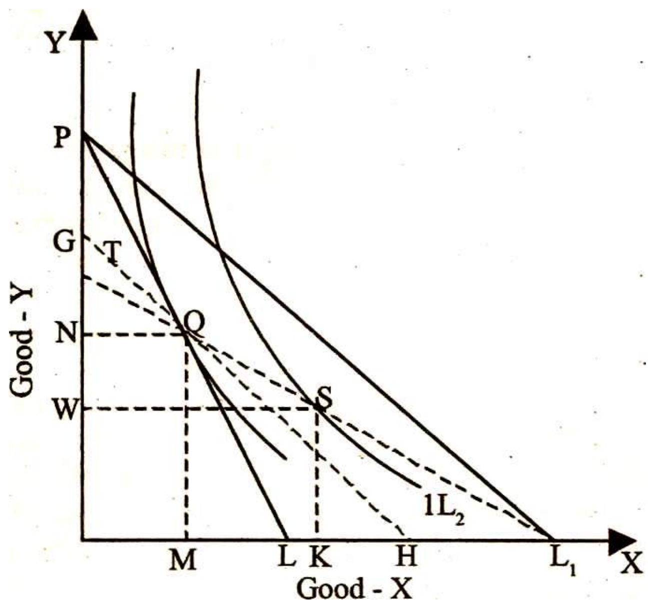


Figure 1.4

Now in order to find out the Slutsky substitution effect, consumers money income must be reduced by the cost-difference, or in other words, by the amount which will leave him to be just able to purchase the old combination Q of the goods if he so desires. For this, a price line GH parallel to PL' has been drawn which passes through the point Q. It means that income equal t PG in terms of Y or L'H in terms of X has been taken away from the consumer and as result he can buy the combination Q if he desires, since Q also lies on the price line GH. Actually, he will not now buy the old combination Q since X has now become relatively cheaper and Y has become relatively dearer than before. The change in relative prices will induce the consumer to rearrange the purchases of X and Y. He will substitute X for Y

But in this Slutsky substitution case, he will not move along the same indifference ICi since the budget line GH, on which the consumer has to remain due to price income circumstances, is nowhere tangent to the indifference curve ICi. The price line GH is tangent to the indifference curve IC2 at point S. Therefore, Hie consumer is now equilibrium at point Son higher indifference curve This movement from Q to S represents Slutsky substitution effect according to which the consumer moves not on the same indifference curve, but from one indifference curve to another. A noteworthy pint is that movement from Q to S as a result of Slutsky substitution effect is due to the change in relative prices alone, since the effect due to the gain in the purchasing power has been eliminated by making a reduction in money income equal to cost difference. It is important note that with slutsky substitution effect consumer chooses new combination on the budget line GH rather than his original combination Q which he could buy if he so desired.

Slutsky method has a distinct advantage in that it is easier to find out the amount of income equal to the cost difference by which income of the consumer is to be adjusted. On the other hand it is not so easy to know the compensating variation in income. The cost difference method has the advantage of being dependent on observable market data, while for knowing the amount of compensating variation in income; knowledge of indifference curve of the consumer between various combinations of goods is required. Pro. J. R. Hicks himself recognizes this merit of Slutsky approach.

PRICE CONSUMPTION CURVE OR PRICE EFFECT:

The price effect shows this reaction of the consumer and measures the full effect of the change in the price of a good on the quantity purchased since no compensating variation in income is made in this case. When the price of good changes the consumer would either better off or worse off than before, depending upon whether the price falls or rises. In other wards as a result of change in price of the good, his equilibrium position would lies at a higher indifference curve I case of the fall in price and at a lower indifference curve in case of the rise in price. Price effect is shown in Figure 1.5**.** With a give prices of goods X and Y, and a given money income as represented by budget line PLi, the consumer is in equilibrium at Q on indifference curve IC1.

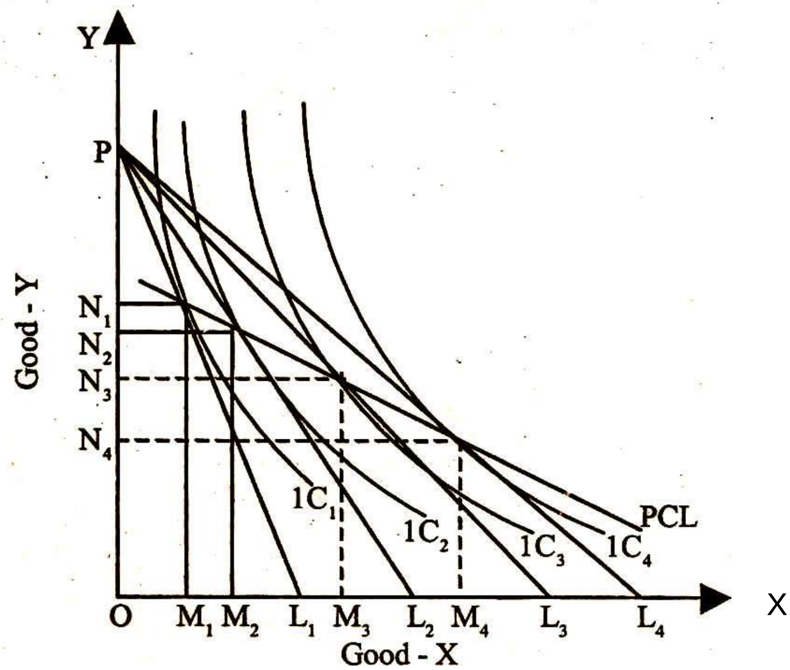


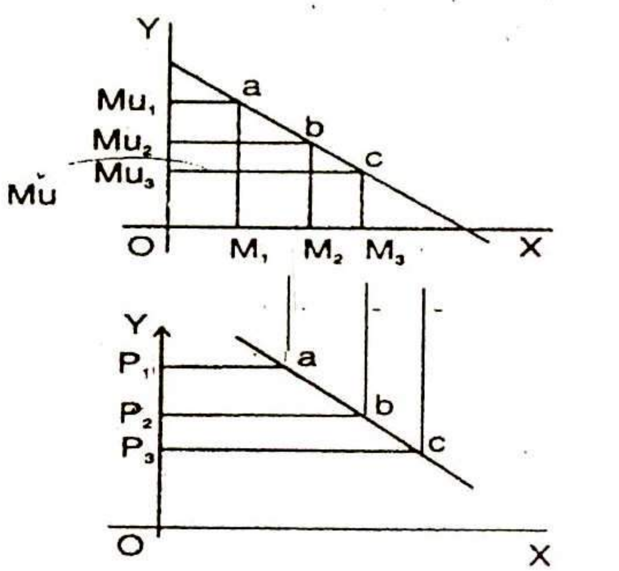
Figure 1.5

In this equilibrium position at Q, he is buying OMi of X and ONi ofY. Let price of good X fall and price of Y and his money income remaining unchanged. As a result of this price change, budget line shifts to the position PL2. The consumer is now in equilibrium at R on the higher indifference IC2 and is buying OM2 of X and ON2 of Y. He has thus become better off, that is his level of satisfaction has increased as a consequence of the fall I the price of good X. Suppose that price of X farther falls the price line shifts towards right like PL3.PL4 and so on. When all the equilibrium points such as Q, R, S, and t are joined together, we will get what is called' Price Consumption Curve' (PCC). The price consumption traces out the price effect.

1.11 DERIVATION OF DEMAND CURVE FROM MARGINAL UTILITY ANALYSIS

The derivation of demand curve depends on the diminishing marginal utility. MU curve is derived from TU curve. MU curve is the locus of the slopes of TU curve. Since MU is declining, as a rule the consumer willing to purchase more commodities at lower price only i.e., since, $MU_x = P_x$ at each quantity purchased and the MU is declining unless price declines consumer will not be induced to buy more.

In this figure, at M, quantity of X commodity, MU is equal MU_1, as a rule this must be equal to P_1,. Therefore, at P_1, demand is OM_1. similarly at M_2, the marginal utility is MU_1, and i price is P_2, and so on.



Derivation of Demand Curve from MU Curve

OX = Quantity
OY = Price

Figure 1.6

1.12 DERIVATION OF THE DEMAND CURVE USING INDIFFERENCE CURVE APPROACH:

Graphical derivation of the demand curve. As the price of a commodity, for example of X, falls the budget line ofthe consumer shifts to the right, frorp its initial position (PL) to a new position (PLi), due to the increase in the purchasing power of the givetl income of the consumer. With more purchasing power in his possession the consumer can buytnore ofX (e.g. curve IC2). The hew equilibrium occurs to the right ofthe original equilibrium showing that as price falls more of the commodity will be bought. If we allow the price of X to fall continuously and we join the points of tangency of successive budget line and higher indifference curves we form the so-called price-Consumption line (in figure 1.7) from which we derive the demand curve for commodity X.

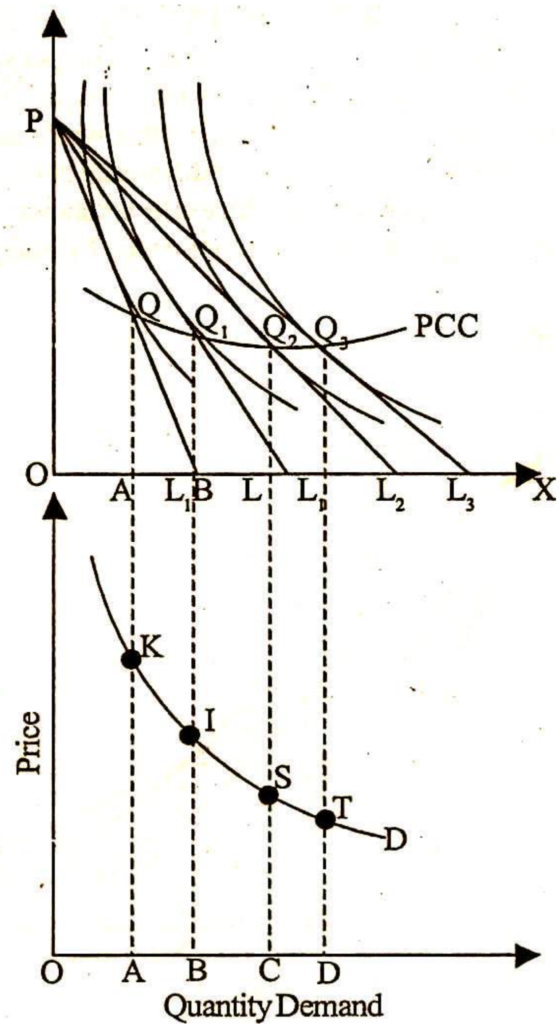


Figure 1.7

The adjoining demand schedule, which has been derived from the indifference curves diagram, can be easily converted into a demand curve with price shown on the Y-axis and quantity demanded on the X-axis. It will be easier if this demand curve is drawn rightly below the indifference curves diagram. This has been done so in fig 1.7. In the diagram at bottom, on the X axis is shown the quantity demanded as in indifference curves diagram .above but on the Y axis in the diagram at bottom is shown price per unit of good X instead of total money. In order to obtain the demand curve, various points K L S and T representing the demand curve the demand decreases when price falls and vice-versa

1.13 HICKSIAN DEMAND FUNCTION

Hicksian demand function is also called as the compensated demand function. It is the demand of a consumer over a bundle of goods that minimizes their expenditure while delivering a fixed level of utility**.** The function is named after John Hicks.

Mathematically the function can be written as

$$h(p, \bar{u}) = \arg\min_{x} \sum_{i} p_i x_i$$

$$\text{such that} \quad u(x) = \bar{u}$$

where *h* (*p*,*u*) is the Hicksian demand function, or commodity bundle demanded, at price level *p* and utility level $\bar{u}$. Here *p* is a vector of prices, and *X* is a vector of quantities demanded so that the sum of all $p_i x_i$, is the total expense on goods *X*.

Relationship to other functions:

Hicksian demand functions are often convenient for mathematical manipulation because they do not require income or wealth to be represented. However, **Marshallian demand functions of the form x (p, w) that describe demand given prices p and income w are easier to observe** directly. The two are trivially related by

$$h(p, u) = x(p, e(p, u)),$$

where *e* (*p*, *u*) is the expenditure function (the function that gives the minimum wealth required to get to a given utility level), and by

$$h(p, v(p, w)) = x(p, w),$$

where *v* (*p*, *w*) is the indirect utility function (which gives the utility level of having a given wealth under a fixed price regime). Their derivatives are more fundamentally related by the **Slutsky equation.**

The Hicksian demand function is intimately related to the **expenditure function**. If the consumer's utility function *u*(*x*) is **locally non satiated** and strictly convex, then $h(p, u) = \nabla_p e(p, u)$.

Properties of Hicksian Demand Functions:

1. The matrix of second derivatives of an expenditure function e (p, u) with respect to he prices is a **negative semi–definite** matrix.

2. Every negative semi–definite matrix must have non–positive numbers on the diagonal.

3. The compensated (Hicksian) demand for any good is a non–increasing function of the good's own price.

1.14 ENVELOP THEOREM

A special application of the theorem occurs in economics as a result of optimization. The theorem from analytic geometry needs to be covered first. A curve in two dimensional space is best represented by parametric equations; i.e., x(t), y(t). For example, the parametric equations for a circle of radius r cantered at the origin are

$$x(t) = r \cos (t),\ y(t) = r \sin (t)$$

This circle can also be represented by the equation as given b $x^2 + y^2 = r^2$

This latter representation is of the form f(x , y) = 0.

A family of curves can be represented in the form as given by g (s,y,c) = 0
Where c is a parameter. There can be families of curves involving more than one parameter. The Envelope Theorem involves one-parameter families of curves.

The envelope of a family of curves g(x, y, c) = 0 is a curve P such that at each point of P, say (x,y), there is some member of the family that touches P tangentially. In other words, for each point of P, (x_0, y_0), there is a value of c, say c_0, such that $g(x_0, y_0, c_0) = 0$. Since, at each point of the envelope curve P there is a corresponding value of the parameter c, the envelope curve can be represented parametrically as (x(c), y(c)). Since, the equation defining the family of curves, g(x, y, c) = 0, is true for all values of c in some range, that equation can be differentiated with respect to c. The result is:

$$(\partial g/\partial x)(\partial x/\partial c) + (\partial g/\partial y)(\partial y/\partial c) + (\partial g/\partial c) = 0$$

For any particular curve of the family the parameter c is constant. Differentiating g(x, y, c) = 0 with respect to x with c held constant gives:

$(\partial g/\partial x) + (\partial g/\partial y)(\partial y/\partial x) = 0$

From the parametric equation for the envelope, (x(c), y(c)), it follows that $(\partial y/\partial x) = (\partial y/\partial c/ (\partial x/\partial c))$

At the point of tangency the envelope curve and the corresponding curve of the family have the same slope. This means that

$(\partial y/\partial x) = (\partial y/\partial c/ (\partial x/\partial c))$ and $(\partial g/\partial x) + (\partial g/\partial y) (\partial y/\partial x) = 0$ imply

$(\partial g/\partial x)(\partial x/\partial c) + (\partial g/\partial y)(\partial y/\partial c) = 0$

When this equation is compared with the equation obtained by differentiating the equation for the families of curves with respect to c; i.e.,

$(\partial g/\partial x)(\partial x/\partial c) + (\partial g/\partial y)(\partial y/\partial c) + (\partial g/\partial c) = 0$ the implication is that $(\partial g/\partial c) = 0$

Thus the way to find the envelope of a family of curves is to solve the two equations:

g (x,y,c) = 0 and (∂g/∂c) = 0 for x and y as functions of c.

1.15 INDIRECT UTILITY FUNCTION:

This function

$V (p_x, p_y , I) \equiv U [x^d (p_x, p_y , I) , y^d (p_x, p_y , I)]$ known as the indirect utility function. This function says how much utility the consumers are getting when they face prices (p_x, p_y) and have income I.

1.16 INDIRECT UTILITY FUNCTION AND DUALITY

The direct utility function describes preferences independent of market phenomena. The indirect utility function reflects a degree of optimization and market prices.

Indirect Utility functions
Let v_i = pi/y. The budget constraint now may be written as

$$1 = \sum_{i=1}^{n} v_i q_i \qquad \text{……………………………(1..1)}$$

Since optimal solutions are homogeneous of degree zero in income and prices, nothing essential is lost by this transformation to "normalized" prices. The utility function $U = f(q_1, q_2, q_3, \ldots\ldots qn)$ together with (1.1) gives the following first-order conditions for utility maximization

$$fi - \lambda v_i = 0 \quad i = 1,2,3, \ldots\ldots n$$

$$1 - \sum_{ii=1}^{n} vi\, qi = 0$$

Ordinary demand functions are obtained by solving 1.2

$$Qi = Di(v_1, \ldots\ldots v_n) \qquad 1.3$$

The indirect utility function $g(v_1 \ldots\ldots\ldots, vn)$ is defined by

$$U = f\{D_1(v_1, \ldots vn) \ldots\ldots Dn(v_1, \ldots\ldots, v_n)\} = g(v_1, \ldots\ldots, v_n) \quad 1.4$$

It gives maximum utility as a function of normalized prices.

1.17 DUALITY THEOREMS:

The relationships between the direct and indirect utility functions may be described by a set of duality theorems. The following illustrative theorems are provided without proof.

Theorm 1 Let f ve a finite regular strictly quasi-concave increasing function which obeys the interior assumption*. The g determined by 1. 4 is a finite regular strictly quasi-convex**decreasing function for positive prices.

Theorm 2 Let g be a finite regular strictly quasi-convex decreasing function in positive prices. The h determined by

$U = g\{V_1(q_1, \ldots.. qn) \ldots. \; Vn(q_1, \ldots\ldots qn)\} = h(q_1, \ldots.. Qn)$ is finite regular strictly quasi-concave increasing function

Theorm: 3 Under the above assumptions

$$h(q_1, \ldots\ldots qn) = g\{V_1(q_1, \ldots\ldots.. q_n) \ldots., Vn(q_1, \ldots\ldots qn)\}$$

And $g(v_1 \ldots\ldots vn) = h\{\{D_1(v_1, \ldots\ldots vn) \ldots.., Dn(v_1, \ldots.. v_n)\}$

The direct utility function determined by the indirect is the same as the directly utility function that determined the indirect.

Duality in consumption forges a much closer link between demand and utility functions for the purposes of empirical demand studies. Duality is also unseful in comparative statics analysis.

Homotheticity, separability, and additivity each have counterparts for the indirect utility function. Consequently, many theoretical analyses can be conducted in terms of either the direct or indirect utility function, whichever is more convenient.

1.18 THE EXPENDITURE FUNCTION

The consumer expenditure function tells us how the consumer minimizes his expenditure, given the prices of commodities and the utility level.

The derivation of the consumer exspenditure function is based on the linear programming technique. The solution to the objective function of minimizing consumer expenditure is

$$\text{Min : } \sum_{i=1}^{n} P_i X_i \ldots\ldots\ldots\ldots\ldots\ldots 1$$

Subject to U(X) > U

Where Pi Xi is the total expenditure which is to be minimized subject to the constraint that the utility level be not less than U. The solution to equation (1) depends on the values of prices and utility level which can be stated as,

$$X_i = f_i (P_i\ U) \ldots\ldots\ldots i=1,\ldots.n \quad \ldots\ldots 2$$

Substituting this function into the objective function in (1) yields a function which represents the minimum level of expenditure that can attain the utility level U, given price p,

$$\sum_{i=1}^{n} P_i f_i (P,U) \ldots\ldots\ldots\ldots\ldots\ldots 3.$$

1.19 SUMMARY:

The neoclassical theory of consumer behaviour rests on the concept of utility, which is subjective want-satisfying power, a property common to commodities. Cardinal Utility is utility assumed to be measurable in principle. Rational behaviour by consumers is the making of deliberate, calculating, and consistent choices aimed at maximizing utility. As consumer acquires, in fact or in contemplation, more units of a commodity, their total utility increases, but a diminishing rate. This is equivalent to diminishing marginal utility. The utility added by the last unit of a quantity of a commodity is the marginal utility of the commodity. When total utility is maximum, marginal utility is zero. Marginal utility diminishes because additional units of a commodity are put to less and less important uses. A consumer allocates units of a commodity among different uses in such a way that marginal utilities in each use are equal-the equi-marginal principle. The size of a consumer's income determines the marginal utility of money to that consumer and, therefore, the marginal utilities represented by the downward because of diminishing marginal utility. The consumer is in equilibrium when the consumer buys that quantity of a commodity whose marginal utility is proportional to its price; in equilibrium. The consumer maximizes his or her satisfaction.

The indifference curve analysis of consumer demand is based on the concept of ordinal utility. Having a choice between two combinations of goods, the consumer either prefers one combination or is indifferent. The indifference curve shows all combinations of two commodities that give the same satisfaction to the consumer. The indifference curve is convex; there is a diminishing marginal rate of substitution between the two commodities. A complete description of a consumer's tastes for two commodities is provided by the indifference map. The consumer's budget and the prices of the two commodities are represented by the budget line. The slope of the budget line is the ratio of the price of X to the price of Y. The position of the line reflects the size of the budget. The consumer is in equilibrium when buying the two commodities in the quantities defined by the tangency of an indifference curve to the budget line. In equilibrium, the ratio of the prices is equal to the marginal rate of substitution.

1.20 BOOKS FOR FURTHER READING

J.R.Hicks, A Revision of Deman Theory (oxford, London,1956.

William J. Baumol, Economic Theory and Operations Analysis, 4th ed.Prentice-Hall, Engle wood Cliffs, N.J., 1977, Chap.22

1.21 SELF ASSESSMENT TEST:

1. Identify the amendments in the neoclassical cardinal utility theory of demand which J.R. Hicks made in his ordinal utility theory.
2. What are the shortcomings of the indifference curve analysis ?
3. Analyse the merits of the approach of the indifference school to consumer's choice.
4. Discuss the qualifications under which the indifference curve may explain the behaviour of a consumer.

5. What is the challenge posed by critics such as Kennedy and Armstrong to the foundations of the indifference cure analysis ? How can it be met ?
6. Explain the consumer's equlibrium with the help of indifference curve analysis
7. State the consumer's equilibrium through ordinal approach and exdplain it superiority.

1.22 GLOSSARY:

Utility: The want –satisfying power.

Marginal Utility: The additional utility got from the consumption of an additional unit of a commodity.

Total Utility: The total amount of satisfaction got from the consumer of a given stock of commodities

Equi-Marginal Utility: Marginal utilities of various commodities consumer becoming equal.

Cardinal Utility: Measurability of utility in terms of numbers.

Marginal Rate of Substitution: The number of units of Y good, which the consumer is ready to forego to get one more unit of Y.

Scale of Preferences: The preference indicated by the consumer towards various combinations of goods.

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2

THE REVELED PREFERENCE THEORY

UNIT STRUCTURE

2.0 Objectives
2.1 Introduction
2.1 The Revealed Preference Hypothesis
2.2 Assumptions
2.3 Derivation of the Demand Curve
2.4 Derivation of the Indifference Curves
2.5 Critique of the Revealed Preference Hypothesis
2.6 Introduction
2.7 Strong and Weak ordering distinguished
2.8 Weak ordering in the Hicks Demand Theory
2.9 The Direct Consistency test
2.10 Demand Theory of Weak Ordering
2.11 Derivation of Law of Demand by the method of cost difference
2.12 Superiority of Hicks Logical Ordering theory of Demand
2.13 Self Assessed Questions
2.14 Books for Reference

2.0 OBJECTIVES

At the end of this lesson you will be able to explain:

PART A

- The Revealed Preference Hypothesis
- Assumptions
- Derivation of the demand curve
- Derivation of the indifference curves
- Critique of the revealed, preference hypothesis

PART B

- Strong and Weak Ordering Distinguished
- Weak Ordering in the Hicks' Demand Theory

- The Direct consistency test
- Demand theory of weak ordering
- Derivation, of law of demand by the Method of Compensating Variation
- Derivation of law of demand by the Method of Cost Difference
- Superiority of Hicks Logical Ordering theory of Demand

2.1 INTRODUCTION

Samuelson's revealed preference theory is regarded as scientific explanation of consumer's behaviour as against the psychological explanation provided by Marshallian and Hicks-Allen theories of demand.

The indifference (or ordinal) theory required less data about the Consumer than the marginal utility (or cardinal) theory. In the indifference theory one did not have to know the quantities utilities of goods. It was enough to know the rankings or preferences of consumers. However, to draw the indifference map one had to know-all the possible combinations of goods. This information had to be supplied by the consumer. If the consumer did not supply if one could not construct his indifference map.

To meet this difficulty Prof. Paul Samuelson offered another theory to explain the consumer's behaviour in the market. According to it, the consumer need not supply data on his preference. We could ourselves find out about his preferences by observing his behaviour by seeing what he buys and at what prices provided his tastes do not change. With this information one may reconstruct his indifference map. This is known as the Revealed Preference Theory. It may be explained as follows.

When consumer buys one set of goods as against others he may have reasons for doing so : (a) he likes that particular set more than the others, (b) that set is cheaper than the others. And between two sets of goods A and B. Suppose the consumer is seen to buying A but not B, this may not mean that he necessarily prefers A to B. He have bought A because it is cheaper than B. Indeed it is possible that even he might have liked B more than A and may regret that he cannot afford B. However if A and B cost same amount of money to the consumer and yet he has bought A and B, the reason could only be that cost he prefers A to B. Generally, if A is preferred to B, C,D etc., But B,C,D are just as expensive as A, we may say then that A is revealed preference to B, C, D or B,C, D are revealed to be inferior to A.

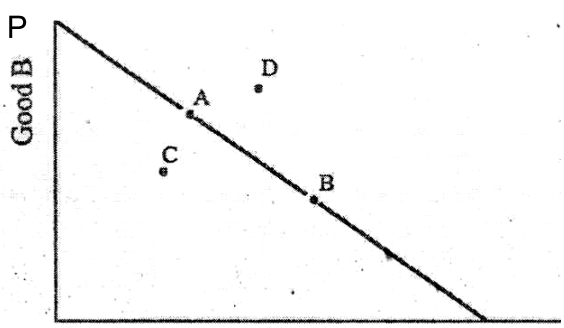
This lesson is divided into two parts. Part A deals with the revealed preference theory. Part B presents Hicks revision of demand theory.

PART A:

REVEALED PREFERENCE THEORY

INTRODUCTION

Revealed preference is an approach, to demand theory, which derives the traditional laws of demand using only information on the choices the consumer makes in different price and income situations coupled with the assumptions that such choices are made rationally. It can be seen as a third approach to consumer behaviour, in contrast to the cardinal approach (marginal utility), which requires there to be an absolute, single, measure of utility, and the ordinal utility approach (based on indifference-curve analysis), which requires there to be some measure of relative utility, albeit one that does not require actual magnitudes of utility to he ascribed to bundles of commodities, The revealed - preference approach holds that only two, types of information are theoretically necessary to predict the behaviour of consumers and derive the laws of demand. The first is the observed spending of a consumer in different price-income situations; -this reveals which bundles of commodities are preferred to others. The second is the assumption that the consumer's behaviour accords to certain axioms of "rationality" - to predict how someone will spend their money, we must know that they will not behave erratically (transitivity). It can be shown that, if such information were available in full* an-indifference map could be constructed for the consumer. Implicitly; therefore, the approach does construct at least a partial indifference map of the form used in indifference- curve theory and should best be seen as an alternative expression of this theory rather than a replacement for it.



0 Good A ¥

Figure 2.1

In this lesson a more advanced nature of topic build on the material on demand *theory.* We have been dealing with two rather exclusive concepts. They are marginal utility and indifference curves. It was known that the use of marginal *utility* requires the measurement of utility in the cardinal sense, while the use of indifference curve analysis requires measurement only in the ordinal sense, however, it was also noted that indifference curves are derived from a utility surface. It has been proposed that a consumers indifference map can he constructed if we assume that an individual's tastes do not change. This proposition is the essence of the revealed preference theory.

The theory is based on a very simple idea: A consumer will decide to boy some Particular basket of goods either because he likes it more than another basket of goods or because it is cheaper when compared with other baskets of goods, Suppose a consumer buys basket of goods A rather than basket of goods R We may not slate that he prefers A to B. It is possible that he could not afford to buy B. Given price information, however, we can make a more definitive statement If A is not less expensive than B and die consumer purchases A, he does so because he likes it better. We say in this situation that A has revealed preferred to B, or B is revealed inferior to A.

In figure 2.1 shows the points represent baskets of goods. Given budget constraint line pp, we see that A is just as expensive as B. If the consumer chooses A, it is revealed preferred to all other points on PP. Also, C is revealed inferior to *A,* for it represents a basket of goods that is less expensive than A. Clearly, any point like D lying above the budget constraint line represents a basket of goods that is more expensive than A and cannot therefore be revealed be revealed inferior to A.

2.1 THE REVEALED PREFERENCE HYPOTHESIS

Samuelson introduced the term ‘revealed preference’ into economics in 1938. Since then the literature in this field has proliferated.

The revealed preference hypothesis is considered as a major break through in the theory- of demand, because it has made possible the establishment of the ‘law of demand' directly (on the basis of the revealed preference axiom) without the use of indifference curves and all their restrictive assumptions. Regarding the ordering of consumers’ preferences, the revealed preference hypothesis has the advantage over the Hicks - Allen approach of establishing the existence and the convexity of the indifference curves (it does not accept them axiomatically), However; the indifference curves are redundant in the derivation of the demand curve, We will first examine the derivation of the 'law of demand'; we will then show how indifference curves can be established.

2.2 ASSUMPTIONS

1. Rationality. The consumer is assumed to behave rationally, in that he prefers bundles of goods that include more quantifies of the commodities.

2. Consistency. The consumer behaves consistently; that is, if he chooses bundle A in a situation in which bundle B was also available to him he will not choose B in any other situation in which A is also available. Symbolically

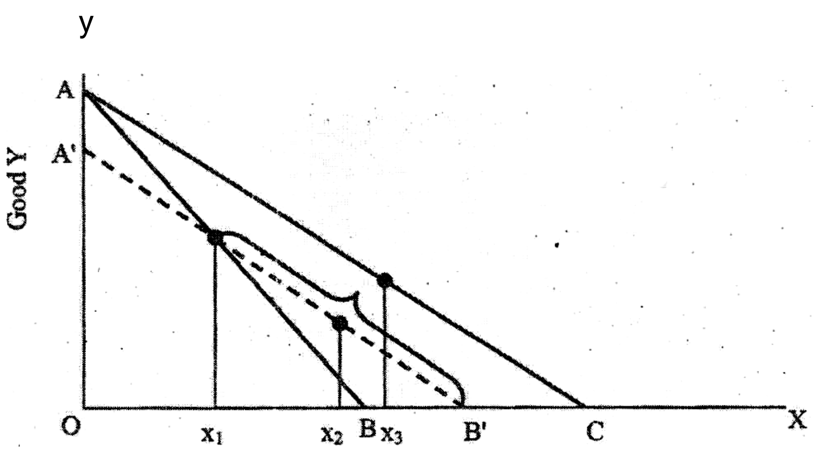
If A > B, B > A

3. Transitivity. If in any particular situation A > B and B > C, Then A > C,

4*. The revealed preference axiom.* The consumer, by choosing a collection of goods in any one budget situation, reveals his preference for that particular collection. The chosen bundle is revealed to be preferred among ail other alternative bundles available under the budget constraint. The chosen 'basket of goods' maximizes die utility of the consumer. The revealed preference for a particular collection of goods implies (axiomatically) the maximization of the utility of the consumer.

2.3 DERIVATION OF THE DEMAND CURVE

Assume that the consumer has the budget line AB in figure 2 and chooses the collection of goods denoted by point Z, thus revealing his preference for this batch. Suppose that the price of x falls so that the new budget line facing the consumer is AC. We will show that the new batch will include a larger quantity of x.



Good X

Figure 2.2

Firstly, we make a 'compensating variation' of the income, which consists in the reduction of income so that the consumer has just enough income to enable Mm to continue purchasing Z if he so wishes. The compensating variation is shown in figure 2 by a parallel shift of the new budget line so that the compensated budget line A^1 passes through Z. Since the collection on Z is still available to Mm, the consumer will not choose any bundle to the left of Z on the segment A^1Z, because his choice would be inconsistent, given that in the original situation all the batches on AZ were revealed inferior to Z. Hence the consumer will either continue to buy Z (in which case the substitution effect is zero) or he will choose a batch on the segment ZB1 such as W, which Includes a larger quantity of x (namely x2). Secondly, if we remove the fictitious reduction in income and allow the consumer to move on the new budget line AC, he will choose a batch (such as N) to right of W (if the commodity x is normal with appositive income effect), the new revealed equilibrium position (N) include a larger quantity of x (i.e. x3) resulting from fall in its price. Thus the revealed preference axiom and the implied consistency of choice open a direct way to the derivation of the demand curve; as price falls, more of x is purchased.

2.4 DERIVATION OF THE INDIFFERENCE CURVES

Although not needed for establishing the law of demand, indifference curves can be derived and their convexity proved by the revealed preference hypothesis.

The indifference curves approach requires less information than the neoclassical cardinal utility theory. But still it requires a lot from the consumer, since the theory expects him to be able to rank rationally and consistently all possible collections of commodities.

Samuelson's revealed preference theory does not require the consumer to rank his preference or to give any other information about his tastes. The revealed preference permits us to construct the indifference map of the consumer just by observing his behaviour (his choice) at various market prices, provided that (a) Ms choice is consistent, (b) his tastes are independent of Ms choices over time and do not change, (c) that the consumer Is rational in the Pareto sense, that is, he prefers more goods to less,

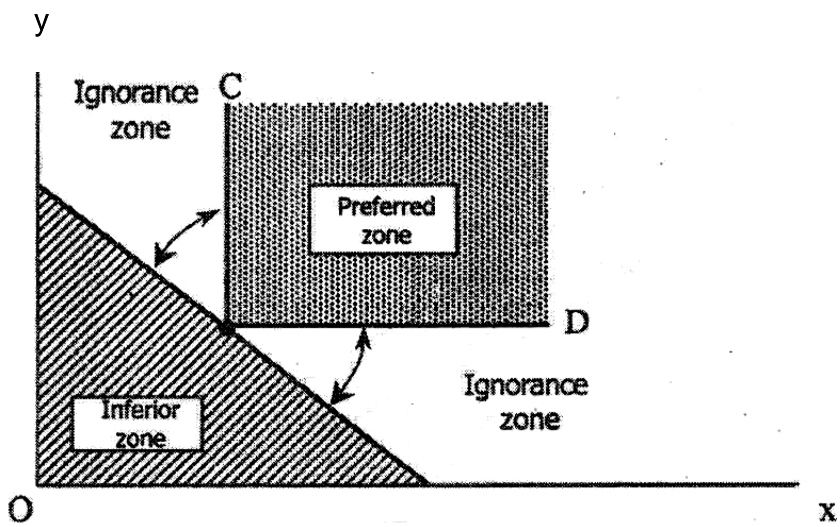


Figure 2.3

Assume that the initial budget line of the consumer is AB in figure 3 and he chooses the batch Z. All the other points on the budget line and below it denote inferior batches to 2L. If we draw perpendiculars through Z*, CZ and ZD, all the bundles on these lines, and m the area defined by them to the right of Z, are preferred to Z' because they contain more of Z. However all bundles above the budget 1 are still not ordered Tills zone (area) CZD is known as preferred zone as it contains preferred bundles. The bundle in the zone O AB are not preferred to Z as they contain less of Z. Hence the zone OAB is known as inferior zone or zone of non preference.

Consumer will be ignorant to say preferred, or not preferred for any bundle in the area CZA or DZB as it contains more of one and less of other when compared to 2. He requires some additional information to opt or not to opt. Hence this zone CZA or DZB is known as ignorance zone or zone of indifference. We may rank them relative to Z by adopting the following procedure. Let the price of x fall so that the new budget line EF passes below Z (figure4)

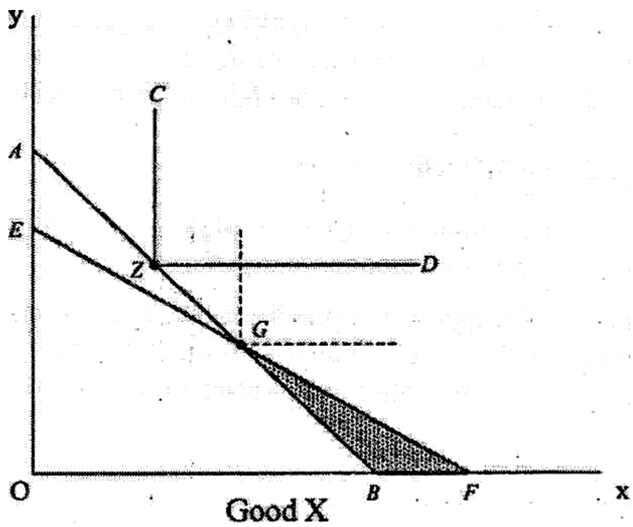
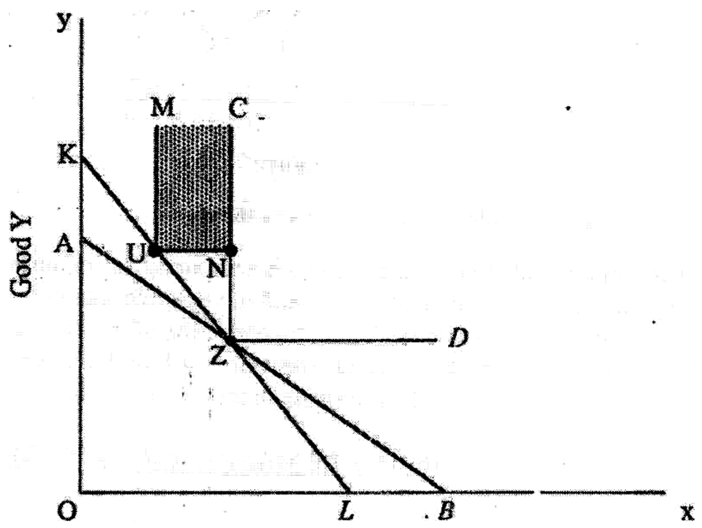


Figure 2.4

The consumer will choose either G or a point to the right of G *(on* GF)» since points on EG would imply inconsistent choice, being below the original budget line and hence inferior to G< Assume that the consumer chooses G, Using the transitivity assumption we have,

Z>G (in the original situation)
G > (GBF) (m the new budget situation.)
Hence Z>(GBF)

In this way we managed to rank all the batches in GBF relative to Z. We may repeat this procedure by drawing budget lines below Z and defining gradually all the batches of the 'lower ignorance zone' that are inferior to Z. Similarly we may rank {relative to Z) all the batches of the "upper ignorance zone". For example, assume that the price of x increase and the new budget line EL passes through 2, The consumer will either stay at Z or choose a point such as U on KL (Figure 5).



Good X

Figure. 2. 5

Using the rationality assumption we find (MUN) > U From the revealed preference principle U > Z And from the transitivity postulate (MUN) > Z

Thus we managed to rank the batches in (MUN) as preferred to Z> Repeating this procedure we may gradually narrow down the ignorance zone" until we locate the indifference curve within as narrow a. range as we wish. Hence the revealed preference axiom permits us to derive the indifference curve from the behaviour (actual choice) of the consumer in various market situations.

The convexity of the indifference curve may he established graphically as follows. Let us redraw the original budget situation (figure 5). We observe that the indifference curve through Z must be some where in the ignorance zone and must be convex, because it cannot have any other shape. The indifference curve cannot be the straight line AB because the choice of Z shows that all the other points on A!3 tire inferior to Z (hence the consumer cannot be at the same time indifferent between them). It cannot be a curve e or line cutting AB at Z, because the points below Z would imply indifference of the consumer win le he has already revealed Ms preference for Z. Finally, the indifference curve cannot be concave through 2, because all its points have already been ranked as inferior to Z (they contain less goods). Hence the only possible shape of the indifference curve is to he convex to the origin.

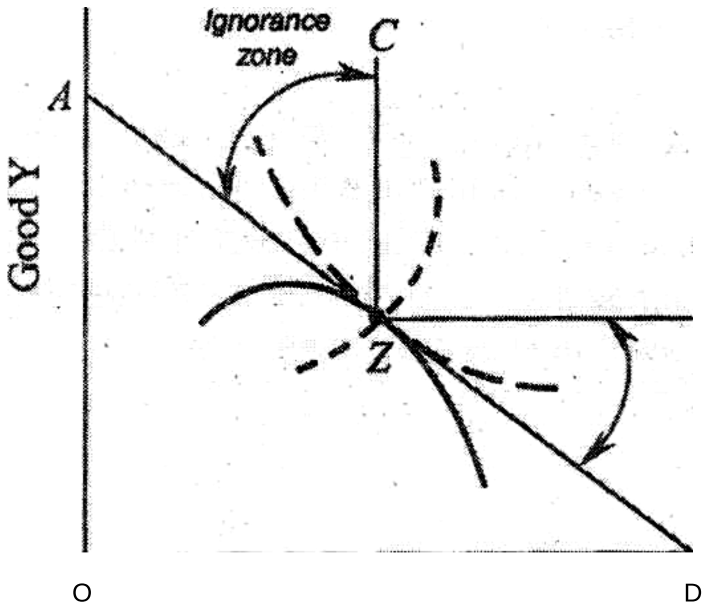


***Figure* 2.6**

2.5 CRITIQUE OF THE REVEALED PREFERENCE HYPOTHESIS

Samuelson's revealed preference theory is a major advancement to the theory of demand. It provides a direct way to the derivation of the demand curve, which does not require the use of the concept of utility. The theory can prove the existence and convexity of the indifference curves under weaker assumptions than the earlier theories. It has also provided the basis for the construction of index numbers of the cost of living and their use for judging changes in consumer welfare. In situations si where prices change.

REVISON OF DEMAND THEORY BY HICKS

2.6 INTRODUCTION

J.R. Hicks ("A Revision of Demand Theory", 1956) revised his demand theory, which he explained in his earlier work "Value and Capital". Hicks realized his earlier explanation to the theory of demand was weak. The main reason to revise earlier theory is the development of Samuelson's revealed preference approach, the rise in the application of econometrics and the emergence of mathematical theories of strong and weak orderings. Hicks was mainly influenced by the revealed preference hypothesis and the logic of strong ordering used by Samuelson and his followers to derive the theory of demand. Thus in his revision of demand theory, he gives emphasis on econometric approach to the theory of demand. He states that the demand theory, which is useful for econometric purpose is definitely superior to the one which does not serve such purposes. According to him, 'There can be no doubt that econometrics is now a major form of economic research, a theory which can he used by econometricians is to that extent a better theory that one which cannot. But still Hicks is of the opinion that only ordinal measurement of utility is possible. He therefore, continues to make use of the concept of ordinal utility in hi s revision of demand theory as well. It is to be noticed that Hicks has not taken the help of indifference curve in any of the theories, which he developed at a later stage. He has pointed out various disadvantages of the indifference curve technique. He says that geometrical method of indifference curve is fully effective and useful for representing only quite simple cases, especially those in which the choice concerns quantities of two goods only. When the problem is extended to more than two goods, then we can not escape mathematics to solve it. Further, he points out that geometrical method, is based upon the assumption of continuity a property which the geometrical field does have hut which the economic in general does not. He therefore, gives up the assumption of continuity in his revision of demand theory. The new method of preference hypothesis explained by Hicks in his revision of demand theory is given below.

In the formulation of his revised demand theory, Hicks assumes preference hypothesis on the part of an ideal consumer. He explains the preference hypothesis as follows: ‘The ideal consumer (who is not affected by any thing else except the price of the commodity) chooses that alternative, out of the various alternatives open to him, which he prefers most, or ranks most highly. In one set of market conditions, he makes ones choice, in others otter choices, but the choices he makes always express the same ordering, and must therefore be consistent with one another; This is the hypothesis made about the behaviour of the ideal consumer’.

It is clear that the consumer in a given market situations chooses the most preferred combinations, but his choices in different market situations will be consistent with each other; Hicks further stated that 'the demand theory which is based upon the preference hypothesis turns out to be nothing else but an economic application of the logical theory of ordering. Hence before deriving his revised demand theory from preference hypothesis, he explains the 'logic of order’. In this context he draws out difference between strong ordering and weak ordering, finally, he develops his demand theory on weak ordering type of preference hypothesis.

2.7 STRONG AND WEAK ORDERING DISTINGUISHED

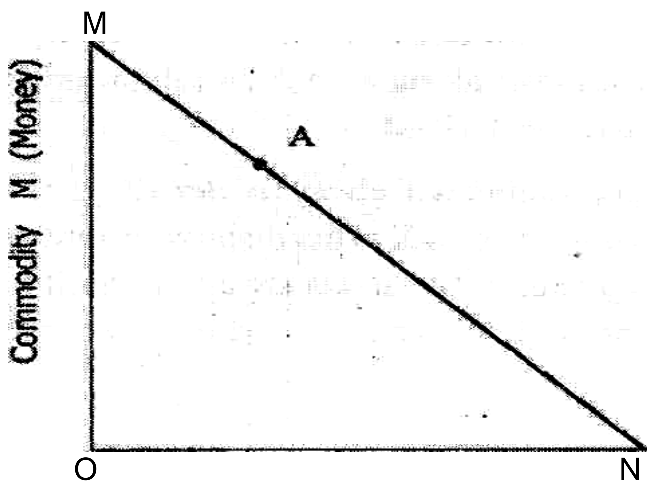
The distinction between strong ordering and weak ordering is as follows.

Strong ordering is a set where items are strongly ordered, Further if each item has place of its own in the order and each item could then be assigned a number and to each number there would be one item and only one item which would correspond.

Weak Ordering is a set of items within a group can he put ahead of the others. Thus a weak ordering implies a division into groups in which sequence of groups is strongly ordered, but in which there is no ordering within the groups,

: The indifference carve is not ordered, though all are equally desirable (or occupy same in the order) but the consumer is indifferent to others. On the other hand, revealed preference approach implies strong ordering because it assumes that the choke of a combination reveals consumer's preference. Choice can reveal preference for a combination only if all the alternative combinations are strongly ordered. It simply means that the ordering, the consumer chooses a point and rejects others open to him, then the rejected points need not be inferior to the point actually selected by him, then the rejected points need not be inferior to the point actually selected by him but may be indifferent to it.

The analysis of strong ordering and weak ordering as applied to the theory of demand can be explained with the help of diagrams. First we explain the strong ordering in figure 7



Commodity X
Figure 2.7

Here Hicks assumes that there are two commodities X and M. Commodity X is an individual good and commodity M (money) is a composite good representing all goods and services, either than: X, Commodity M (money) is measured along the vertical axis while commodity X is measured along the horizontal axis. Given the income of the consumer and the price of X and M, the price income situation of the ideal consumer (who acts according to preference hypothesis) is shown by the line MN. Now the consumer may opt any combination in or on the triangle MON. The point A on the line MN represents the actual choice of the consumer. Now the question is how his act of choice of A from among the available alternative in and on the triangle MON is to be interpreted. If the available alternatives are strongly ordered, then the choice of A by the consumer will show that he prefers A over all other available alternatives. In Samuelson's language he ‘reveals his preference’ for A over all other possible alternatives which are rejected. Thus, under strong ordering, the consumer shows definite preference for the selected alternative, there is no question, of any indifferent positions to the selected one. But such type of strong ordering has been criticized by Hicks.

(1) In Samuelson's version of the preference hypothesis in its strong ordering form, it can not be assumed that ail the geometrical point, which lie within or on the triangle MON represent effective alternatives.

(2) Hicks states that if commodities are assumed to be available only in discrete units, so that the diagram is to be conceived as being drawn on squared paper and the only effective alternatives are the points at the comers of the squares and, therefore the selected point must also lie at the corners of the strong ordering hypothesis is acceptable. Since in real world such a situation never arise because of the use of money. Thus, in making a choice between commodity X available in discrete units and the finely divisible commodity, M (money), strong ordering has to be given up. It is on the basis of this reining of Hicks that he rejects Samuelson's strong ordering hypothesis.

2.8 WEAK ORDERING IN THEHICKS' DEMAND THEORY

Hicks mikes use of weak ordering hypothesis in. his demand theory Hicks argued that "if the * consumer's scale of preference is weak iy ordered, then his choice of a particular position A docs not show (or reveal) that A is preferred to any rejected position which is preferred to A, It is perfectly possible that some rejected position maybe indifferent to A; the choice of A instead of that rejected position is then a matter of choice.

If preference hypothesis in its weak ordering form is adopted, then it provides very little information, about the consumer's behaviour that the basic propositions of demand theory can not be derived, from it Thus Hicks introduces an additional hypothesis along with the weak ordering hypothesis, so that basic proposition of the demand theory can easily be derived.

Additional hypothesis is that 'the consumer will, always prefer a large amount of money to a smaller amount of money, provided that the amount of good X at his disposal is unchanged' It is not necessary to adopt this additional hypothesis is very reasonable and is always implicit in economic analysis, This additional hypothesis along with weak ordering preference hypothesis is adopted, we may get the positive solution, as shown in figure. 7.

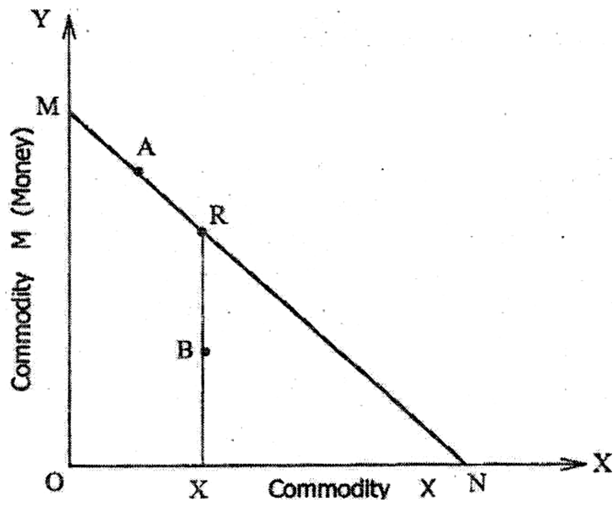


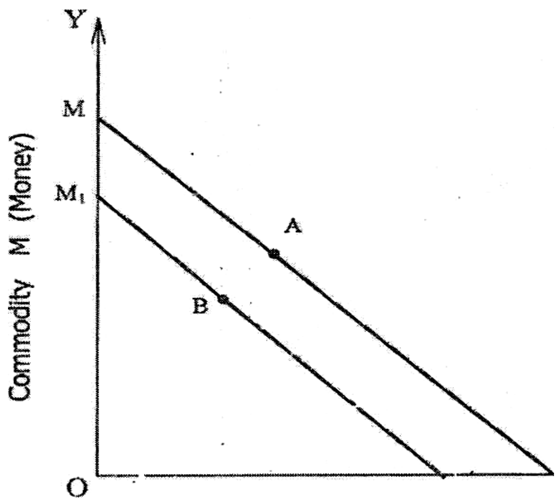
Figure 2.8

We assume that from all available combinations in and on the triangle MON, the consumer chooses A.

Under weak ordering hypothesis alone, the choice of A rather than B means that either A is preferred to B, or A and B are positions of indifference. But this proposition of weak ordering does not hold if another position R is taken on the line MN through the vertical line XB. Under the additional hypothesis, R is preferred to B because the consumer prefers more of money (i.e., commodity M) at point R to less of money (i.e., commodity M) at point B, given the same quantity of OX of commodity X. T f A and B are points of indifference, it follows from the transitive that point R is preferred to point A. But according to the weak ordering hypothesis, no other position is preferred to point A within and on die triangle MON which means that R has already been rejected in favour of A. Therefore points A and R maybe of indifference, it follows that if A and R are points of indifference; than the alternative that A and B points are indifferent must be ruled out. Therefore the logic of weak ordering states that the point A on the line MN is preferred to B which lies within the triangle MON. But what can not be explained is that point A is preferred to point C or any other point on the line MM. Thus drawing the difference between the implications of strong and weak ordering, Hicks states, "the difference between the consequence of *Strong* and weak ordering so interpreted amounts to no more than this; that under strong ordering the chosen position is shown to be preferred to all oilier positions *within* and on the triangle, while under weak ordering, it is preferred to all positions within the triangle, but may be different to other positions on the same boundary as itself".

2.9 THE DIRECT CONSISTENCY TEST

Following Samuelson, Hicks also assumes consistency of choice behaviour on the part of the ideal consumer who reveals an unchanged scale of preferences. Hicks calls this consistency test as the direct consistency test Here the direct consistency is the economic expression of the two-term *consistency condition* on the theory of logic of order. Besides the consistency of consumer's choices, Hicks proves the hypothesis on the basis of inconsistency also. It can be explained with the help of diagram.



Commodity X Nj

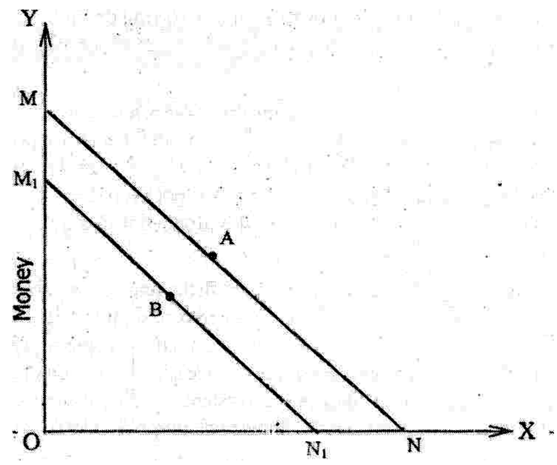
Figure 2.8

In figure 8 commodity X is measured along the horizontal axis and the composite commodity M (money) is measured on the vertical axis. Given the income of the consumer and the price of X, the choices open to the consumer are shown by the area of the triangle MON. The point A on the line MN represents the actual choice of the consumer. The preference hypothesis in its strong ordering form (Samulson's version) represents A to be preferred to all other combinations within or on the triangle MON. While in its weak ordering form, it implies that point A is preferred to all combinations within the triangle and is either preferred or indifferent to other points on the line MN.

We take another market situation represented by the price-income lineM1N1, where the price of X is different and the income of the consumer may or may *not* be different. The various alternative combinations open to the consumer in the new situation are represented by the triangle M1ON1. Point B on the line M1N1 represents the actual choice of the consumer in the new situation. Similar kind of preference, as in situation a, will follow in situation B under- strong and weak; form of preference hypothesis.

Since the consumer acts according to the unchanged scale of preference in both situations, fee preferences by Mm in the two situations must be consistent with each other. The behaviour of the consumer will be inconsistent if he reveals his preference for combination A over combination B in situation A, when both the combinations A and B are available in both the situations. But in the ease of weak ordering, the possibility of indifference has also to be considered. Thus the various cases of consistency or inconsistency that may arise under strong ordering and weak ordering may be analysed in the following manner.

(1) First Case. The first situation may be when one of the two price-income lines wholly outside the other.



Commodity X
Figure 2. 9

In figure 9, we assume that the line MN lies above M1N1 throughout its length. In situation A, point B lies; within the triangle MON and therefore the consumer prefers point A to B both under strong and weak ordering. In the B situation, point A is not available (as it lies outside the triangle M1ON1). Therefore the consumer's choice of A in the A situation is quite consistant with his choice B in the B situation. But there is no inconstancy in these two situations. This is feet is the Samuelson's version of revealed preference hypothesis.

(2) Second Case. The second situation may be when one of two price -income line does not lie wholly outside the other, that is, the two lines intersect each other at the same point. This case has been presented in figure 10

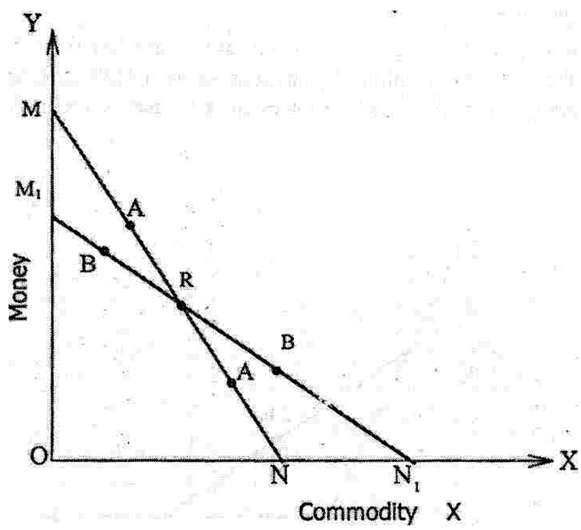


Figure 2.10

When line MN lies outside the line M1N1 on the left of the cross point R (so that it represents a Higher income) and lies inside the prfee4ncoiiie line MINI on the right of the cross point R (so that it represents a lower income). When two price-lines intersect each other then the following possibilities are created.

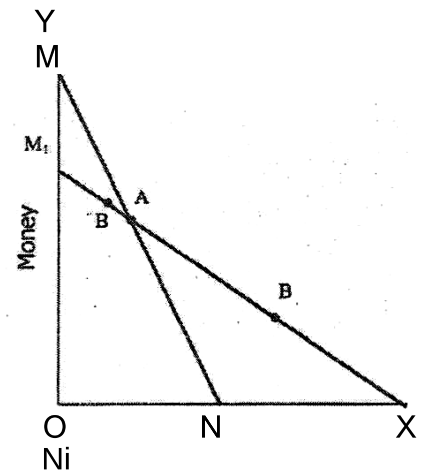
i) The first possibility is one when the choices available to the consumer in the two situations. A and B lie to the left of the cross point R (or above point R). In the A situation, B lies within the triangle MOM. Thus the consumer prefers A to B and is indifferent to B (weak ordering). In the B situation, A is not available to him, because it lies outside the triangle M1ON1. Thus his choice of B in situation B is consistent with his preference for A in situation A.

ii) The second possibility is that when both the selected positions A and B respectively in the two situations lie to the right of the cross point R (or below RX In this case, point B is preferred to point A because A lies within the triangle $M1_x$ $ON1_l$ and the consumer is also indifferent to A, Here point B is not possible to the consumer in situation A; Thus Ms choice of A in the situation A is consistent with Ms preference for B in situations, Thus in this case also the consumer's choice behaviour is quiet consistent,

iii) The third possibility is that when A lies outside to the left of the cross point R, and B outside to the right of it (in the area MARBMj), In this ease, in A situation A is chosen because B lies outside his approach, and in B situation only B is chosen because A lies outside his approach. The question of preference of one over the other does not arise in this case, Thus the choice are consistant in both the situations tinder both strong .and weak ordering whatever is adopted by the consumer.

iv) The forth possibility is that when the two selected points A and B lie inside the cross point R i.e., B to the left of R and A to the right of R (as shown by the area M_1BRAN in fig.11 In this case there is inconsistency in the behaviour of the consumer, hi situation point A is preferred to B because B lies within the triangle MON. Similarly in situation point B is preferred to A because A lie within, the triangle M1,ON1. In either of the situations, the consumer is indifferent to the other choice. But he can not prefer A to B and B to A simultaneously. Therefore we find inconsistence in the consumer's behaviour under both strong and weak orderings..

(3) Third Case. Here we may show a group of special cases when the two price-income lines intersect each other but one of the two choice positions lies at the cross point while the other choice position may lie either outside or inside the cross point. These cases can be explained with the help of a diagram, as in Fig.11.



Commodity X
Figure 2.11

i). The first-case is one where point A lies on the cress and B outside the cross to the left of A, In the A situation, point A is preferred to B because B lies within the triangle MON aid the consumer is indifferent to point B. But in B situation on the line $M1_{!}N1_{t}$ point A is also there. Under: strong ordering B reveals a preference over A in the B situation, through A reveals a preference over B in the A situation, Thus there is inconsistency because both can not he chosen to each other Unclear weak ordering in B .situation, either B is preferred to A or B is indifferent to A. Thus there is again inconsistency in the consumer behaviour:

(ii) The second case is that when A lies on the crossand B out side the cross to it right as in Fig.8. In this case B lies outside the reach of the consumer in situation A. Therefore, there is no inconsistency in choice A in the situation; In the B situation B is preferred to A. Under weak ordering, it implies, that either B is preferred to A or Bis indifferent to A. So there is no inconsistency because B is not available in the A situation.

(4) Fourth Case. Here both A and B lie at the cross (can not he presented in fig.), the consumer *can* not have any preference for one situation over the other and hence there can he no inconsistency in his choice.

It is clear from the above analysis that we reach the same conclusions *in* all cases from the direct consistency test whether we are applying strong ordering or weak ordering hypotheses. On either hypothesis, there is inconsistency in the following two eases:

- when both points A and B lie within the cross.
- When- one point lies at the cross and the other within the cross

Though the weak and the strong forms of preference hypothesis give the same result is regard to the consistency tests, hut it should be remembered that the arguments by which they achieve the result are different

2.10 DEMAND THEORY OF WEAK ORDERING

Prof. Hicks develops his revised theory on the basis of weak ordering (along with additional hypothesis) and the direct consistency test, He first builds the theory of demand for a single commodity i.e., for the consumer confronted with a market in which the price of only-one commodity changes while the prices of other goods are held constant* He derives the law of demand by dividing the effects of a price change into substitution effect and income effect. The main is to separate out the substitution effect.

The substitution effect *cm* be separated from income effect by the following two methods. (1) The method of Compensating variation, and (2), The method of Cost Difference.

Derivation of law of demand by the Method of Compensating Variation

In Figure 12 we measure money along the vertical axis and commodity X along the horizontal axis, Given a certain price of the commodity X and income of the consumer, the price income line is drawn as MN, Now assume that on this initial line MN, the consumer selects the combination A. Now suppose; that the price of commodity X falls and money income (i.e., prices of other composite items) remaining unchanged; the price-income line becomes MN1 . Now[1] the consumer has to select a new combination on line MN1. It follows from the consistency test that so long as some quantity of X is consumed, any position (point) on line MN, must be preferred to point A. In other words, whether the selected new position B on MN1 lies to the left of A or to the right of A or exactly above A, it will be preferred to A; this is because of the fact that A lies within the triangle MON1. Now the question is where the position B on MN1 will lie; that is whether it will lie; to the right of A, or to the left of A or exactly above A. The position B lying to the right of A means that the quantity demanded for commodity X rises as a result of fall in its price; and the position B lying to the left of A implies that die quantity demanded for commodity X falls with the fall in its price, and further the position B lying exactly vertical above A would mean, that the consumer does not change the consumption of X even if its price falls.

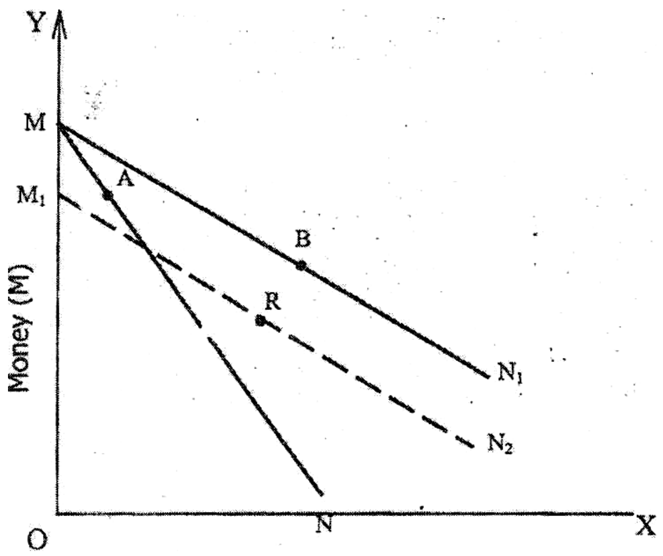


Figure 2.12

Thus now the question is whether the quantity of X consumed rises, falls or remains the same with a fall in the price of X, The answer to tine question, as a whole, cannot be deduced from the consistency test. As stated above. It is perfectly consistent for there to be arise, %11 or no change in quantity of X as a result of a fall in the price of X, However, if the fall in the price v associated with a reduction in income, then it can be shown from the consistency test that the: quantity demanded for X must rise or remain the same, it can not fall When the income of the consumer is redir, [1] by an. appropriate amount along with a fill in the price of X, the remaining effect of the change in price r J in price) on the quantity demanded for X will be due to substitution effect. It is clear now that it can be shown from the consistency test that due to substitution effect to fall in the price of X, the consumption of X must rise or remain the same, it e n not diminish. The rest of the total effect of change in. price is the income effect. In which, direction (i .e negative or positive) the income effect of the fall in price of X works, can. not be shown with the help of consistency test. In fact it will depend upon the nature of the commodity which is based on the observation of the consumer.

It follows from, the above fact that in order to indicate the substitution, effect on the demand for a commodity we make a suitable reduction in income along with the fall in price of commodity X. The movement from point A, on line MN to point B on line MN1 as a result of the change (fall) in price represent the price effect. In order to .find out the substitution effect from the price effect, we have to reduce the gain in real income accruing to the consumer due to a fall in the price of X. In other words, income of the consumer is reduced by so much, amount that the intermediate position R is arrived at the new lower price but lower income where the consumer is indifferent *to* the initial position A,

According to the indifference curve hypothesis, the movement of the consumer form point A to R is the substitution effect which is positive, It can be shown from the consistency theory that in what direction the substitution effect works, Since the consumer is indifferent between the positions A and R, the opportunity line $M1N2_2$ must intersect the line MN. This is because of the line $M1_lN2_2$ were to lie wholly outside MN, then. R would be shown to be preferred to be preferred to A; and if line $M1_tN2$ were to lie wholly inside MN, then a would he shown to he preferred, to IL Similarly if A and R are to be indifferent, A and R can not lie both to the left or both to the light of cross point (not shown, in the diagram) of the lines on which they lie. Again, if the two positions A and R lie within the cross, or one at the cross and other within the cross, inconsistency of choice will come into the picture. Thus, the only alternatives left are: (i) both positions A aid R lie outside the cross; (ii) form positions A arid R, one lies at the cross and the other outside the cross ; and (iii) both positions A and R lie at the cross. These three are the only possible cases

if A and R ate to be indifferent ami if the consumer's choice is to be consistent. In any of these eases, it should be noted that either the consumption of X increases or remains the same. This is one part of the price effect (i.e., fall in the price o f X). Thus, here we conclude that when the price of X fells, the consumer moves from position. A to B and the consumption of X tends to rise, The movement from A to B is the price effect which is composed of the movement form A to R via the substitution effect aid from R to 8 through the income effect Thus the theorem is proved that the demand curve is downward sloping,

2.11 DERIVATION OF LAW OF DEMAND BY THE METHOD OF COST DIFFERENCE

Although the above method of compensation is perfectly valid for the derivation of law of demand, yet the alternative method of cost difference is more convenient for this purpose. This method was evolved by Sultsky and has been used by Hicks to derive the law of demand According to the method, when the price of commodity X falls, the real income of the consumer is reduced in such a way that he is just able to buy the original combination A as shown in Fig, 13 .Hicks says that income is accordingly reduced by the difference between the cost of Ms previous (A) consumption of X at the old price and at the new price, It can be illustrated in Fig, 13.

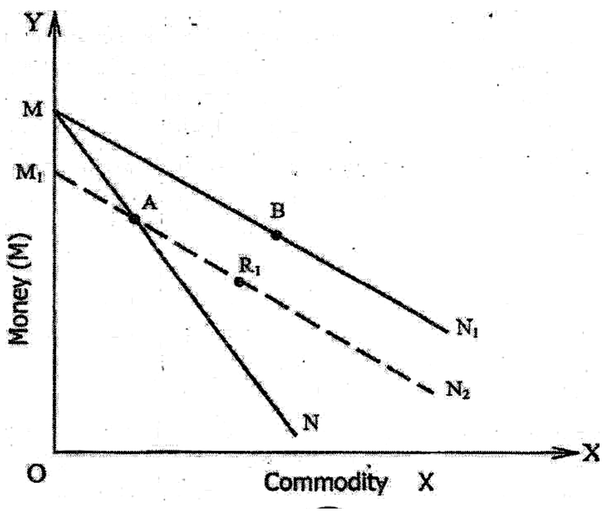


Figure 2.13

We assume that MN is the original price-income line and A is the initially selected combination, When the price of X falls, the consumer goes to point B on new price line MN_1, The shift from A to B is known as price effect, which is to be divided into substitution effect and income effect by the cost difference method. Under the cost difference method, the fall in price of X accompanied by the reduction in income of the consumer by such an amount which will put the consumer just able to buy the original combination A. Hence the cost difference in Fig, $13_:$ is MN1 and the income of the consumer is reduced by M1,N2 in such a way that the line M1N2 passes through the old (initial) combination A. On the line $M1_?N1_:$ the consumer will he at posit ion $R1_?$ Thus the movement from A to $R1_;$ is the substitution effect, For a consistent choice the possibilities open to the consumer are: (i) that the situation $R1_l$ lies to the right of A; and (ii) that A and R1, coincide (when under changed situation $M1_fN2$, the consumer chooses 'point A instead of $R1_x$):

Since in case (i) the consumer of X will tend to rise from position A to R_1 and incase (ii) it (textile consumption of X) will tend to be constant as a result of the situation effect. As in the compensation variation method, if the reduced income of the consumer is returned to him, he will again move to point B on MN1, line. Since point B lies above and to the right to R1, the income effect from $R1_t$ to B is positive; the consumer consumes more of X as a result of the income effect Here X is a superior or normal good. Thus the law of demand holds.

Given different bundles of two goods the consumer chooses one bundle. Suppose he chooses bundle Q^1 and there is another bundle Q^0 which was affordable but not chosen. It means that bundle Q^1 is to Q^0. This is the principle of revealed preference.

Assume that there are n commodities. A particular set of prices is denoted by P^0, and the corresponding quantities bought by the consumer by Q^0. The consumer's total expenditure are given by P^0Q^0 which is defined as the sum $\sum_{i=1}^{n} P_1^0 Q_1^0$.

Consider an alternative batch of commodities that could have been purchased by the consumer but was not. The total cost of Q^1 at prices P^0 must be no greater than the total cost of Q^0.

$$P^0Q^1 \leq P^0Q^0 \text{ ------------------------ } 1$$

Since Q^0 is at least as expensive a combination of commodities as Q^1, and since the consumer refused to choose combination Q^1, Q^0 is revealed to be preferred to Q^1. When the consumer reveals that he prefers Q^1 to Q and Q to Q^2 etc how we know that he is making optimum choice. Two conditions must be satisfied to know this.

Weak Axiom of Revealed Preference (WARP)

If Q^0 is revealed to be preferred to Q^1, the latter must never be revealed to be preferred to Q^0.

The only way in which Q^1 can be revealed to be preferred to Q^0 is to have the consumer purchase the combination Q^1 in some price situation in which he could also afford to buy Q^0. In other words, Q^1 is revealed to be preferred if

$P^1Q^0 \leq P^1Q^1$ ---------------------------------- 2

The axiom states that (2) can never hold if (1) does. Consequently implies the opposite of (2) or

$P^0Q^1 \leq P^0Q^0$ implies that $P^1Q^0 > P^1Q^1$

These two conditions are the two basic axioms of the revealed preference theory. These are :

1) Weak Axiom of Revealed Preference and
2) Strong Axiom of Revealed Preference

Strong Axiom of Revealed Preference (SARP)

If Q^0 is revealed to be preferred to Q^1, which is revealed to be preferred to Q^2, ………. which is revealed to be preferred to Q^k, Q^k must never be revealed to be preferred to Q^0. This axiom ensures the transitivity of revealed preferences, but is stronger than the usual transitivity condition.

If bundle Q^1 is revealed preferred to Q^2, directly or indirectly, Q^2 is different from Q^1, then bundle Q^2 cannot be, directly or indirectly, revealed preference to bundle Q^1. It implies that revealed preference of the consumer must be transitive.

SARP is both necessary and the sufficient condition for optimum choice. It is necessary condition because it assures that the consumer is always choosing to the best bundle he can afford. It is a sufficient condition because we can always find nice-well behaved preferences that could have generated the observed choices leading to optimized behaviour.

2.12 SUPERIORITY OF HICKS LOGICAL ORDERING THEORY OF DEMAND

Hicks" Revision of Demand theory is superior to the cardinal and indifference approaches in following respects

1. Different from Samuelson's Approach, Hicks does not follow Samuelson's behaviouristic approach to study consumer's behaviour but instead uses the technique of logical ordering on the pert, of the consumer to establish the theorems of demand. The cardinal; theory fails to analyse the effect of price changes into substitution effect aid income effect.

2 Sound Methodology. Fritz Machiup points out that "Methodological position underlying Hicks approach is eminently sound. He is free from positivist behaviotiristic restrictions on the study of consumer's behaviour; and he also avoids contentions about the supposedly empirical assumptions regarding rational action. Instead, he starts from a fundamental postulate, the preference hypothesis"[5].

3. Avoids Unrealistic Assumptions. Hicks's revised theory of demand is free from unrealistic assumptions of cardinal and ordinal approaches, namely continuity and maximizing behaviour on the part of the consumer. He has not used indifference curves in his analysis' and therefore avoids the assumption of continuity Further, instead of assuming that consumer always tries to maximize his satisfaction, he now, like Samuelson, relies on consistency in the behaviour of the consumer; which is a more realistic assumption,

4. Applicability to More than, two goods, The previous theory of indifference curves could. 'be applied only in the context of two goods, hut now the revised theory of demand, can be applied even to more than two goods. It is done, by deducing from preference hypothesis and logic of order.

5. *Distinction between weak ordering and strong ordering is possible.* For the first time Hicks makes the distinction between weak; ordering and strong ordering forms of preference hypothesis. He has based his revised theory of demand oil weak ordering which reeongnises the possibility of indifference in consumer's scale of preferences.

6. Distinction between inferior goods and Giffen goods has b6en drawn. Hicks has clearly - . explained the distinction between normal goods, inferior goods, and Giffen goods through . his new logical weak ordering theory This he has done by classifying the

income effect [1] and the substitution effect from the price effect. Samue1son's revealed preference theory has not been able to distinguish between inferior goods and .Giffen goods,

2.13 SELF ASSESSMENT QUESTIONS

1 Critically examine Samuelson's Revealed Preference Theory of consumer behaviour?

2. Examine the revealed preference theory and show how it is an improvement over the indifference curve analysis.

3. Explain with the help of diagram, the Hicks' revision of demand theory

4 How does, JR Hicks derive the law of demand directly from the assumption of weak ordering without using indifference curves?

5. Demand theory which is based upon the preference hypothesis, turns out to be nothing else but an economic application of the logic of ordering itself" - JJFL Hicks. Comment, on the statement

6. Explain clearly "the direct .consistency test" as developed by J R Hicks. Derive his demand theorem. on this basis.

7. Distinguish between weak ordering and strong ordering. Which one would you select for deriving a demand curve and why?

State clearly but briefly

1. Rationality
2. Consistency
3. Transitivity
4. The Revealed Preference Axiom
5. Strong Ordering Preference

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3

NEUMANN-MORGENSTERN UTILITY ANALYSIS

UNIT STRUCTURE

3.0 OBJECTIVES

In this lesson we shall learn about the theory of consumer choice under risk and uncertainty as developed by authors like von-Neumann and Morgenstern.
Measures of risk aversion.

3.1 INTRODUCTION

So far we have assumed certainty. In the marginal utility theory of Marshal, the indifference curve approach of Hicks and the revealed preference theory of Samuelson the consumer is sure of his end. Given a choice between goods he may have one or the other without doubt. But is this true always ? What if there is uncertainty? There may be just a probability of his getting one good against another. Then how will he choose ? If there were certainty he wood so choose as to maximise his satisfaction. In the cardinal theory utility can even be measured. In the ordinal theory it may only be compared. Thus one prefers more of a good to less of it. Two units are better than one. This is reasonable. But is this so when the prospect of getting one good is less certain and of another more certain ?.

Traditional economic theory implicitly assumed a riskless world. However, most economic choices involve risk or uncertainty. For example, an individual may decide to become a lawyer or to go into business, where incomes can be either very high or only modest. Similarly, a homeowner may insure him or herself against the small chance of a heavy loss through fire and also purchase a lottery ticket offering a small chance of a large win. Traditional economic theory could not explain choices involving risk because of its strict adherence to the principle of diminishing marginal utility. Such an apparently conflicting behavior as the same individual purchasing insurance and also gambling can be rationalized by a total utility curve that first rises at a decreasing rate (so that Marginal Utility declines) and then at an increasing rate (so that Marginal Utility rises).

The standard utility theory described in the previous lesson with consumer's choice of commodities when the benefit of each commodity to the consumer is known and is certain. In some circumstances, however, a consumer buys a commodity that yields different benefits with known probabilities. For example, a lottery ticket may cost Rs.5 and yield Rs.500 with a probability of 1/100 and Rs.0 with a probability of 99/100. The risk that goes with buying the lottery ticket is known because the distribution of probabilities across all outcomes is known. Many commodities and services contain an element of risk, important examples being gambling, insurance, and investments Neumann and Morgenstern in their famous work, "Theory of Games and Economic Behaviour" gave a method of cardinally measuring expected utility from win and prizes. On the basis of such a cardinal utility index called N-M index, rational decisions are made by the individuals in case of risky situations. Thus, Neumann-Morgenstern method seeks to assign a utility number, or in other words, construct a N-M utility index of the marginal utility of money which a person gets from extra amounts of money income. The choices by an individual under risky and uncertain situations depend on the N-M utility index(i.e., expected numerical utilities) If risk is measurable, utility can be measured. Here utility can be inferred from the behaviour of consumers, It is inferred from several observations, not just one, as in the revealed preference theory. However if behaviour were consistent, there is no room for indifference but only preference. But if there were indifference, choice cannot be limited to one instance, but Samuelson postulated single choice. Morgenstern and Savage assume several implying the possibility of indifference and changes in it with the changes in money income.

3.2 CHOICE AND RISK

Often one may have to select from a number of a alternatives which differ in the risk the consumer has to bear. This is seen in insurance and gambling. When you take an insurance policy (say. against fire in your house), you lose your premium (a small amount) to avoid the risk of losing your house (or a large value). However, it is not certain that your house will catch fire. It may or may not. The loss of the house is probable and therefore uncertain..But it is certain that you lose your premium when you have paid it. Here you prefer the certainty of small loss to uncertainty of a large loss.

The **risk** refers to a situation when the outcome of a decision is uncertain but when the probability of each possible outcome is known or can be estimated. The analysis of decision making and choice involving risk or uncertainty requires that the individual known all the possible outcomes and all have some idea of the probability of occurance of each possible outcome. The greater the variability of possible outcome, the greater the risk involved in making Investment decision.

The **uncertainty** refers to the situation when there is more than one possible outcome of a decision but where the probability of occurance of each particular outcome is not known or even cannot be estimated

When you buy a lottery ticket (and gamble), you may win a prize. Then you will get back the money you had spent on the ticket but the chance of your winning a prize is small. Therefore there is a risk in buying the ticket and or losing. But you prefer to take the risk and face uncertainty or winning you may avoid the risk of not winning the prize and losing the ticket money. But you prefer the uncertainty of winning the prize to the certainty of saying your ticket money.

Thus in insurance one prefers certainty to uncertainty (If you do not buy insurance, you face uncertainty). In gambling one prefers uncertainty. (If you do not gamble, you enjoy certainty).

Risky choices, illustrated by insurance and gambling are found in occupations. Occupations differ in the incomes they yield. In the civil service the income is clearly fixed and it varies within narrow limits. In management(accountancy), incomes may vary more widely but not very much. In film acting the variations are extreme and fluctuate between very high and very low levels. The degree of risk varies from civil service to Management (accountancy)and from Management (accountancy) to the film acting.

Securities differ in their degree of risk. Government bond yield a steady but low interest. Ordinary company shares yield high dividends but they are less certain. Different lines of business are more or less risky, yielding different rates of profit.

When you choose between different occupations, securities and lines of business, it is like choosing between certainty in insurance and gambling.

In choosing between risky alternatives, are you consistent in your choice ? Do you take-into account the risk involved or ignore it ? If risk does influence your choice, how does it ? These are some questions connected with risky choices and we may consider them to explain consumer behaviour when risk is present.

In economic theory the subject of risk has been considered in regard to earnings in different occupations and profits in different lines of business by Adam Smith (wealth of Nations) and Alfred Marshall (Principles).

In choosing between riskless alternatives you select the one, which has the maximum utility. You may prefer that combination of goods which has the large amount of utility, amount all combinations that are open to you: Now what about the risky alternatives ? How do you choose between them.

If occupations very in their risk, it was said by Adam Smith and Marshall, that is a more risky job was preferred to a more secure one, it was due to a spirit of adventure, confidence in one's own ability or faith in one's good luck.

The principle of maximum utility cannot explain choice among risky alternatives (which it could for riskless ones) due to diminishing marginal utility. If marginal utility of money diminishes, marginal utility of money cannot .be maximised if one takes part in a fair game of chance that is, one in which one has an equal chance of winning or losing a rupee. If you have Rs.5 and you take one rupee in order to win one more rupee, if you lose your rupee its utility will be more than the utility of the rupee you may win viz., the sixth rupee. Hence the gain in utility from winning a rupee will be less than the lose in utility from rupee. Therefore the expected utility from the game is negative. Then why should one participate in such games of chance. In reality, however people do indulge in them. Likewise, people pursue risky occupation and investments. How is one to account for this behaviours ?

Marshall rejected utility maximisation to explain risky choices as shown by the game of chance cited above. But others like Daniel Bernoulli used utility maximisation to explain such choice.

Diminishing marginal utility of money as we have seen. According to Bernoulli, a rational individual will take decisions under risky and uncertain situations on the basis of expected utility rather than on the basis of expected monetary value. He further states that the marginal utility of income decreases with rising incomes. Since the marginal utility decreases as money income increases, a rational individual will not play the game at equal odds that is, he will not make a bet. It is in this way that Bernoulli resolved St. Peter'sburg paradox.

Bernoulli's hypothesis can be explained with the help of the following Figure.

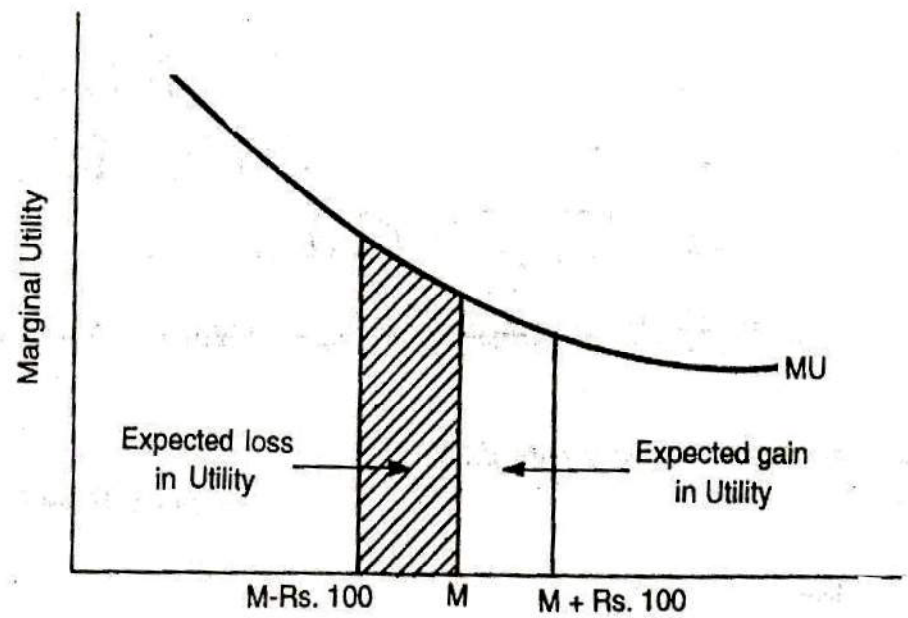


Fig.3.1

Suppose that the individual possesses Rs. M and is contemplating a gambling activity that offers an even chance of winning or loosing Rs.100. If he wins, he will have Rs.M+100. The gain of utility from Rs.100 will be added to Rs. M. If he loses, he will have with him M- Rs.100. The loss of utility from Rs.100 will be subtracted from Rs. M The expected gain in utility is shown by the white area, and his expected loss in utility by the cross shaded area. Since the expected gain is smaller than the expected loss in utility, a rational individual will never gamble at fair or even odds.

The above analysis is based on the assumption that the individual derives no pleasure from gambling.

On the other hand Morgenstern and Neumann have questioned the rejection of utility maximisation to explain risky choices to them even in risky choices the maximisation of utility could occur and explain choice.

If you prefer A to B and B to C the utility of A must be more than that of B and the utility of B more than that of C to you. Further, if you prefer a 50-50 chance of A to B, the expected utility of A must be more than that of B to you. But will you always prefer A to B. Suppose you prefer A to B (with 50 per cent chance of A) but the next time you prefer B to A (again 50 per cent chance of A. With such risky choices as A against B we may say you are sometimes inconsistent. On the other hand you may be consistent if you always choose A instead of B given 50-50 percent chance of A. If therefore, we can apply the maximisation rule here, we can explain behaviour of consumers under risk. For this however, we have to drop the assumption of diminishing marginal utility if gambling is any evidence people gamble because the marginal utility of money increases and does not diminish with an increase in the amount of money.

John von Neumann and Oskar Morgenstern used in their well-known Theory of games and Economic Behavior to construct an index of the marginal utility of money. The Neumann-Morgenstern theory can be applied to gambling, insurance and economic phenomena involving risk. In economic life risk is found in the use of resources, talent and energy in the choice of occupations, capital and enterprise in the choice of business and savings in the choice of investments. The use of resources may be divided, according to the degree of risk, into

1. Use of resources without any risk in regard to money returns. In occupations, teaching, civil service, clerical work in business ; Public utilities in investment government bonds.
2. Use of resources involving moderate risk in occupations, in business competitive trades, in investments, preferred shares.
3. The use of resources involving high risk, in occupations, aviation, racing, medicine and the law, in business untried fields and in investments, speculative stocks.

The chances of high gains or losses are much greater in three than in two above. Among the three above kinds people may prefer 1 or 3 or 2. Their choice of 1 is similar to their propensity to gamble. People either prefer no risk as in insurance or high risk as in gambling in the matter of occupations business and investments.

It may seem a little inconsistent for the same person it insures to avoid risk and gamble to bear risk. This may happen since there are different kinds of insurance and of gambling. A man may then take one type of insurance policy and indulge in a certain form of gambling at the same time without conflict. The risk of losing in a lottery is reduced by the offer of several attractive prices. Hence insurance and gambling may be reconciled with one another. There is abundant evidence for the willingness of people to buy insurance. These belong to all income groups. The premium paid by an insured is more than the costs of operations of the insurance company and hence it is more than what he receives as compensation for loss. The excess he pays must be for escaping risk, Likewise the willingness of the public to gamble is amply proved by the popularity of lotteries such as government raffles to raise public revenue, in India and the numbers game in the United States among the lower income group. The people who insure and those who gamble are not sharply divided as the same person often does both. While there is no direct evidence for this, it may be inferred from the various forms of investment involving different degrees of risk. Relatively poor people are found buying highly speculative stock (as is evident from the laws against them) and at the same time depend more on interest and rent from their capital than on dividends.

The traditional theory of consumer behaviour does not include an analysis of uncertain situations. Von Neumann and Morgenstern showed that under some circumstances it is possible to construct a set of numbers for a particular consumer that can be used to predict her choices in uncertain situations. Great controversy has centred around the question of whether the resulting utility index is ordinal or cardinal. It will be shown that von Neumann-Morgenstern utilities possess at least some cardinal properties.

3.3 ASSUMPTIONS OF CONSTRUCTING N-M UTILITY INDEX

Before explaining the Neumann – Morgenstern method of measuring utility form money or the construction of N-M utility index, it will be better to describe the assumptions on which the method is based.

Firstly, it is assumed that the individual possesses a scale of preferences that is quite comprehensive and complete. This is similar to the assumption of indifference curve analysis of demand that the individual known fully his indifference map depicting his scale of preferences. But unlike the indifference curve analysis of demand, the question here is the choice of “events”. The events refer to the amounts of money some of which are “certain” and others uncertain, monetary amounts with probabilities or odds attached to them.

Secondly, it is assumed that the individual can always say whether he prefers one event to another or he is indifferent between the two. This means that he can make probability calculations and on their basis can make comparison between the alternative events. For instance, he can compare the event of receiving Rs.5,000 for sure, or Rs.10,000 with 60-40 odds or any other probability, and can say whether he prefers one to the other or is indifferent between the two.

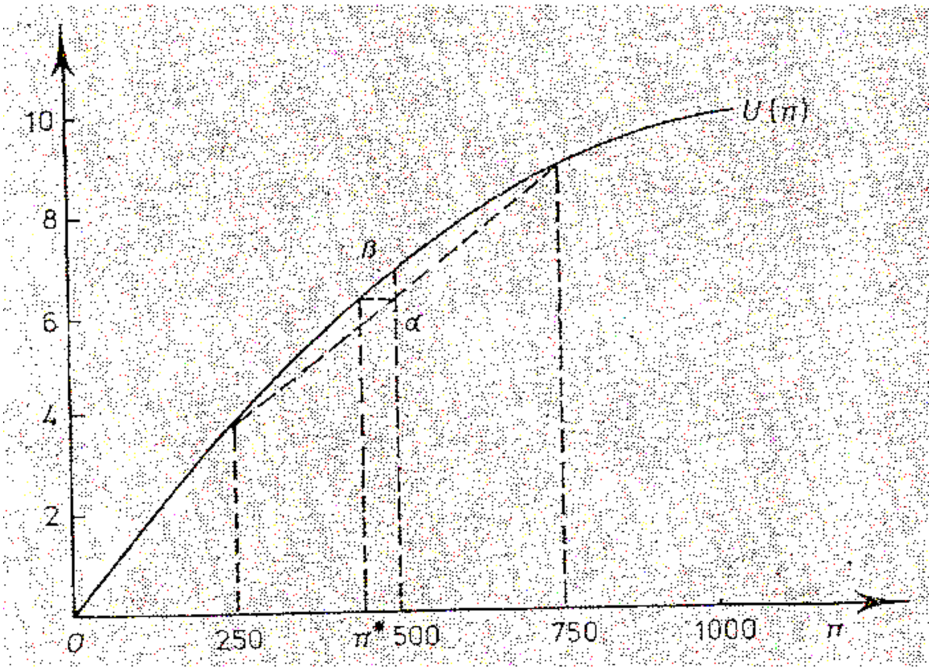
Lastly, it is assumed that individual’ choices are consistent.

If certain behavioural assumptions are satisfied, Von Neumann and Morgenstern have shown that an index can be constructed which enables one to predict behaviour in risky situations. By using the index, behaviour can be forecasted for situations which have not been previously observed. Their theory is not empty for it could conceivably be contradicted by observations. However, it is not a hedonistic theory, despite the fact that their index is unfortunately called a utility index, but a purely behavioural one. This is true even though their index possesses cardinal properties.

The axioms do appear to be reasonable ones. If they hold, consistency requires that an individual act to maximise the expected value of his "utility" function.

An individual's "utility" function can be constructed by first assigning an arbitrary number to his least preferred alternative and a higher arbitrary number to his most preferred alternative. Numbers are then assigned to all other alternatives by observing the probability of the best and worst alternatives required in a lottery, in order to make the individual indifferent about the lottery and the alternative under consideration. For example, consider four alternatives, *A, B, C* and *D* where the preference relation is $A > B > C > D$. Assign a number *r* to *D* and a number s to *A* where s exceeds *r.* The utility number to be assigned to *B* depends upon the chances required in a lottery in which *A* and *D* are the prizes to make the individual indifferent about the lottery and the certainty of *B.* Imagine that the individual is indifferent if *A* has a probability of p and therefore, *D* has a probability of (I —*p).* The utility (number) assigned to *B* is then $ps+(1-p)r$. In a similar fashion a number can be assigned to *C* and the individual utility function can be constructed. Once, numbers are assigned in this manner, they can be used to predict behaviour. Faced with any set of alternatives, the consistent individual (one acting in accordance with von

Neumann and Morgenstern's axioms) must choose the alternative which maximises the expected value of his utility index.



Pay in Rs.
Figure 3.2

The expected utility of any alternative is equal to the sum of its possible utility values, times their probability of occurrence. Thus, ***having set the arbitrary constants*** and, for example, having assigned utilities as follows: *U(A)* = 10; *U{B)*= 6: *U(C)* =5; and *U(D*) = 1, we could conclude that the individual would prefer (0.5 chance of *B* or C) to (0.2 chance of *A* and 0.8 chance of *D).* At least we are predicting this to be so,

0.5U(B) + 0.5U(C) > 0.2U(A) + 0.8U(D).

Because 3 + 2.5 > 2 + 0.8.

But, to reiterate, the index is not unique because two constants are arbitrarily determined. Given one index, any other index which is a linear transform of it also given the same predictive results.

A more concrete example may be of value. Imagine that the individual is faced with alternative money payoffs, returns or income levels and prefers those of higher value. Suppose that payoffs between Rs.0 and Rs. 1000 are possible and assign a utility of 0 units to Rs.O and 10 units to Rs.1000. This last assignment is arbitrary; we might have used 1 and 5, or 3 and 19, etc. Having assigned these values, the utility values for other payoffs can be calculated. Suppose that the individual is indifferent between Rs.500 with certainty and the chance of Rs.1000 with a probability of 0.7 or Rs.0 with a probability of 0.3. The appropriate utility assignment for Rs.500 is then

$$0.3 \times 0 + 0.7 \times 10 = 7.$$

In a similar fashion, utilities can be assigned to all payoffs between zero and Rs. 1000. On completion, the utility function might be like the one shown in Fig. 3.1 by $U(\Pi)$ which is a strictly concave function.

The utility function can be used to make forecasts. For example, the individual prefers the certainty of Rs.500 to an 0.5 probability of Rs.250 or Rs.750. The expected utility in the former case is β and α in the latter and as the diagram indicates, $\beta > \alpha$. He would always be prepared to trade the lottery alternative for the certain one. Indeed, he would be prepared to trade the lottery alternative for the certainty of Π^* dollars or more. The utility associated with Π^* just equals α, the utility for the risky alternative. Note that the individual is willing to exchange his risky alternative for less than its actual value (Rs.500). This occurs because his utility function is strictly concave. He is a risk-averter

The possible applications of this theory are great. It has been applied to the demand for insurance and can be applied to all trading in income rights and in this respect it sheds new light on the notion of liquidity preferences. However, there is not room to consider these fascinating applications here. No doubt, it could be also adapted to help explain a consumer's decision to try a new product when he is uncertain of its characteristics.

Before considering demand theory which emphasizes the demand for characteristics, we might note that certain criteria for choice under uncertainty are inconsistent with von Neumann and Morgenstern's axioms. Furthermore, the form of a utility function may imply that certain, apparently unrelated, rules for choice under uncertainty maximize expected utility. For example, if the individual's utility function is linearly dependent upon his money gain, then the maximization of expected gain also maximizes expected utility. If the utility function is quadratic, then preferences based on expected gain and the variance of gain (provided that these preferences take a particular form) maximize expected utility. But, at this stage, it is impossible to explore these matters. The Characteristics Approach and Other Developments: A limitation of traditional demand theory is its inability to predict the demand for new or differentiated products. This stems principally from its specification of preferences in terms of products rather than the intrinsic properties of products. Recently, economists such as Lancaster have been exploring the alternative of basing demand theory upon the demand for the properties inherent in products. The difficulties involved in this approach need no emphasis. For example, how do we define and measure a relevant characteristic? But it is not always easy to define a product either.

According to the N-M theory, if the consumer satisfyies certain crucial axioms like complete ordering, continuity, independence, Unequal probability and complexity", then this utility function can be derived by presenting him with a series of choices between a certain outcome on the one hand and a probabilistic combination of two uncertain outcomes on the other. The utility function thus derived is unique up to linear transformation and provides a ranking of alternatives in situations that do not involve risk. A consumer maximizes expected utility, and these utilities are cardinal in the sense that they can be combined to compute expected utilities and can be used to compare differences in utilities. The expected utility so calculated can be used to determine the consumer's choice and demand decisions in situations involving risk.

3.4 THE AXIOMS

It is possible to construct a utility index of a consumer which can be used to predict choice in uncertain situations provided the following axioms are observed

a) The axiom of Complete-ordering: According to the axiom for the two alternatives A and B one of the following must be true: the consumer prefers A to B, she prefers B to A, or she is indifferent between them. The consumer's evaluation of alternatives is transitive: if she prefers A to B and B to C, she prefers A to C.

b) The axiom of Continuity : The axiom assumes that A is preferred to B and B is preferred to C. The axiom asserts that there exists some probability P,0 <P<1, such that the consumer is indifferent between outcome B with certainty and a lottery ticket (P,A,C).

c) The axiom of Independence: The axiom assumes that the consumer is indifferent between A and B and that C is any outcome whatever. If one lottery ticket L_1 offers outcomes A and C with probabilities P and 1 – P respectively and another L2 the outcomes B and C with the same probabilities P and 1 –P, the consumer is indifferent between the two lottery tickets. Similarly, if she prefers A to B, she will prefer L1 to L_2.

The axiom of Unequal- Probability: The axiom assumes that the consumer prefers A to B. Let L_1 = (P_1, A,B) and L_2=(P_2,A,B). The consumer will prefer L_2 to L_1 if and only if $P_2>P_1$.

Compound-lottery axiom. Let L_1=(P_1,A,B), and L_2=(P_2,L_3, L_4) where L_3=(3,A,B) and L_4= (P_4,A,B), be a compound lottery in which the prizes are lottery tickets.L_2 is equivalent to L_1 if $P_1 = P_2P_3+(1- P_2)P4$. Given L_2 the probability of obtaining L_3 is P_2. Consequently, the probability of obtaining A through L_2 is P_2P_3. Similarly, the probability of obtaining L_4 is (1 - P_2), and the probability of obtaining A through L_4 is (1 - P_2)P_4. The probability of obtaining. A with L_2 is the sum of the two probabilities. The consumer evaluates lottery tickets only in terms of the probabilities of obtaining the prizes, and not in terms of how many times he is exposed to a chance mechanism.

A utility index explains the preference of an individual. It shows the expected utility of different outcomes. The Neumann and Morgenstern method of measuring utility is as follows:

"Consider three events, C, A and B for which the order of individual's preferences is the one stated. Let α be a real number which lies between 0 and 1, such that A is exactly equally desirable with the combined event consisting of a chance of probability (1 –α) for B and the remaining chance of probability α for C. Then we suggest the use of α as a numerical estimate for the ratio of the preference of A over B to that of C over B."

We may put this method in the form of a mathematical formula:

$$A= B (1-\alpha + \alpha C)$$

Where α denotes the probability of the event occurring. If P is substituted in the place of α, the above equation may be written as A = B (1-P) + PC.

If the preferences (choices) are given, we may construct a cardinal utility index with the help of the above formula.

Suppose there are the three situations are C,A and B respectively. Let us take that the outcomes B and C are uncertain while A is certain. Then the consumer while making the choice will use the utility derived from these two events jointly; that is, if P is the probability of occurring C and (1-P) is the probability of B then

$$PU_c = (1-P) U_b.$$

$U_{(a)}$, $U_{(b)}$, $U_{(c)}$ are the respective utility for events A,B and C. By applying the respective utilities to the above events the formula becomes $U_{(a)} = PUc + (1-P)U_b$.

i) In order to construct a utility index based on the Neumann and Morgenstern equation, we have to assign utility values to c and b. These utility values are arbitrary except for the fact that higher value should be assigned to preferred event(lottery). Suppose we assign the following artitrary utility values Uc = 100 utils, Ub =0 util, and P = 4/5 or 8, then

$$Ua = (4/5)\ 100 + (1 - 4/5\ (0)$$

$$= 80 + (1/5)\ (0)$$

$$= 80$$

ii) In order to construct a utility index on this N-M equation, we have to assign utility values to any two events.

Suppose the utility of C=100 utils and of B = 10 utils and the value of P(probability) is 0.20 then U(A) = 0.20 (100) + 1-0.20) 10 =28 utils.

Proceeding with this, we can find utility values for U_a, U_b, U_c etc., for all possible combinations starting from two arbitrary situations involving probabilities or risks. Consumer choices can thus be predicted by looking at the utility index numbers.

The average yield from a risky commodity is summarized in the expected value of the return. The expected value of a risky commodity is the probability of each outcome multiplied by the payoff of the respective outcomes, summed over all the outcomes. The expected value of the payoff of the lottery ticket above is Rs.5 (1/100 xRs.500 + 99/100 x Rs.0). If the price of the lottery ticket was exactly Rs.5, the lottery would be called fair. A fair lottery, therefore, is one where the expected value of the payoff is equal to the price of the ticket. But the expected value of a lottery ticket is nearly always less than the price of the ticket. The difference between the expected value and the price of the ticket is the source of profit to those running the lottery.

Would a rational consumer every buy a lottery ticket? Clearly the purchase of a lottery ticket is rational only if the utility from the payoff exceeds the utility given up in paying for the ticket. If the lottery is fair, the price of the ticket and the expected value of the lottery are equal. If there are, do the price of the ticket and the expected payoff yield the same utility? The answer depends on the marginal utility of money. If the marginal utility of money for a consumer is constant – that is, the utility of the first rupee is the same as the utility of the one hundred thousandth rupee- then the utility of the ticket price and the expected utility of the payoff will be the same . A consumer with a constant marginal utility of money will be indifferent between buying or not buying a fair lottery ticket. Such a consumer would rationally never buy a lottery ticket whose price exceeded its expect value.

Why a consumer for whom the marginal utility of money diminishes, and who is rational, would not buy a ticket in a fair lottery, or make any kind of fair bet. Suppose you have Rs.1000 and can toss a coin to win or lose Rs.100. If you win you will have Rs.1,100 and the gain of utility from Rs.100 added to Rs.1000. If you lose you will have Rs.900 and the loss of utility subtracted from Rs.1000. With diminishing marginal utility the gain of utility is smaller than the loss, even though the amounts of money are equal. The marginal utility of money as a stock of wealth. Only if the marginal utility of money increases at higher incomes will a rational consumer favour the fair lottery. That is, the fari lottery is a good buy for a consumer might buy a lottery ticket even when the price exceeds the expected value of the payoff, provided of course that there could be a gain of utility.

The consumer's evaluation of the lottery ticket can be stated more exactly by defining the expected value of the utility of the payoff, or the expected utility. Suppose that a zero payoff is worth zero utils; that when considering Rs.5, an extra rupee is worth ten utils; and that when considering Rs.500, an extra rupee is worth twelve utils. The expected utility other lottery ticket is 99/100 x 0 = 1/100 x Rs.500 x 12 utils = 60 utils. The lottery ticket costs Rs.5 x 10 utils = 50 utils. The lottery ticket is therefore a good buy for this consumer when the expected utility of the payoff exceeds the utility of the ticket, because the marginal utility of money increases for this consumer.

Suppose a consumer (as an individual or a family) behaves as if he had a consistent set of preferences, these preference can be expressed in terms of utility and the object of the consumer is to maximise his utility. Then he will choose according to a scale of preferences. He can say of the various alternatives which one he prefers or if he is indifference between them. An alternative may be a combination with different probabilities. A and B may constitute a combination in which 40-percent chance of A and 60 percent chance of B will be alternative.

If A is preferred to B, A plus C is preferred to B plus C (if B equals C in probability) .Conversely if A plus C is preferred to B plus C (if B equals C in probability) A is preferred to B.If A is preferred to B and B is preferred to C the consumer will be indifferent between A plus C and B if there is certain probability of A combined with a certain probability of C.

There is no difference between indifference curves for riskless choices and the above.

Suppose for simplicity the alternatives are expressed as money incomes. If $I_{1,}$ I_2 and I_3 are alternative incomes the consumer chooses the one which has the greatest utility, that is the largest income. This means that the higher the income the larger the utility. This is true of riskless choices. If, however risk is involved there can only be a probability in regard to each. If the chance of getting I_3 is a the chance of getting I_2 is I-a But I may be a certainty.

First we examine the different views or preferences toward risk of different individuals and then use this information to examine consumers" choices in the face of risk. We will see that in making choices under risk or certainty the consumer maximizes utility or satisfaction. When risk or uncertainty is present, the consumer maximizes expected utility.

3.5 DIFFERENT PREFERENCES TO RISK

Most individuals, faced with two alternative investments of equal expected value or profit, but differently standard deviation or risk, will generally prefer the less risky investment (i.e., the one with the smaller standard deviation). That is, most individuals seek to minimize risks or are risk averters. Some individuals, however, may very well choose the more risky investment (i.e., are risk seekers or risk lovers), while still others may be indifferent to risk (i.e., are risk neutral). The reason is that different individuals have different preferences toward risk. Most individuals are risk averters because they face diminishing marginal utility of money. The meaning of diminishing, constant, and increasing marginal utility of money can be explained with the aid of Fig 3.3.

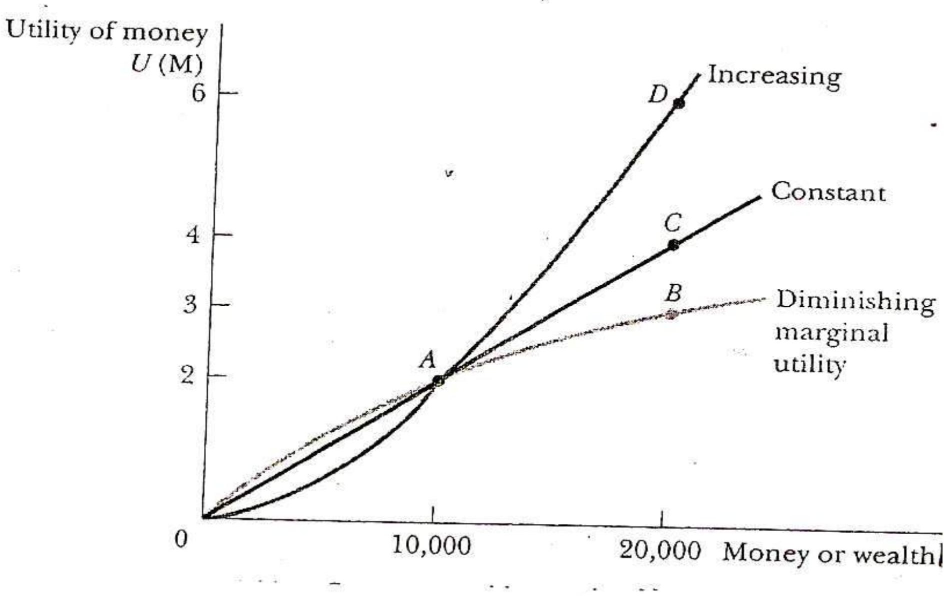


Fig.3.3

Fig.3.3 Diminishing, Constant, and Increasing Marginal Uitility of Money .A Rs.10,000 money or wealth provides 2 utils of utility to a particular individual (point A), while Rs.20,000 provides 3 utils(point B) if the total utility of money curve of the individual is concave or faces down(so that the marginal utility of money declines), 4 uitls (point C) if the total utility curve is straight line(so that the marginal utility is constant, and 6utils (point D) if the total utility curve is convex or faces up (so that marginal utility increases). The individual would then be, respectively, a risk averter, risk neutral or a risk seeker.

In fig3.3 money income or wealth is measured along the horizontal axis while the utility or satisfaction of money (measured in utils) is plotted along the vertical axis. From the figure, we can see that Rs.10,000 in money or wealth provides 2 utils of utility to a particular individual (point A), while Rs.20,000 provides 3 utils (point B), 4 utils (point C), or 6Utils (point D), respectively, depending on the total utility of money curve for this individual being concave or facing down, a straight line, or convex (facing up).

The total utility curve is concave or faces down, doubling the individual's income or wealth from Rs.10,000 to Rs.20,000 only increases his or her utility from 2 to 3 utils, so that the marginal utility of money (the slope of the total utility curve) diminishes for this individual. If the total utility of money curve is a straight line, doubling income also doubles utility, so that the marginal utility of money is constant. Finally, if the total utility of money curve is convex or faces up, doubling income more than doubles utility, so that the marginal utility of money income increases.

Most individuals are risk averters and face diminishing marginal utility of money(i.e.,. their total utility curve is concave or faces down –see Fig.3.3.). To see why this is so, consider an offer to engage in a bet to win Rs.10,000 if "head" turns up in the tossing of a coin or to lose Rs.10,000 if "tail" comes up. Since the probability of a head or a tail is 0.5 or 50% and the amount of the win or loss is Rs.10,000, the expected value of the money won or lost from the gamble is

0.5(Rs.10,000) + 0.5 (-Rs.10,000) = 0

Even though the expected value of such a fair game is zero, a risk averter (as an individual facing diminishing marginal utility of money) would gain less utility by winning Rs.10,000 than he or she would lose by losing Rs.10,000. Starting from point A in Figure 3. we see that by loosing Rs.10,000, the risk-averting individual loses 2 utils of utility if he or she losesRs.10,000 but gains only 1 utility of utility if he or she winsRs.10,000. Even though the bet is fair(i.e., there is a 50 -50 chance or winning or losing Rs.10,000 the expected utlity of the bet is negative. That is,

Expected utility = E(U) = 0.5 (1util) + 0.5(- 2 utils) = -0.5

In such case, the individual will refuse a fair bet.* From this, we can conclude that a risk averting individual will not necessarily accept an investment with positive expected monetary value. To determine whether or not the individual would undertake the investment, we need to know his or her utility function of money or income.

Risk averse individual marginal utility of money diminishes as he has more money, while for a risk-seeker marginal utility of money increases as money with him increases. In case of risk-neutral individual marginal utility of money remains constant as he has more money.

*With constant utlity, E(U) = 0.5 (2utils) + 0.5 (- 2 utils) = 0 and the individual is risk neutral and indifferent to the bet. With increasing marginal utility, E(U) = 0.5(4utils) + 0.5 (-2utils) = 1 and the individual is a risk seeker and would accept the bet.

3.6 MAXIMIZING EXPECTED UTILITY

To determine whether or not an individual should undertake an investment, he or she needs to determine the expected utility of the investment. For example, suppose that an investment has a 40

% probability of providing profit of Rs.20,000 and a 60% probability of resulting in a loss of Rs.10,000. Since the expected monetary return of such a project is positive (see Table:3.1), a risk-neutral or a risk-seeking individual would undertake the project. However, if the individual is risk averse (the usual case) and his or her utility function is as indicated from the4 investment is negative (see table1+).

Thus, even if the expected monetary return is positive, a risk-averse manager will not make the investment if the expected utility of the investment is negative. The general rule is that the individual seeks to maximize utility in a world of no risk or uncertainty, but maximizes expected utility in the face of risk*. Need less to say, even

Table 3.1 Expected Return from the Investment

States of Nature	**Probability (1)**	**Monetary Outcome (2)**	**Expected Return (1) X (2)**
Success	0.40	Rs. 20,000	Rs. 8,000
Failure	0.60	- Rs. 10,000	-6,000 Rs. 2,000

The Utility Function of a Risk – Averse Individual > An investment with a 40 % probability of providing a return of Rs.20,000 (3utils of utility) and a 60% probability of resulting in a loss of Rs.10,000 (-4utils of utility) has an expected utility of (0.4)(3utils) + (0.6) (-4utils) = -12 utils, and it would not be made by the individual.

*only for a risk –neutral individual does maximizing the expected monetary value or return correspond to maximizing expected utility.

Table 3.2 Exptected Utility from the Investment

States of Nature	Probability (1)	Monetary outcome (2)	Associated Utility (3)	Expected Return (4) (1 x 3)
Success	0.40	Rs.20,000	3	1.2
Failure	0.60	Rs.10,000	-4	-2.4

Expected Utility -1.2

different risk-averse individuals have different utility functions and face different marginal utilities of money, and so even they can reach different conclusions with regard to the same investment. Being risk averse, it would seem irrational for most individuals to engage in gambling.

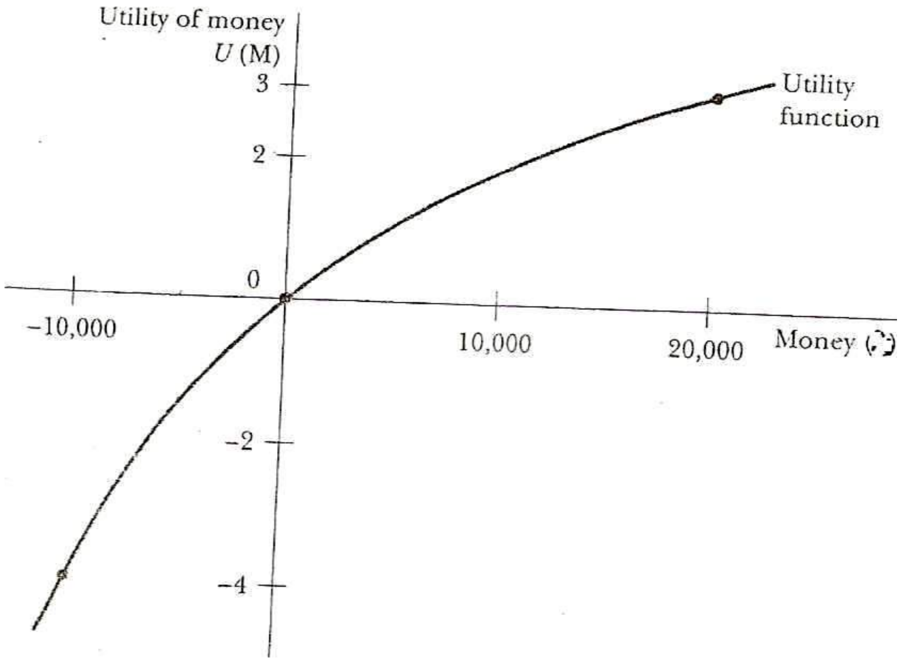


Fig3.4

Summing up, a consumer behaves as if,

1. He had a consistent set of preferences.
2. The preferences can be expressed in terms of utility.
3. When risk is absent, the alternative with the highest utility is preferred.
4. When risk is present, the alternative is preferred for which the expected Utility is the highest and
5. It is assumed that utility rises with income the marginal utility of money income is positive.

3.7 SUMMARY

Modern utility theory deals with choices subject to risk. Rational decisions look to expected utility, not expected money value, when risks are present. The Neumann-Morgenstern method of measuring the utility of money to a person is to find the probabilities the person will accept in deciding whether to put a sum of money to risk, as in a gamble. If the person insists on favourable odds, then for this person the marginal utility of money diminishes. If even odds are accepted, the marginal utility of money is constant, at least over some range. And if a person willingly accepts unfavourable odds, the marginal utility of money increases, over some range. N-M cardinal utility is not identical with the older neo-classical cardinal utility. The method does not measure the strength of feelings toward goods and services. All that the N-M method can do is to illuminate the actions of a person making choices in the face of risk. But by opening up new possibilities of measurement, N-M have given new strength to the older idea of neo classical cardinal utility.

3.8 IMPORTANT CONCEPTS

Risk: The risk refers to a situation when the outcome of a decision is uncertain but when the probability of each possible outcome is known or can be estimated.

Uncertainty: The uncertainty refers to the situation when there is more than one possible outcome of a decision but where the probability of occurence of each particular outcome is not known or even cannot be estimated.

Risk Aversion: A person who prefers a certain given income to a risky job with the same expected income is called risk averter or risk-averse.

Risk Lover; A person is risk preference or risk loving who prefers a risky outcome with the same expected income as a certain income.

Risk Neutral: A person is called risk neutral, if he is indifferent between a certain given income and an uncertain income with the same expected value.

3.9 SUGGESTED READINGS

D. Bernoulli, Exposition of a New Theory on the Measurement of Risk, Econometrica, Jan.1954,pp.23-36.

Dominick Salvatore: Micro Economics Theory and Application, Oxford University Press, 2003.

J.M. Henderson and R.E.Quandt , “Micro Economic theory”(A mathematical approach)(2nd ed).

Von Neumann,J and Oskar Morgenstern, Theory of Games and Economic Behaviour, 2nd ed., Princeton University Press, Princeton, 1947.

3.10 QUESTIONS

1. Discuss the part played by risk and uncertainty in economic life.
2. How do you explain classical and neo-classical thinkers explain risky choices in economic life?
3. Analyse the Neumann-Morgenstern approach to choice under conditions of uncertainty and the maximization of utility.

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4

TECHNOLOGY OF PRODUCTION AND PRODUCTION FUNCTION

UNIT STRUCTURE

4.0 OBJECTIVES

After going through this module you will come to know the concepts, like –

- Technology of production,
- Specification of technology,
- Input Requirement Set and production function,

- Convex Technology
- Leontief – Technology
- Technical Rate of substitution (TRS)
- Elasticity of Substitution
- Returns to Scale (Long-Run Production Function)
- Efficient Production
- Homogeneous Production Function
- Homothetic production Function
- The CES Production Function

4.1 INTRODUCTION

The simplest and the most common way to describe the technology of a firm is the production function, which is generally studied in intermediate courses. However, there are other ways to describe firm technologies that are both more general and more useful. We will discuss several of these ways to represent firm production possibilities in this unit, along with ways to describe economicaly relevant aspects of a firm's technology.

4.2 TECHNOLOGY OF PRODUCTION

A firm produces outputs from various combinations of inputs. In order to study firm choices we need a convenient way to summarise the production possibilities of the firm, i.e., which combinations of inputs and outputs are technologically feasible.

A certain amount of inputs are used to produce certain amount of outputs per unit time period. We may also want to distinguish inputs and outputs by the calendar time in which they are available, the location in which they are available, and even the circumstances under which they become available. By defining the inputs and outputs with regard to when and where they are available, we can capture certain aspects of the temporal or spatial nature of production.

The level of detail that we will use in specifying inputs and outputs will depend on the problem at hand, but we should remain aware of the fact that a particular input or output good can be specified in arbitrarily fine detail.

4.2.1 SPECIFICATION OF TECHNOLOGY:

Suppose the firm has 'n' possible goods to serve as inputs and /or outputs. If a firm uses y_j^i units of a good j as an input and produces y_j^o of the good as an output, then the net output of good j is given by $y_j = y_j^o - y_j^i$. If the net output of good j is positive, then the firm is producing more of good j than it uses as inputs; if the net output is negative, then the firm is using more of good j than it produces.

A production plan is simply a list of net outputs of various goods. We can represent a production plan by a vector y in R^n where y_j is negative if the j^{th} good serve as a net input and positive if the j^{th} good serve as a net output. The set of all technologically feasible production plans is called the firm's production- possibilities set and will be denoted by Y, a subset of R^n. The set Y gives us a complete description of the technological possibilities facing the firm.

When we study the behaviour of a firm in certain economic environments, we may want to distinguish between production plans that are "immediately feasible" and those that are "eventually feasible". For example, in the short run, some inputs of the firms are fixed so that only production plans compatible with these fixed factors are possible. In the log run, such factors may be variable so that the firm's technological possibilities may well change.

We will generally assume that such restrictions can be described by some vector z in R^n. For example, z could be a list of maximum amount of the various inputs and outputs that can be produced in the time period under consideration. The restricted or short-run production possibilities set will be denoted by Y(z); this consists of all feasible net output bundles consistent with the constraint level z.

4.2.2 INPUT REQUIREMENT SET:-

Suppose we are considering a firm that produces only one output. In this case we write the net output bundle as (y,-x) where x is vector of inputs that can produce y units of output. We can then define a special case of a restricted production possibilities set, i.e., the input requirement set, as-

$$v(y) = \left\{ x \text{ in } R^n : (y, -x) \text{ is in } Y \right\}$$

The input requirement set is the set of all input bundles that produce at least y nits of outputs.

Here the input requirement set measures inputs as positive numbers rather than negative.

4.2.2 (i) ISOQUANT

The isoquant gives all input bundles that produce exactly y units of output. In other words, an isoquant is the combination of all inputs that produce same level of output i.e., y.
An isoquant can also be defined as:

$$Q(y) = \left\{ x \text{ in } R^n : x \text{ is in } V(y) \text{ and } x \text{ is not in - } V(y') \text{ for } y' > y \right\}$$

4.2.2 (ii) SHORT-RUN PRODUCTION POSSIBILITY SET

Suppose a firm produces some output from labour and capital. Production plans then look like $(y, -\ell, -k)$ where y is the level of output, ℓ the amounts of labour input, and k the amount of capital input. We know that the labour can be varied immediately in the short run but the capital remains fixed at the level $\bar{k}$. Then the short-run production possibility set can be expressed as –

$$Y(\bar{k}) = \left\{ (y, -\ell, -k) \text{ in } Y : k = \bar{k} \right\}$$

4.2.2 (iii) PRODUCTION FUNCTION

The production function for a firm which has only one output can be defined as –

$$f(x) = \{ y \text{ in } R : y \text{ is the maximum output} - \text{associated with} - x \text{ in } y \}$$

4.2.2 (iv) TRANSFORMATION FUNCTION

A production plan y in Y is technologically efficient if there is no y' in Y such that $y' \geq y$ and $y' \neq y$; in other words, a production plan is efficient if there is no other way to produce more output with the same inputs or to produce the same output with less inputs.

The set of technologically efficient production plans can be described by a transformation function:

$$T : R^n \to R$$

Where T(y)=0 if and only if y is efficient. The transformation function gives the maximal vectors of net outputs.

4.2.3 COBB-DOUGLAS TECHNOLOGY

Let ‘a’ be a parameter such that $0 < a < 1$. Then the Cobb-Douglas technology can be defined as –

1. Production possibility set -

$$Y = \left\{(y, -x_1, -x_2) \text{ in } R^3 : y \le x_1^a \, x_2^{1-a}\right\}$$

2. Input requirement set –

$$V(y) = \left\{(x_1, x_2) \text{ in } R^2 : y \le x_1^a \, x_2^{1-a}\right\}$$

3. Isoquant

$$Q(y) = \left\{(x_1, x_2) \text{ in } R^2 : y = x_1^a \, x_2^{1-a}\right\}$$

4. Short-run production possibility set –

$$Y(z) = \left\{(y, -x_1, -x_2) \text{ in } R^3 : y \le x_1^a \, x_2^{1-a}, x_2 = z\right\}$$

5. Transformation function –

$$T(y, x_1, x_2) = y - x_1^a \, x_2^{1-a}$$

6. Production function –

$$f(x_1, x_2) = x_1^a \, x_2^{1-a}$$

4.2.4 LEONTIEF TECHNOLOGY

Let a>o and b>o be parameters. Then the Leontief Technology can be defined as –

1. Production possibility set -

$$Y = \left\{(y, -x_1, -x_2) \text{ in } R^3 : y \le \min(ax_1, bx_2)\right\}$$

2. Input requirement set –

$$V(y) = \left\{(x_1, x_2) \text{ in } R^2 : y \le \min(ax_1, bx_2)\right\}$$

3. Isoquant

$$Q(y) = \left\{(x_1, x_2) \text{ in } R^2 : y = \min(ax_1, bx_2)\right\}$$

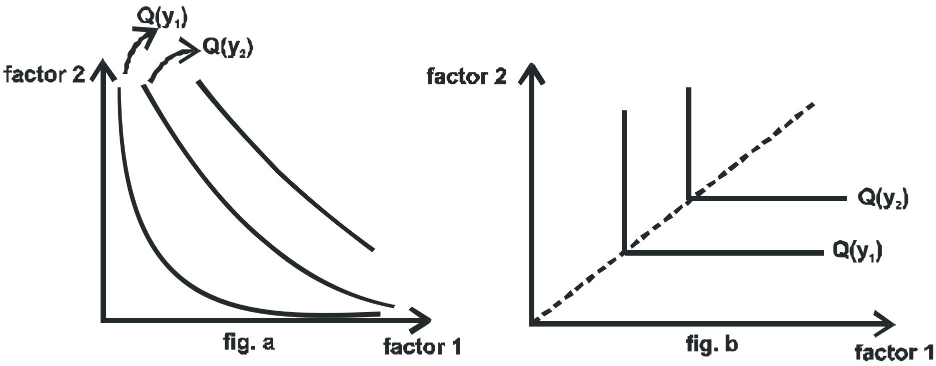
4. Transformation function –

$$T(y, x_1, x_2) = y - \min(ax_1, bx_2)$$

5. Production function –

$$f(x_1, x_2) = \min(ax_1, bx_2)$$

The general shape of Cobb-Douglas and Leontief technology can be depicted diagrammatically as in the figures (a) and (b) respectively.



4.3 ACTIVITY ANALYSIS

The most straightforward way of describing production sets or input requirement sets is simply to list the feasible production plans. For example, suppose that we can produce an output good using factor inputs 1 and 2. There are two different activities or technologies by which this production can take place.

Technique A: One unit of factor 1 and two units of factor 2 produces one unit of output.

Technique B: Two units of factor 1 and one unit of factor 2 produces one unit of output.

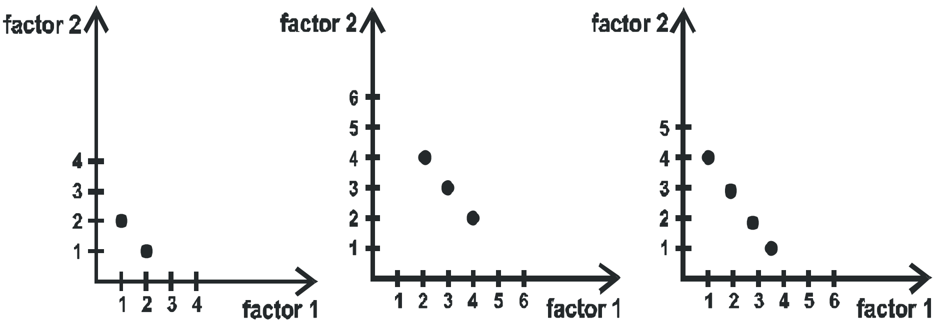
Let the output be good 1; and factors be goods 2 and 3. Then we can represent the production possibilities implied by these two activities by the production set –

$$Y = \{(1,-1,-2),(1,-2,-1)\}$$

or the input requirement set –

$$V(1) = \{(1,2),(2,1)\}$$

This input requirement set is depicted in the **figure 4.2(A).**



It may be the case that to produce y units of output we could just use y times as much of each input for y=1,2, …. In this case one might think that the set of feasible way to produce y units of output would be given by

$$V(y)=\{(y,2y),(2y,y)\}$$

However, this set does not include all the relevant possibilities. It is true that (y, 2y) will produce y units of output if we use technique A and that (2y, y) will produce y units of output if we use technique B- But what if we use a mixture of technique A & B.

In this case we have to let y_A be the amount of output produced using technique A and y_{AB} be the amount produced using technique B. The $V(y)$ will be given by the set –

$$V(y)=\{(y_A+2y_B, y_B+2y_A): y=y_A+y_B\}$$

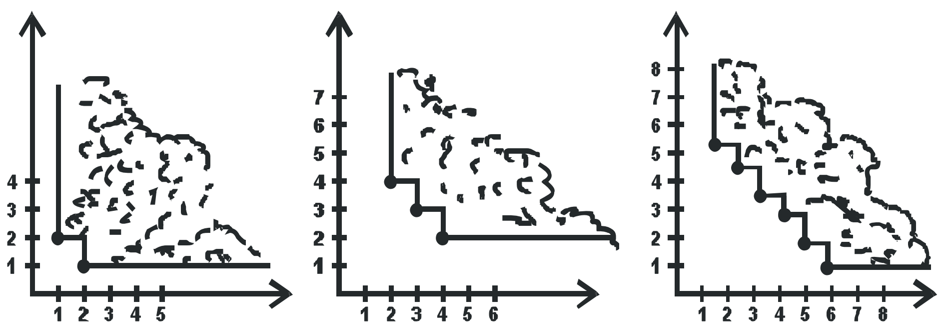
So, for example, $V(2)=\{(2,4),(4,2),(3,3)\}$. Both $V(y) \,\&\, V(2)$ are depicted in the above figures.

4.4 MONOTONIC TECHNOLOGY

Suppose that we had an input vector (3,2). Is this sufficient to produce one unit of output? We may argue that since we could dispose of 2 units of factor 1 and be left with (1,2), it would indeed be possible to produce 1 unit of output from the inputs (3,2). Thus, if such free disposal is allowed, it is reasonable to argue that if x is feasible way to produce y units of output and x' is an input vector with at least as much of each input, then x' should be a feasible way to produce y. Thus, the input requirement set should be monotonic in the following sense.

Monotonicity: x is in $V(y)$ and $x' \geq x$ is in $V(y)$.

If we assume monotonicity, then the input requirement sets depicted in figure 4.2 become the sets depicted in figure 4.3.



This assumption of monotonicity is often an appropriate assumption for production sets as well. In this context we generally want to assume that if y is in Y and $y' \le y$, then y' must also be in Y. That is to say that, if y in Y is feasible then y' in Y is also feasible.

4.5 CONVEX TECHNOLOGY

Let us now consider what the input requirement set looks like if we want to produce 100 units of output. As a first step we might argue that if we multiply the vectors (1,2) and (2,1) by 100, we should be able just to replicate what we were doing before and thereby produce 100 times as much. It is clear that not all production processes will necessarily allow for this kind of replication, but it seems to be plausible in many circumstances.

If such replication is possible, then we can conclude that (100, 200) and (200, 100) are in V(100). Are there any other possible ways to produce 100 units of output? Well we could operate 50 processes of technique I and 50 process of activity II. This would use 150 units of good 1 and 150 units of good 2 to produce 100 units of output; hence (150 ,150) should be in the input requirement set. Similarly, we could operate 25 process of activity I and 75 processes of activity II. This implies that

$\bullet 25(100,200) + \bullet 75(200,100) = (175,125)$ should be in V(100). More generally,
$t(100,200) + (1-t)(200,100) = (100t + 200\,(1-t), -200t + (1-t)100)$
Should be V(100) for t = 0, .01, .02 ….

We might as well make the obvious approximation here and let t take on any fractional value between 0 and 1. This leads to a production set of the form depicted in figure 2.4 A. Thus,

Convexity: If x and x' are in V(y), then $tx + (1-t)x'$ is in V(y), for all $o \le t \le 1$. That is, V(y) is a <u>Convex set</u>.

We applied the arguments given above to the input requirement sets, but similar arguments apply to the production sets. It is common to assume that if y and y' are both in Y, then $ty + (1-t)y'$ is also in Y for $o \le t \le 1$; in other words Y is a convex set.

Now we will describe a few of the relationships between the convexity of V(y) and the convexity of Y.

Convex production set implies convex input requirement set. i.e., if the production set Y is a convex set, then the associated input requirement set, V(y), is a convex set.

Convex input requirement set is equivalent to quasiconcave production function. V(y) is a convex set if and only if the production function f(x) is a quasiconcave function.

4.6 REGULAR TECHNOLOGY

Finally, we will consider a weak regularity condition concerning V(y)

V(y) is a closed, nonempty set for all $y \geq o$

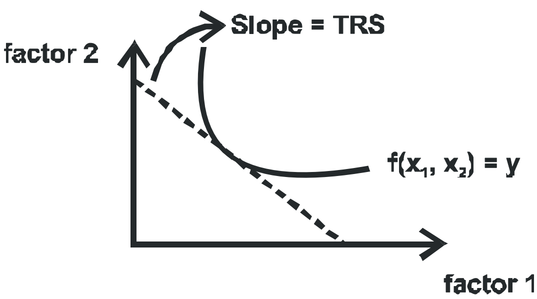
The assumption that V(y) is nonempty requires that there is some conceivable way to produce any given level of output. This is simply to avoid qualifying statements by phrases like "assuming that y can be produced"

The assumption that V(y) is closed is made for technical reasons and is innocuous in most contexts. Roughly speaking, the input requirement set must include its own boundary.

4.7 THE TECHNICAL RATE OF SUBSTITUTION

Assume that we have some technology summarized by a smooth production function and that we are producing at a particular point $y^* = f(x_1^*, x_2^*)$. Suppose that we want to increase the amount of input 1 and decrease the amount of input 2 so as to maintain a constant level of output. How can we determine this technical rate of substitution between these two factors?

In the two dimensional case, the technical rate of substitution is just the slope of an isoquant; how one has to adjust x_2 to keep output constant when x_1 changes by a small amount, as depicted in figure 4.4



In the 'n' –dimensional case, the technical rate of substitution is the slope of an isoquant surface, measured in a particular direction.

Let $x_2(x_1)$ be the (implicit) function that tells us how much of x_2 it takes to produce y if we are taking x_1 units of the other input. Then by definition, the function $x_2(x_1)$ has to satisfy the following identity - $f(x_1, x_2(x_1)) \equiv y$

Actually, we require an expression for - $\partial x_2(x_1^*)/\partial x_1$

Then, differentiating the above identity, we get –

$$\frac{\partial f(x^*)}{\partial x_1} + \frac{\partial f(x^*)}{\partial x_2} \bullet \frac{\partial x_2(x_1^*)}{\partial x_1} = 0$$

$$\frac{\partial x_2(x_1^*)}{\partial x_1} = -\frac{\partial f(x^*)/\partial x_1}{\partial f(x^*)/\partial x_2}$$

This gives us an explicit expression for the technical rate of substitution.

Here is the another way to derive the technical rate of substitution. Think of a vector of small changes in the input levels which we write as $dx = (dx_1, dx_2)$. The associated changes in the output is approximated by $dy = \frac{\partial f}{\partial x_1} dx_1 + \frac{\partial f}{\partial x_2} dx_2$ this expression is known as the total differential of the function f(x). Consider a particular change in which only factor 1 and factor 2 changes, and the change is such that output remains constant. That is dx_1 and dx_2 adjust "along an isoquant".

Since output remains constant, we have

$$o = \frac{\partial f}{\partial x_1} dx_1 + \frac{\partial f}{\partial x_2} dx_2,$$

which can be solved for -

$$\frac{dx_2}{dx_1} = -\frac{\partial f/\partial x_1}{\partial f/\partial x_2}$$

Either the implicit function method or the total differential method may be used to calculate the technical rate of substitution.

4.8 TRS FOR A COBB-DOUGLAS TECHNOLOGY

Given that $f(x_1, x_2) = x_1^a x_2^{1-a}$, we can take the derivatives to find -

$$\frac{\partial f(x)}{\partial x_1} = a x_1^{a-1}\ x_2^{1-a} = a\left[\frac{x_2}{x_1}\right]^{1-a}$$

$$\frac{\partial f(x)}{\partial x_2} = (1-a) x_1^{a}\ x_2^{-a} = (1-a)\left[\frac{x_1}{x_2}\right]^{a}$$

It follows that,

$$\frac{\partial x_2(x_1)}{\partial x_1} = \frac{\partial f / \partial x_1}{\partial f / \partial x_2} = -\frac{a}{1-a}\frac{x_2}{x_1}$$

4.9 THE ELASTICITY OF SUBSTITUTION

The technical rate of substitution measures the slope of an isoquant. The elasticity of substitution measures the curvature of an isoquant. More specifically, the elasticity of substitution measures the percentage change in the factor ratio divided by the percentage change in the TRS, with output being held fixed.

If we let $\Delta(x_2 / x_1)$ be the change in the factor ratio and $\Delta\,\text{TRS}$ be the change in the technical rate of substitution, then the elasticity of substitution denoted by $'\sigma'$ can be given as –

$$\sigma = \frac{\dfrac{\Delta(x_2 / x_1)}{x_2 / x_1}}{\dfrac{\Delta \text{TRS}}{\text{TRS}}}$$

The elasticity of substitution, which is a relatively natural measure of curvature, asks how the ratio of factor inputs changes as the slope of the isoquant changes. If a small change in slope gives us large change in factor input ratio, then the isoquant is relatively flat which means that the elasticity of substitution is large.

In practice we think of the percentage change as being very small and take the limit of this expression as Δ goes to zero. Then, the expression for σ becomes –

$$\sigma = \frac{TRS}{(x_2/x_1)} \frac{d(x_2/x_1)}{d\,TRS}$$

It is often convenient to calculate σ using the logarithmic derivative. In general, if y=g(x), the elasticity of y with respect to x refers to the percentage change in y induced by a small percentage change in x.

That is, $\in = \dfrac{\frac{dy}{y}}{\frac{dx}{x}} = \dfrac{dy}{dx}\dfrac{x}{y}$

Provided that x and y are positive, this derivative can be written as

$$\in = \frac{d\ln y}{d\ln x}$$

To prove this, note that by the chain rule $\dfrac{d\ln y}{d\ln x}\dfrac{d\ln x}{dx} = \dfrac{d\ln y}{dx}$

Carrying out the calculations on the left-hand and right-hand side of the equals sign, we have –

$$\frac{d\ln y}{d\ln x}\frac{1}{x} = \frac{1}{y}\frac{dy}{dx}$$

$$\frac{d\ln y}{d\ln x} = \frac{x}{y}\frac{dy}{dx}$$

Alternatively we can use total differential to write –

$$d\ln y = \frac{1}{y}dy$$

$$d\ln x = \frac{1}{x}dx,$$

So that,

$$\in = \frac{d\ln y}{d\ln x} = \frac{dy}{dx}\frac{x}{y}$$

Applying this to the elasticity of substitution, we can write –

$$\sigma = \frac{d\ln(x_2/x_1)}{d\ln|TRS|}$$

Here, it should be noted that the absolute value sign in the denominator is to convert the TRS to a positive number so that the logarithm makes sense.

The Elasticity of Substitution for the Cobb-Douglas Production Function:

We have seen above that –

$$TRS = -\frac{a}{1-a}\frac{x_2}{x_1}$$

or

$$\frac{x_2}{x_1} = -\frac{1-a}{a}TRS$$

It follows that,

$$\ln\frac{x_2}{x_1} = \ln\frac{1-a}{a} + \ln|TRS|$$

This in turn implies –

$$\sigma = \frac{d\ln(x_2/x_2)}{d\ln|TRS|} = 1$$

Hence, it is clear from the above expression that the elasticity of substitution for the Cobb-Douglas production function is equal to one.

4.10 RETURNS TO SCALE AND EFFICIENT PRODUCTION

Suppose that we are using some vector of inputs x to produce some output y and we decide to scale all inputs up or down by some amount $t \geq o$. What will happen to the level of output?

In the case we described earlier, where we wanted only to scale output up by some amount, we typically assumed that we could simply replicate what we were doing before and thereby produce 't' times as much output as before. If this sort of scaling is always possible, we will say that the technology exhibits constant returns to scale. More formally, a technology is said to exhibit constant returns to scale if any of the following are satisfied.

(1) y in Y implies ty is in Y, for all $t \geq o$;

(2) x in V(y) implies tx is in V(ty), for all $t \geq o$;

(3) $f(tx) = tf(x)$ for all $t \geq o$; i.e., the production function $f(x)$ is homogeneous of degree 1.

The replication argument given above indicates that constant returns to scale is often a reasonable assumption to make about technologies. However, there are situations where it is not a plausible assumption.

One circumstance where constant returns to scale may be violated is when we try to "subdivide" a production process. Even if it is always possible to scale operations up by integer amounts, it may not be possible to scale operations down in the same way.

Another circumstance where the constant returns to scale may be violated is when we want to scale operations up by non-integer amounts. Certainly, replicating, what we did before is simply enough, but how do we do one and one half times what we were doing before.

A third circumstance where constant returns to scale is inappropriate is when doubling all inputs allows for a more efficient means of production to be used. Replication says that doubling our output by doubling our inputs is feasible, but there might be a better way to produce output. Consider, for example, a firm that builds an oil pipeline between two points and uses labour, machines and steel as inputs to construct the pipeline. He may take the relevant measure of output for this firm to be the capacity of resulting pipeline. Then it is clear that if we double all inputs to the production process, the output may more than double since increasing the surface area of a pipe by 2 will increase the volume by a factor of 4. in this case when output increases by more than the scale of the inputs, we say the technology exhibits increasing returns to scale.

A technology exhibits increasing returns to scale if,

$$f(tx) > t\,f(x) \text{ for all } t > 1.$$

A fourth situation where constant returns to scale may be violated is by being unable to replicate some inputs.

Consider for example, a 100 acre farm. If we wanted to produce twice as much output, then we could use twice as much of each input. But this would imply using twice as much land as well. It may be that this is impossible to do since more land may not be available. Even though the technology exhibits constant returns to scale if we increase all inputs, it may be convenient to think of it as exhibiting decreasing returns to scale with respect to the inputs under our control.

More precisely, we have a technology that can be said to exhibit decreasing returns to scale if,

$$f(tx) < t\,f(x) \text{ for all } t > 1.$$

The most natural case of decreasing returns to scale is the case where we are unable to replicate some inputs. Thus, we should expect that the restricted production possibility sets would typically exhibit decreasing returns to scale. It turns out that it can always be assumed that decreasing returns to scale are due to the presence of some fixed factor input.

Finally, it should be noted that the various kinds of returns to scale explained above are global in nature. It may well happen that a technology exhibits increasing returns to scale for some values of x and decreasing returns to scale for other values.

4.10.1 THE ELASTICITY OF SCALE

The elasticity of scale measures the percent increase in output due to a one percent increase in all inputs – that is, due to an increase in the scale of operations.

Let y=f(x), be the production function. Let t be a positive scalar, and consider the function y(t)= f(tx). If t=1, we have the current scale of operations; if t >1, we are scaling all inputs up by t; and if t<1, we are scaling all inputs down by t.

The elasticity of scale is then given by –

$$e(x) = \frac{\dfrac{dy(t)}{y(t)}}{\dfrac{dt}{t}}$$

evaluated at t=1

Rearranging this expression, we have -

$$e(x) = \left.\frac{dy(t)}{dt}\frac{t}{y}\right|_{t=1} = \left.\frac{df(tx)}{dt}\frac{t}{f(tx)}\right|_{t=1}$$

from the above expression, we may say that the technology exhibits – locally;

(1) Increasing returns to scale, if e(x) >1;

(2) Constant returns to scale, if e(x) =1; and

(3) Decreasing returns to scale, if e(x)<1.

4.10.2 RETURNS TO SCALE AND COBB-DOUGLAS TECHNOLOGY

Suppose that $y = x_1^a x_2^b$.

Then,

$$f(tx_1, tx_2) = (tx_1)^a (tx_2)^b$$
$$= t^{a+b} x_1^a x_2^b$$
$$= t^{a+b} f(x_1, x_2)$$
$$\therefore f(tx_1, tx_2) = t^{a+b} f(x_1, x_2)$$

Hence,

$f(tx_1, tx_2) = t\, f(x_1, x_2)$ if and only if a+b=1. It, therefore, implies that the,

(1) Technology exhibits constant returns to scale, if a+b =1;
(2) Increasing returns to scale, if a+b >1; and
(3) Decreasing returns to scale if a+b < 1.

In fact, the elasticity of scale for the Cobb-Douglas technology turns out to be precisely a+b. To see this consider the definition of elasticity of substitution –

$$\frac{d(tx_1)^a (tx_2)^b}{dt} = \frac{dt^{a+b} x_1^a x_2^b}{dt}$$

$$= (a+b)t^{a+b-1} x_1^a x_2^b$$

Evaluating this derivative at t=1 and dividing by

$$f(x_1, x_2) = x_1^a x_2^b =$$
$$= \frac{(a+b)1^{a+b-1} x_1^a x_2^b}{x_1^a x_2^b}$$
$$= a+b$$

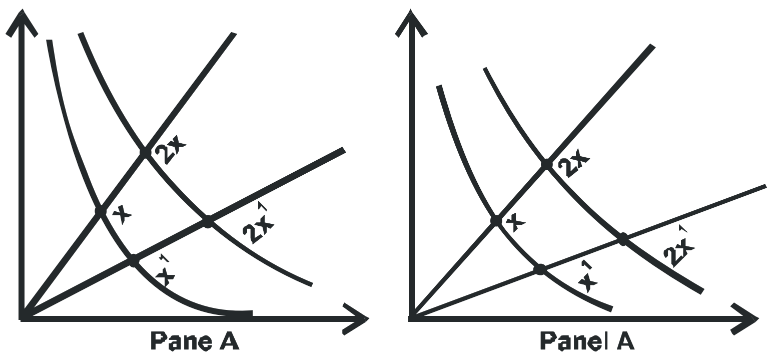
4.11 HOMOGENEOUS AND HOMOTHETIC TECHNOLOGY

A function f(x) is homogeneous of degree k if $f(tx) = t^k f(x)$ for all t>o. The two most important “degrees” in economics are the zeroth and first degree. A zero degree homogeneous function is one for which f(tx) = f(x), and first degree homogeneous function is one for which f(tx)= t f(x).

Comparing this definition to the definition of constant returns to scale we see that a technology has constant returns to scale if and only if its production function is homogeneous of degree one.

A function $g: R \rightarrow R$ is said to be a positive monotonic transformation if g is strictly increasing function; that is, a function for which x > y implies that g(x) > g(y).

A homothetic function is a monotonic transformation of a function that is homogeneous of degree one. In other words, f(x) is homothetic if and only if it can be written as f(x)= g(h(x)), where $h(\bullet)$ is monotonic function. Both, homogeneous and homothetic functions are depicted in the figure 4.5.



Panel A of the figure 4.5 depicts the function that is homogeneous of degree one. That is, if x and x^1 can both produce y units of output, then 2x and $2x^1$ can both produce 2y units of output.

Panel B of the figure 4.5 depicts a homothetic function. That is, if x and x^1produce the same level of output, y, then 2x and $2x^1$can produce the same level of output, but not necessarily 2y.

Homogeneous and homothetic functions are of interest due to the simple ways that their isoquants vary as the level of outputs varies. In the case of a homogeneous function the isoquants are all just “blown up” versions of a single isoquant. If f(x) is homogeneous of degree one, then if x and x^1produce y units of output, it follows that tx and tx^1can produce ty units of output, as depicted in figure 4.5A.

A homothetic function has almost the same property: if x and x^1 produce the same level of output, then tx and tx^1 can also produce the same level of output – but it wont necessarily be t times as much as the original output. The isoquants for a homothetic technology look just like the isoquants for homogeneous technology, only the output levels associated with the isoquants are different.

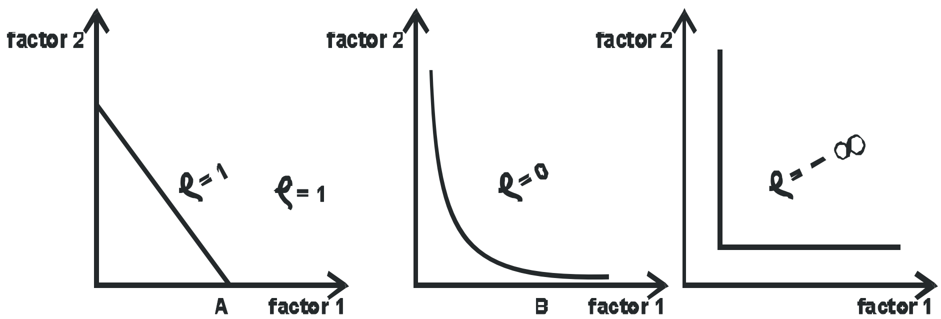
Homogeneous and homothetic technologies are of interest since they put specific restrictions on how the technical rate of substitution changes as the scale of production changes. In particular, for either of these functions the technical rate of substitution is independent of the scale of production.

4.11.1 THE CES PRODUCTION FUNCTION

The constant elasticity of substitution or CES production function has the following form;

$$y = \left[a_1 x_1^{\rho} + a_2 x_2^{\rho}\right]^{1/\rho}$$

It is quite easy to verify that CES function exhibits constant returns to scale. The CES function contains several other well-known production functions as special cases, depending on the value of the parameter ρ. These are illustrated in **figure 4.6**.



In figure 4.6 above, panel A depicts the case where ρ =1, panel B the case where $\rho = 0$ and the panel C the case where $\rho = -\infty$.

The production function contained in the CES function can be described as –

1) The linear production function (ρ =1).
   Simple substitution yields – $y = x_1 + x_2$

2) The Cobb-Douglas production function (ρ =0). When ρ =0 the CES production function is not defined, due to division by zero. However, we will show that as ρ approaches zero, the isoquants of the CES production function looks very much like the isoquants of the Cobb-Douglas production function.

This is easiest to see using the technical rate of substitution. By direct calculation –

A ρ approaches zero, this tends to a limit of $TRS\frac{x_2}{x_1}$

Which is simply the TRS for the Cobb-Douglas production function.

3) The Leontief production function $\rho = -\infty$. We have just seen that the TRS of CES production function is given by equation (1) above, A P approaches $-\infty$, this expression approaches -

$$TRS = -\left(\frac{x_1}{x_2}\right)^{-\infty} = -\left(\frac{x_2}{x_1}\right)^{\infty}$$

If $x_2 > x_1$ the TRS is negative infinity; if $x_2 < x_1$ the TRS is zero. This means that Q as ρ approaches $-\infty$, a CES isoquant looks like an isoquant associated with the Leontief technology.

The CES production function has a constant elasticity of substitution. In order to verify this, remember that the technical rate of substitution is given by –

$$TRS = \left(\frac{x_1}{x_2}\right)^{\rho-1}$$

So that,

$$\frac{x_2}{x_1} = |TRS|^{\frac{1}{1-\rho}} = |TRS|^{\frac{1}{1-\rho}}$$

Taking logs we see that,

$$\ln\frac{x_2}{x_1} = \frac{1}{1-\rho}\ln|TRS|$$

4.12 SUMMARY

In short production is the creation of utility by transforming physical units of inputs into physical units of output. Production function is the technology of combining physical units of inputs to produce the given level of output.

4.13 QUESTIONS

Q.1 Explain the concept of technology of production.

Q.2 Elaborate the concept of input requirement set.

Q.3 Define and explain the concepts of Cobb-Douglas and Leontief Technology.

Q.4 Discuss the concept of monotonic, convex and Regular technology.

Q.5 What is technical rate of substitution? Explain

Q.6 Explain returns to scale and the concept of efficient production.

Q.7 Explain the concept of CES production function.

❖❖❖❖

5

COST FUNCTION

UNIT STRUCTURE

5.0 OBJECTIVES

After going through this unit you will able to explain the concepts, like –

- Cost Function,
- Average and marginal costs,
- Long-run and Short-run costs,
- Properties of the cost function,
- Shephard's Lemma,
- The Envelope Theorem for Constrained Optimisation,
- Duality of cost and Production function,
- Geometry of Duality

5.1 INTRODUCTION :-

People without a background in economics usually make a mistake between cost and price. Price is the amount paid by the consumer and received by the producer. Cost is the amount spent by the producer in manufacturing the commodity or the service.

Cost can be understood in a variety of ways. The opportunity cost is the returns from the next best alternative. There are implicit costs which may not be seen in the accounts statements and explicit costs which could be clearly understood.

An important division of costs is between Fixed and Variable Costs. Fixed costs are those which do not depend on the quantity of output produced, they include costs like rent, payment of loan installments, permits, etc. Variable costs depend upon the quantity of output produced and increase with output (for total variable costs).

Another concept of classifying costs is total, average and marginal costs. Total cost is divided into total fixed and total variable costs. The total cost refers to the cost incurred in producing the given quantity of output. The usual total cost function is of a cubic form. Average cost is the per unit cost of producing the commodity which can be obtained by dividing the total cost with quantity of output. Marginal cost is the rate of change in total cost with respect to output and so there can not be any marginal fixed cost by definition.

5.2 COST FUNCTION

The cost function measures the minimum cost of producing a given level of output for some fixed factor prices. As such it summarizes information about the technological choices available to the firms. The behaviour of the cost function can tell us a lot about the nature of the firm's technology.

Just as the production function was our primary means of describing the technological possibilities of production, the cost function will be our primary means of describing the economic possibilities of a firm. Here we will investigate the behaviour of the cost function $c(w, y)$ with respect to its price and quantity arguments.

5.2.1 AVERAGE AND MARGINAL COST

Let us consider the structure of the cost function. In general, the function can always be expressed simply as the value of the conditional factor demands.

$$c(w, y) \equiv wx(w, y)$$

This just says that the minimum cost of producing y units of output is the cost of the cheapest way to produce y.

In the short run some of the factors of production are fixed at predetermined levels. Let x_f be the vector of fixed factors, x_v be the vector of variable factors, and break up $'w'$ into $w = (w_v, w_f)$, the vectors of prices of the variable and fixed factors. The short-run conditional factor demand functions will generally depend on x_f, so we write them as $x_v(w, y, x_f)$. Then the short-run cost function can be written as –

$$c(w, y, x_f) = w_v x_v(w, y, x_f) + w_f x_f$$

The term $w_v x_v(w, y, x_f)$ is called short-run variable cost (SVC), and the term $w_f x_f$ is the fixed cost (FC).

From these basic units, we can define various derived cost concepts, as follows –

Short run total cost (STC)

$$STC = w_v x_v(w, y, x_f) + w_f x_f$$

Short run average cost (SAC)

$$SAC = \frac{c(w, y, x_f)}{y}$$

Short run average variable cost (SAVC)

$$SAVC = \frac{w_v x_v(w, y, x_f)}{y}$$

Short run average fixed cost (SAFC)

$$SAFC = \frac{w_f x_f}{y}$$

Short run marginal cost (SMC)

$$SMC = \frac{\partial c(w, y, x_f)}{\partial y}$$

When all factors are variable, the firm will optimize in the choice of x_f. Hence, the long-run cost function only depends on the factor prices and level of output as indicated earlier.

We can express this long-run function in terms of the short-run cost function in the following way. Let $x_f(w, y)$ be the optimal choice of the fixed factors, and let $x_v(w, y) = x_v(w, y, x_f(w, y))$ be the long-run optimal choice of the variable factors. Then the long-run cost function can be written as –

$$c(w,y) = w_v x_v(w,y) + w_f x_f(w,y) = c(w,y,x_f(w,y))$$

The long-run cost function can be used to define cost concepts similar to those defined above:

Long run average cost $= LAC = \dfrac{c(w,y)}{y}$

Long run marginal cost $= LMC = \dfrac{\partial c(w,y)}{\partial y}$

It should be noted here, that the "long-run average cost" equals "long-run average variable cost" since all costs are variable in the long-run; and the "long-run fixed costs" are zero.

5.2.2 THE SHORT-RUN COBB-DOUGLAS COST FUNCTION :

Suppose the second factor in a Cobb-Douglas technology is restricted to operate at a level 'k'. Then the cost minimizing problem is –

min $w_1 x_1 + w_2 k$

Such that $y = x_1^a k^{1-a}$

Solving the constraint for x_1 as a function of y and k gives,

$$x_1 = \left(yk^{a-1}\right)^{\frac{1}{a}}$$

Thus,

$$c(w_1, w_2, y, k) = w_1\left(yk^{a-1}\right)^{\frac{1}{a}} + w_2 k$$

The following variations can also be calculated –

Short-run average cost $= w_1\left(\dfrac{y}{k}\right)^{\frac{1-a}{a}} + \dfrac{w_2 k}{y}$

Short-run average variable cost $= w_1 \left(\frac{y}{k}\right)^{\frac{1-a}{a}}$

Short-run average fixed cost $= \frac{w_2 k}{y}$

Short-run marginal cost $= \frac{w_1}{a}\left(\frac{y}{k}\right)^{\frac{1-a}{a}}$

5.2.3 THE GEOMETRY OF COSTS

The cost function is the single most useful tool for studying the economic behaviour of the firm. In a sense, the cost function summarizes all economically relevant information about the technology of the firm.

Since, we have taken factor prices to be fixed, costs depend only on the level of output of a firm. The total cost curve is always assumed to be monotonic in output : the more you produce the more it costs. The average cost curve, however, can increase or decrease with output, depending on whether total cost rise more than or less then linearly. It is often thought that the most realistic case, at least in the short-run, is the case where the average cost curve first decreases and then increases. The reason for this is as follows –

In the short-run the cost function has two components : fixed costs and variable costs. We can therefore write short-run cost as–

$$SAC = \frac{c(w, y, x_f)}{y} = \frac{w_f x_f}{y} + \frac{w_v x_v(w, y, x_f)}{y}$$
$$= SAFC + SAVC$$

In most applications, the short-run fixed factors will be such things as machines buildings, and other types of capital equipments while the variable factors will be labour and raw material. Let us consider how the costs attributable to these factors will change as output changes.

As we increase output, average variable costs may initially decrease, if there is some initial region of economies of scale. However, it seems reasonable to suppose that the variable factors required will increase more or less linearly until we approach some capacity level of output determined by the amounts of the fixed factors. When we are near to capacity, we need to use more than a proportional amount of the variable inputs to increase output. Thus, the average variable cost function should eventually increase as output increases, as depicted in figure 2.7A. Average fixed costs must of course decrease with output, as indicated in figure 2.7B. Adding together the average variable cost curve and the average fixed cost curve gives us the U shaped average cost curve as is depicted in figure 2.7C.

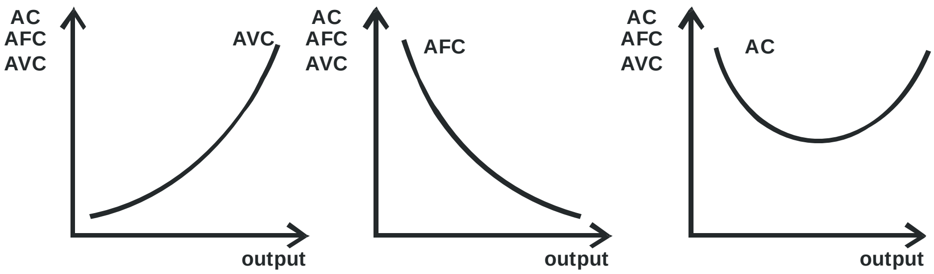


Fig: 2.7A **Fig: 2.7B** **Fig: 2.7C**

The initial decrease in the average cost is due to the decrease in average fixed costs; the eventual increase in the average cost is due to the increase in average variable costs. The level of output at which the average cost of production is minimized is sometimes known as the minimal efficient scale.

In the long-run all costs are variable costs; in such circumstances increasing average costs seems unreasonable since a firm could always replicate its production process. Hence, the reasonable, long-run possibilities should be either constant or decreasing average costs. On the other hand, certain kinds of firms may not exhibit a long-run constant-returns-to-scale technology because of long-run fixed factors. If some factors do remain fixed even in the long-run, the appropriate long-run average cost curve should presumably be U-shaped.

Let us now consider the marginal cost curve. What is its relationship with the average cost curve? Let $y*$ denote the point of minimum average cost; then to the left of $y*$ average costs are declining so that for $y \le y*$

$$\frac{d}{dy}\left(\frac{c(y)}{y}\right) \le 0$$

Taking the derivatives, it gives,

$$\frac{yc'(y)-c(y)}{y^2} \leq 0 \text{ for } y \leq y*$$

This inequality says that marginal cost is less than average cost to the left of the minimum average cost point. A similar analysis shows that,

$$c^{'}(y) \geq \frac{c(y)}{y} \text{ for } y \leq y*$$

Since both inequalities must hold at $y*$, we have

$$c^{'}(y*) = \frac{c(y*)}{y*} \;;$$

That is marginal cost equal average cost at the point of minimum average cost.

The Cobb-Douglas Cost Curves

The generalized Cobb-Douglas technology has a cost function of the firm,

$$c(y) = Ky^{\frac{1}{a+b}} \qquad a+b \leq 1$$

Where, k is a function of factor prices and parameters. Thus,

$$AC(y) = \frac{c(y)}{y} = Ky^{\frac{1-a-b}{a+b}}$$

$$MC(y) = c^{'}(y) = \frac{K}{a+b}\, y^{\frac{1-a-b}{a+b}}$$

If $a+b<1$, the cost curves exhibit increasing average costs; if $a+b=1$, the cost curves exhibits constant average costs.

5.2.4 LONG-RUN AND SHORT-RUN COST CURVES

Let us now consider the relationship between long-run cost curves and the short-run cost curves. It is clear that the long-run cost curves should never lie above any short-run cost curves, since the short-run cost minimization problem is just a constrained version of the long-run cost minimization problem.

Let us write the long-run cost function as $c(y) = c(y, z(y))$. Here we have omitted the factor prices since they are assumed fixed and we let $z(y)$ be the cost minimizing demand for a single fixed factor. Let $y*$ be some given level of output, and let $z* = z(y)$ be the associated long run demand for the fixed factor. The short run cost, $c(y, z*)$, must be at least as great as the long run cost, $c(y, z(y))$, for all levels of output, and the short-run cost will equal the long-run cost at output $y*$ so $c(y*, z*) = c(y*, z(y*))$. Hence, the long-run and the short-run cost curves must be tangent at $y*$. This is just the geometric restatement of the envelope theorem. The slope of the long-run cost curve at $y*$ is –

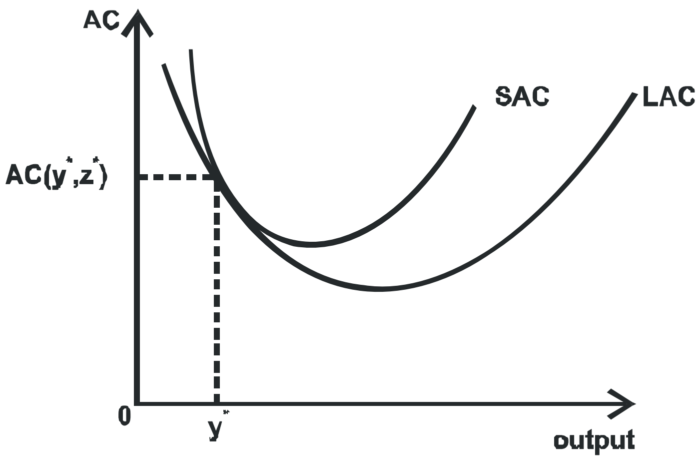
$$\frac{dc(y*, z(y*))}{dy} = \frac{\partial c(y*, z*)}{\partial y} - \frac{\partial c(y*, z*)}{\partial z}\frac{\partial z(y*)}{\partial y}$$

But since $z*$ is the optimal choice of the fixed factors at he output level $y*$, we must have –

$$\frac{\partial c(y*, z*)}{\partial z} = 0$$

Thus, long-run marginal costs at $y*$ equal short-run marginal costs at $(y*, z*)$.

Finally, we note that if the long-run and short run cost curves are tangent then the long-run and short-run average cost curves must also be tangent. A typical configuration is illustrated in figure 2.8



5.3 FACTOR PRICES AND COST FUNCTIONS

We turn now to the study of the price behaviour of cost functions. Several interesting properties follow directly from the definition of the functions. These properties of the cost functions are summarized as below –

Properties of the Cost Functions –

1) Non-decreasing in w :

If $w' \geq w,$ then $c(w^1, y) \geq c(w, y)$

2) Homogeneous of degree 1 in w :

$c(tw, y) = tc(w, y)$ for $t > 0$

3) Concave in w :

$c(tw + (1-t)w^{'}, y \geq tc(w, y) + (1-t)c(w^{'}, y)$ for $0 \leq t \leq 1$

4) Continuous in w :

$c(w, y)$ is continuous as a function of w, for $w \geq 0$

Proof :

1) Cost function is non-decreasing in w :

Let x and $x^{'}$ be cost minimizing bundles associated with w and w^1. Then $wx \leq wx^{'}$ by minimization and $wx^{'} \leq w^{'}x^{'}$. Since, $w \leq w^{'}$. Putting these inequalities together gives $wx \leq w^{'}x^{'}$ as required.

2) Cost function is homogeneous of degree 1 in w :

We show that if x is the cost minimizing bundle at price w, then x also minimizes costs at prices tw. Suppose this is not so, and let x' be a cost minimizing bundle at tw so that $twx' < twx$. But this inequality implies $wx' < wx$, which contradicts the definition of x. Hence, multiplying factor prices by a positive scalar t does not change the composition of a cost minimizing bundle, and thus, costs must rise by exactly a factor of t :

$$c(tw, y) = twx = tc(w, y)$$

3) Let (w, x) and (w', x') be two cost-minimizing price factor combinations and let $w'' = tw + (1-t)w'$ for any $0 \leq t \leq 1$. Now,

$$c(w'', y) = w''x'' = twx'' + (1-t)w'x''$$

Since x'' is not necessarily the cheapest way to produce y at price w' or w, we have $wx'' \geq c(w, y)$ and $w'.x'' \geq c(w', y).$ Thus,

$$c(w'', y) \geq tc(w, y) + (1-t)\, c\, (w', y)$$

5.4 SHEPHARD'S LEMMA

Let $x_i(w, y)$ be the firms conditional factor demand for input $'i'$. Then if the cost function is differentiable at (w, y), and $w_i > 0,$ for $i = 1, ..., n$ then

$$x_i = (w, y) = \frac{\partial c(w, y)}{\partial w_i}$$

$$i = 1, \ldots\ldots.., n$$

Proof :

Let x^* be a cost – minimizing bundle that produce y at prices w^*. Then define the function,

$$g(w) = c(w, y) - wx^*$$

Since, $c(w, y)$ is the cheapest way to produce y, this function is always non-positive, at $w = w^*,\ g(w^*) = 0.$

Since, this is the maximum value of $g(w),$ its derivative must vanish:

$$\frac{\partial g(w^*)}{\partial w_i} = \frac{\partial c(w^*, y)}{\partial w_i} - x_i = 0$$

$$i = 1, \ldots\ldots..., n$$

Hence, the cost minimizing input vector is just given by the vector of derivatives of the cost function with respect to the prices.

5.5 THE ENVELOPE THEOREM

Shephard's Lemma is another example of the envelope theorem. However, in this case we must apply a version of the envelope theorem that is appropriate for constrained optimization problems.

Consider a general parameterized constrained maximization problem of the form –

$$M(a) = \max_{x_1, x_2} g(x_{1,} x_{2,} a)$$

Such that $h(x_{1,} x_{2,} a) = 0$

In the case of the cost function –

$$g(x_{1,} x_{2,} a) = w_1 x_1 + w_2 x_2, h(x_{1,} x_{2,} a) =$$

$= f(x_{1,} x_{2)-} y,$ and $'a'$ could be one of the prices.

The Langrangian of this problem is

$$\mathcal{L} = g(x_{1,}x_{2,}a) - \lambda\, h(x_{1,}x_{2,}a)$$

and the first order conditions are –

$$\frac{\partial g}{\partial x_1} - \lambda \frac{\partial h}{\partial x_1} = 0$$

$$\frac{\partial g}{\partial x_2} - \lambda \frac{\partial h}{\partial x_2} = 0 \text{ ------------------------------------ (1)}$$

$$h(x_{1,}x_{2,}a) = 0$$

These conditions determine the optimal choice functions $(x_1(a), x_2(a)),$ which in turn determine the maximum value function

$$M(a) \equiv g(x_1(a), x_2(a), a) \text{ ------------------------ (2)}$$

The envelope theorem gives us the formula for derivative of the value function with respect to a parameter in the maximization problem. Specifically, the formula is –

$$\frac{dM(a)}{da} = \left.\frac{\partial \mathcal{L}(x,a)}{\partial a}\right|_{x=x(a)}$$

$$= \left.\frac{\partial g(x_{1,}x_{2,}a)}{\partial a}\right|_{x_i=x_i(a)} - \lambda \left.\frac{\partial h(x_{1,}x_{2,}a)}{\partial a}\right|_{x_i=x_i(a)}$$

These partial derivatives are the derivatives of g and h with respect to a holding x_1 and x_2 fixed at their optimal values.

Application of the Envelope Theorem to the Cost Minimization Problem :

In this problem the parameter $'a'$ can be chosen to be one of the factor prices, w_i. The optimal value function $M(a)$ is the cost function $c(w, y)$.

The envelope theorem asserts that –

$$\frac{\partial c(w,y)}{\partial w_i} = \frac{\partial \mathcal{L}}{\partial w_i} = x_i\big|_{x_i = x_i(w,y)} = x_i(w,y)$$

Envelope Theorem: Marginal Cost Revised:

It is another application of the envelope theorem, consider the derivative of the cost function with respect to y. According to the envelope theorem, this is given by the derivative of the Langrangian with respect to y. The Lagrangian for the cost minimization problem is

$$\mathcal{L} = w_1x_1 + w_2x_2 - \lambda\,[\,f(x_{1,}x_{2)} - y]$$

Hence,

$$\frac{\partial c(w_{1,}w_{2,}y)}{\partial y} = \lambda$$

In other words, the Lagrange multiplier in the cost minimization problem is simply marginal cost.

5.6 DUALITY

Suppose, set $VO(y)$ is an "outer bound" to the true input requirement set $V(y)$. Given data (w^t, x^t, y^t) $VO(y)$ is defined to be

$$VO(y) = \{x : w^t x \geq w^t x^t \text{ for all t such that } y^t \leq y\}$$

It is straightforward to verify that $VO(y)$ is a closed, monotonic and convex technology. Furthermore, it contains any technology that could have generated the data (w^t, x^t, y^t) for t = 1,………,T

If we observe choices for many different factor prices, it seems that $VO(y)$ should "approach" the true input requirement set in some sense. To make this precise, let the factor prices vary over all possible price vectors $w \geq 0$. Then the natural generation of VO becomes –

$$V^*(y) = \{x : wx \geq wx(w,y) = c(w,y) \text{ for all } w \geq 0\}$$

Relationship between $V^*(y)$ will contain $V(y)$ and the true input requirement set $V(y)$:

Of course $V^*(y)$ will contain $V(y)$. In general, $V^*(y)$ will strictly contain $V(y)$. For example, in figure 2.9A we see that the shaded area can not be ruled out of $V^*(y)$ since the points in this area satisfy the condition that $wx \geq c(w, y)$.

The same is true for figure 2.9B.

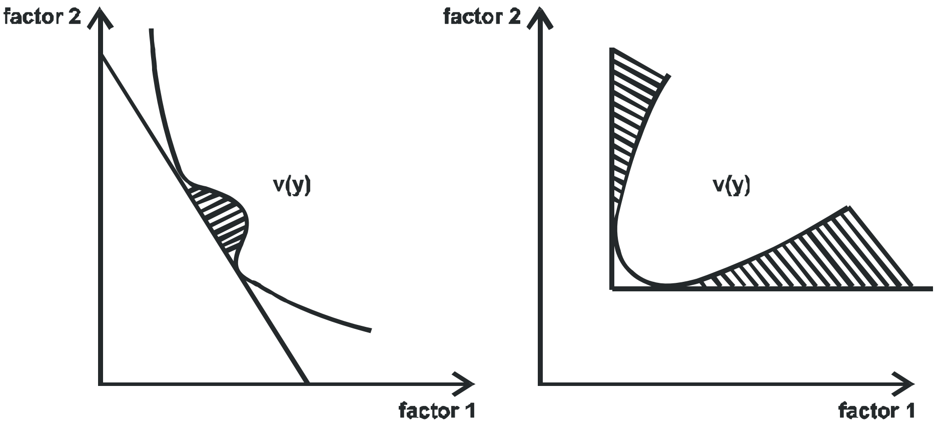


Fig : 2.9A **Fig : 2.9B**

The cost function can only contain information about the economically relevant sections of $V(y)$, namely, those factor bundles that could actually be the solution to a cost minimization problem, i.e. that could actually be conditional factor demands.

However, suppose that our original technology is convex and monotonic. In this case $V^*(y)$ will equal $V(y)$. This is because, in the convex monotonic case, each point on the boundry of $V(y)$ is a cost minimizing factor demand for some price vector $w > 0$. Thus, the set of points where $wx \geq c(w, y)$ for all $w \geq 0$ will precisely describe the input requirement set more formally –

When $V(y)$ equals $V^*(y)$. Suppose $V(y)$ is regular, convex, monotonic technology.

Then $V^*(y) = V(y)$

Proof: We already know that $V^*(y)$ contains $V(y)$, so we only have to show that if x is in $V^*(y)$ then x must be in $V(y)$.

Suppose that x is not an element of $V(y)$. Then since $V(y)$ is a closed convex set satisfying the monotonicity hypothesis, we can apply a version of separating hyperplane theorem to find a vector $w^* \geq 0$ such that $w^*x < w^*z$ for all z in $V(y)$. Let z^* be a point in $V(y)$ that minimizes cost at the prices w^*. Then in particular we have $w^*x < w^*z^* = c(w^*, y)$. But then x can not be in $V^*(y)$, according to the definition of $V^*(y)$.

This proposition shows that if the original technology is convex and monotonic then the cost function associated with the technology can be used to completely reconstruct the original technology. If we know the minimal cost of operation for every possible price vector w, then we know the entire set of technological choices open to the firm.

This is a reasonably satisfactory result in the case of convex and monotonic technologies but what about less well-behaved cases? – Suppose we start with some technology $V(y)$, possibly non-convex. We find its cost function $c(w, y)$ and then generate $V^*(y)$. We know from the above results that $V^*(y)$ will not necessarily be equal to $V(y)$, unless $V(y)$ happens to have the convexity and monotonicity properties. However, suppose we define –

$c^*(w, y) = \min wx$
Such that x is in $V^*(y)$

What is the relationship between $c^*(w, y)$ and $c(w, y)$?

When $c(w, y)$ equals $c^*(w, y)$. It follows from the definition of the functions that $c^*(w, y) = c(w, y)$

Proof: It is easy to see that $c^*(w, y) \leq c(w, y)$; since $v^*(y)$ always contains $v(y)$, the minimal cost bundle in $v^*(y)$ must be at least as small as the minimal cost bundle in $v(y)$. Suppose that for some prices w', the cost minimizing bundle x' in $v^*(y)$ has the property that $w'x' = c^*(w', y) \leq c(w', y)$. But that can not happen, since by definition of $v^*(y) - w'x' \geq c(w', y)$.

This proposition shows that the cost function for the technology $v(y)$ is the same as the cost function for its convexification $V^*(y)$. In this sense, the assumption of convex input requirement sets is not very restrictive from an economic point of view.

In short, it can be stated that –

(1) Given a cost function we can define an input requirement set $V^*(y)$

(2) If the original technology is convex and monotonic, the constructed technology will be identical with the original technology.

(3) If the original technology is non-convex or non-monotonic, the constructed input requirement will be convexified, monotonized version of the original set, and most importantly, the constructed technology will have the same cost function as the original technology.

The above three points can be summarized succinctly with the fundamental principle of duality in production : <u>the cost function of a firm summarizes all the economically relevant aspects of its technology.</u>

5.7 SUFFICIENT CONDITIONS FOR COST FUNCTIONS

We know that the cost function summarizes all the economically relevant information about a technology. We also know that all cost functions are non-decreasing, homogeneous, concave, continuous functions of prices. The question arises : suppose that you are given a non-decreasing, homogeneous, concave continuous function of prices – is it necessarily the cost function of some technology?

The answer is yes, and the following proposition shows how to construct such a technology.

When $\phi(w, y)$ is a cost function. Let $\phi(w, y)$ be a differentiable function satisfying –

1) $\phi(tw, y) = t\phi(w, y)$ for all $t \geq 0$;
2) $\phi(w, y) \geq 0$ for $w \geq 0$ and $y \geq 0$;
3) $\phi(w', y) \geq \phi(w, y)$ for $w' \geq w$;
4) $\phi(w, y)$ is concave in w.

Then $\phi(w, y)$ is the cost function for the technology defined by $V^*(y) = \{x \geq 0 : wx \geq \phi(w, y),$ for all $w \geq 0\}$

Proof: Given $w \geq 0$ we define

$$x(w, y) = \left(\frac{\partial \phi(w, y)}{\partial w_1}, \ldots\ldots, \frac{\partial \phi(w, y)}{\partial w_n} \right)$$

And note that since $\phi(w, y)$ is homogeneous of degree 1 in w, Euler's law implies that $\phi(w, y)$ can be written as

$$\phi(w, y) = \sum_{i=1}^{n} w_i \frac{\partial \phi(w, y)}{\partial w_i} = wx(w, y)$$

Here it should be noted that the monotonicity of $\phi(w, y)$ implies $x(w, y) \geq 0$

Her we need to show that for any given $w' \geq 0$, $x(w', y)$ actually minimizes $w'x$ over all x in $V^*(y)$:

$$\phi(w', y) = w'x(w', y) \leq w'x \text{ for all } x \text{ in } V^*(y):$$

First, we show that $x(w', y)$ is feasible; that is, $x(w', y)$ is in $V^*(y)$. By the concavity of $\phi(w, y)$ in w we have –

$$\phi(w', y) \leq \phi(w, y) + D\phi(w, y)(w' - w)$$
$$\text{- for all } w \geq 0$$

Using Euler's law as above it reduces to

$$\phi(w', y) \leq w'x(w, y) \text{ for all } w \geq 0$$

It follows from the definition of $V^*(y)$, that $x(w', y)$ is in $V^*(y)$.

Next we show that $x(w, y)$ actually minimizes wx over all x is in $V^*(y)$, then by definition it must satisfy.

$$wx \geq \phi(w, y)$$

But by Euler's law,

$$\phi(w, y) = wx(w, y)$$

The above two expressions imply –

$$wx \geq wx(w, y)$$

for all x in $V^*(y)$ as required.

5.8 SUMMARY

Concept of cost plays a vital role in determining the performance of a firm. One requires to known the cost of production together with the revenue to find the total amount of profits or losses if any. Per unit cost of production i.e. average cost and average revenue has a greater role in determining the profits or losses. Marginal cost of production is necessary in knowing the equilibrium level of output.

5.9 QUESTIONS

1) What is cost function?
2) Discuss the concept of average and marginal costs.
3) What is geometry of costs?
4) Explain long-run and short-run cost curves.
5) Explain the Shephard’s Lemma.
6) Explain the Envelope Theorem.
7) Discuss the duality of costs.

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6

DISTRIBUTION

UNIT STRUCTURE

6.0 OBJECTIVES

After going through this unit you will come to know –

- The concept of technical progress
- Exogenous and endogenous technical progress
- The concept of factor share
- Degree of monopoly theory or Kalecki's model of distribution
- Neo – Keynesian model or Kaldor's model of distribution.

6.1 INTRODUCTION :-

In this unit we will enquire into the determination of the distribution of income in a capitalist economy. Our concern will be with distribution as rewards to factors of production. The essential point to note is that the reward to a factor of production may be regarded as the price paid to the owner for the use of the factor: rent to the landowner for the use of his land, wage to the labourer for the use of his labour, interest to the capitalist for the use of his capital. These prices, like commodity price, are determined by demand for and supply of factors. Hence, an explanation of the price of factors requires an investigation into the conditions of demand for and supply of the factor concerned.

The neo-classical theory uses the marginal principle to explain the demand for and the return to each factor of production separately. Whereas, it is the emphasis on classes which explains the inclusion in this unit of Kalecki's and Neo-Kenesian theories of income distribution.

6.2 TECHNICAL PROGRESS

So far in the analysis of production and of the production function, we have assumed a given technology. Now, we take explicit account of technical progress. At any point of time, there is for an economy, a certain technology which may be defined as a pool of knowledge relating to the art of production. Given the technology, there will be a number of techniques, a techniques being method of combining inputs to produce a Specific quantity of a good. Technical progress encompasses all improvements in knowledge which have a nearing on production. Blaug defines technical progress as, “an addition to existing technical knowledge. Since the production function already takes account of the entire spectrum of known technical possibilities – known in the sense of being practice somewhere in the system – innovating activity ought to denote the adoption of untried methods.” It should be noted that technological change and technical change are not the same. Technological change means that a new set of production alternatives has been created. On the other hand, technical change refers to a change in the methods of production from the prevailing set of alternatives. According to Feller’s observation, technical change refers to a, “change in the choice of techniques out of the existing art. Such a change represents Factor Substitution. It does not represent the introduction of new method of production into the set of existing techniques.”

From the above observation two points can be observed i.e.

(I) Technical progress means that given inputs produce a larger output, or that a given output can be produced with a smaller quantity of inputs. In other words, technical progress reduces the per unit cost of output even with unchanged input prices.

(II) As production function represents the optimal organization of production, reorganization of inputs cannot be thought to lead to technical progress.

Technical progress may be in respect of a new product i.e. a product innovation, when the quality or the degree of sophistication of an existing product is raised, or a new product is discovered; or in respect of method of production i.e. a process innovation, when the output is expanded by more efficient use of inputs. It is not possible to distinguish between process and product innovation in reality. Thus, product innovation in one industry may be a process innovation for another industry. Now, for convenience for exposition, we will assume that distinguish is possible and concentrate on process innovation.

6.2.1 EXOGENOUS TECHNICAL PROGRESS

In general, an increase in output per head may be due to an increase in the quantity of capital per head and to technical progress. An increase in output per head due to the increase in capital per head is depicted as the movement along the same production curve; an increase in output per head due to technical progress is depicted as a movement on to a new production curve, in **figure. 5.1** we have per capita production function:

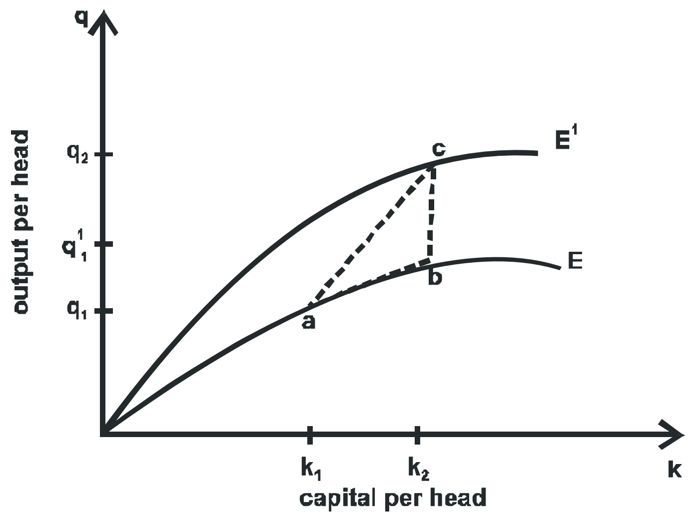

Fig: 5.1

On x-axis k for capital per head; on y-axis, q for output per head. The curve OE shows the relation between output per head and capital per head in the absence of technical progress. OE' represents the relation between output per head and capital per head subsequent to technical progress. If initially, with no technical progress, capital per head is k_1, output per head is q_1, and if capital per head is k_2, output per head is q'_1. This is indicated by a movement from a to b along OE. If, however, over a period of time, when capital per head has increased from k_1 to k_2, there has been technical progress, output per head will increased from q_1 to q_2. The increase in output due to technical progress, q'_1 to q_2, is shown by a movement from b on OE to c on OE'. Thus, it is possible to represent technical progress by a shift in the production curve. This shift in the production function can be neutral, labour-saving or capital saving. In this sense, technical progress is exogenous to the economic system. The classical empirical work is solow's study on technical progress.

6.2.2 ENDOGENOUS TECHNICAL PROGRESS

Arrow assumes technical progress to be endogenous to the economic system. He attempts to examine the underlying concept of knowledge implicit in a production function. Learning means the acquisition of knowledge. It may come through education and research, or as a by-product of the activity of production. It is this latter source of learning with which Arrow concerns himself, though he concedes the importance of the other sources. Thus, he is able to view technical progress as a huge and extended process of learning about the "environment in which we operate." He, therefore, advances the hypothesis that learning is the product of experience, and increases in productivity is the result of experience acquired in production. Technical progress, he says, "can be ascribed to experience, it is very activity of production, which gives rise to problems for which favourable responses are selected over time." He takes the cumulative gross investment as the index to represent this experience. Each new capital good introduced into production can change the environment of production, so that learning becomes a continuous process. Arrow follows Solow and Johansen, in suggesting that technical progress is fully embodied in the new capital goods. All the knowledge available at any point of time is fully incorporated into new capital goods. However, once the machines are built, their productive efficiency can not be changed by later learning.

CONTRIBUTION OF KALECKI AND KALDOR:

Kalecki and Kaldor have developed the theories of distribution, called, "Degree of Monopoly theory OR Kalecki's Model of Distribution" and "Neo-Kenesian Model OR Kaldor's Model of Distribution".

6.3 DEGREE OF MONOPOLY THEORY OR KALECKI'S MODEL OF DISTRIBUTION

This theory is due to Michal Kalecki. It states that the profit share is a function of the degree of monopoly of an enterprise. This proposition was subsequently generalized to apply to the entire economy. The theory was propounded in the thirties against the background of the increasing attacks on perfect competition by P. Sraffa, Joan Robinson and E.H.Chamberlin and against the background of increasing unemployment in the industrial economies. Kalecki realized the irrelevance of the theory of perfect competition for the manufacturing sector as compared with its validity for the agricultural sector. He assumed that excess capacity is a normal feature of a capitalistic enterprise and that, as a consequence, production take place largely under condition of decreasing or constant marginal costs. He assumed a reverse L-shaped cost curve, so that average variable costs will be constant and the short-sun marginal and average cost will be equal over a long range up to capacity output. He also assumed that price is determined by the full-cost principle, that is, by adding up a mark-up to prime costs, such as interest, depreciation and salaries.

Kalecki first formulated his model of distribution in 1939 and in response to criticism. He revised the model in 1954. This first version was based on Lerner's concept of degree of monopoly. We will concern ourselves here only with second version.

The formula for mark-up pricing for single firm may be expressed as,

$$P = k.\,AVC \quad \text{--------------------------------------- (I)}$$

Where, P = Price
AVC = Prime Cost (Average Variable Cost)
k = Ratio by which Prime Costs are marked up,

$$i.e.,\ k = \frac{aggregate\ proceeds}{aggregate\ prime\ \cos ts}$$

$$k = \frac{W + R + O + E}{W + R} \quad \text{----------------------------- (II)}$$

Where, W = Wage bill
R = Cost of Raw Material
O = Total overheads
E = Aggregate entrepreneurial income including dividends.

From the above, we have,

$$O + E = (k - 1)(W + R) \quad \text{-------------------------- (III)}$$

Where the left had side of the equation represents the sum of overheads and profits. Now, let A be the value added by the production process.
Therefore,

$$A = W + O + E$$
$$A = W + (k - 1)(W + R)$$

$$\therefore \frac{W}{A} = \frac{W}{W + (k-1)(W + R)} \quad \text{-------------------------- (IV)}$$

$$\therefore \frac{W}{A} = \frac{1}{1+(k-1)(1+\frac{R}{W})}$$

$$= \frac{1}{1+(k-1)(1+J)} \qquad \text{-------------------------- (V)}$$

The above equation indicates that the share of wages in the national income is a function of two variables: k and j, the ratio of costs of raw materials to the wage bill.

In Pure Competition, price is equal to the short-run marginal (and average) cost of production. Therefore, in such markets, k=1. But in markets which deviate from pure competition, k will be greater than one. The more imperfect the market, the greater will be the gap between price and marginal costs and consequently, the greater will be K. Therefore k is presumed to represent the "degree of monopoly" in the market. The share of wages in the national income will be higher, the lower is the degree of monopoly and the lower is the ratio of raw material costs to wage costs. Kalecki believed, in accordance with the Marxian view, that overtime, there will be a growing concentration of industry, so that k will be on increasing function of time. This will tend to depress the share of wages in wages in the national income. However, this trend is likely to be offset by the secular fall in the terms of trade of agricultural products vis-à-vis manufactures possibly through the exploitation of the poorer nations. Hence, the wage share will bend to be stable over time.

Kalecki attempted, initially at least, to test this hepothesis by using the data for the ratio of raw material costs to wage costs and the wage share in the national income and deducing what the degree of monopoly may have been in the period in question. This is a questionable procedure. The correct procedure would have been to we the data for the ratio of raw material costs to the wage costs and some measure of the degree of monopoly to estimate what the wage share would have been during the relevant period. This could then have been compared with the actual share of wages in the national income. Again, Kalecki distinguished between wages in the national economic basis for such a distinction is weak. In a theory of factor shares, both ought to have been included in the category of wages. True, Kalecki separated wages and salaries because he was seeking a basis to distinguish between variable and fixed costs. However, it is not true that wages correspond to variable, and salaries to fixed cost of production.

Besides, it may not always be possible to indicate the extent of imperfection in a market by the degree of monopoly. Even if the concept is accepted at the level of enterprise, little meaning can be attached to the concept of a weighted degree of monopoly for the manufacturing sector as a whole. Moreover, since k, is ratio of aggregate proceeds to aggregate prime costs, it is suggested that the explanation is tautological. However, it is important to note that kalecki did try to explain the factors which have a bearing on the degree of monopoly.

There are:

(a) the increase in the concentration in industry and the tacit agreements and formal collusion which characterize their functioning,

(b) the growing replacement of price competition by advertising campaigns,

(c) the increase in overhead costs, which lead firms to raise prices relative to prime cost in order to protect profits and

(d) the growing influence of trade unions.

The first three factors will tend to reduce it. Riach defended Kalecki's theory against the charge of tautology and the consequent implication that it cannot be tested empirically. He argued that although Kalecki measures the degree of monopoly as the ratio of aggregate proceeds to prime costs, he does not define it as this ratio, that is, the ratio of aggregate proceeds to prime costs merely reflects what the degree of monopoly is. Thus, since there are forces, which we have seen above, which have a bearing on K, the theory is not tautological and can be subjected to empirical verification.

Bauer also criticized the lumping together of overheads and profits. He argued that as production becomes more capital intensive, the "degree of monopoly" will also show an increase, whereas the return to capital will normally fall. It has also been pointed out that the theory fails if applied to a situation of pure competition. In such a situation, since the degree of monopoly is zero, wages will absorb the entire income. That is, there will be no profits. Yet, capital is scarce and productive and should receive a return. This criticism is misplaced since Kalecia's model is designed for a situation of imperfection and can not be applied where competitive conditions prevail. Despite all the limitations, Kalecki's theory remains an intuitively appealing one. As Kaldor has observed so well, Kalecki's hypothesis relates the distribution of income between wages and profits to the extent of competition in the prevailing market structure. As such, there is nothing tautological, but in fact, a good deal of realism in such a proposition. Factor shares are seen as a function of the extent to which labour costs can be marked up by a firm and passed on to the consumers. The question of market power, inclusive of institutional arrangements, such as collective bargaining and monopoly, becomes relevant. Also relevant are the effects of changes in the aggregate demand on the market for goods and labour. Not withstanding this, the theory has, as with the neoclassical theory of distribution, one serious limitation, namely, that is relies on a microeconomic concept to explain a macroeconomic phenomenon.

6.4 NEO-KEYNESIAN MODEL OR KALDOR'S MODEL OF DISTRIBUTION

This is a full-fledged macro model which use concepts of aggregate investment, savings and income to explain the distribution of income. Unlike, in Kalecki's case, this model assumes conditions of full employment. Investment, an exogenously given constant, is given by the state of confidence in the economy and is not dependent on the distribution of income. There are two income categories: wages and profits. Wages include payment to manual labour as well as salaries. Profits include, besides return to entrepreneurs, the income of property owners. Constant saving propensities are assumed so that the average and the marginal propensities to save of each group are the same. However, the propensity to save out of wage income is small relative to that out of profit income. With these assumptions, Kaldor proceeded to establish the constancy of relative factor shares and the capital-output ratio. The model can be expressed using the following symbols:

Y – Aggregate income
I – Investment
W – Wage bill
S – Savings
s_w – Average and marginal propensities to save out of wages
P - Profits
s_P – Average and marginal propensities to save out of profits
S_W – Aggregate savings out of wage income = s_wW
S_P – Aggregate savings out of Profits = s_PP

Now,

$$Y = W + P \text{ -- (I)}$$

In equilibrium,

$$I = S \text{ -- (II)}$$

And

$$S = S_W + S_P \text{ ------------------------------------- (III)}$$

Substituting Eq. (III) in Eq. (II)

$$I = S_W + S_P$$

$$I = s_w W + s_p P$$

$$I = s_w (Y - P) + s_p P \quad \text{--------------------------- (IV)}$$

$$I = (s_p - s_w) P + s_w Y$$

Divide both sides by Y,

$$\frac{I}{Y} = (sp - sw)\frac{P}{Y} + sw$$

$$\therefore \frac{P}{Y} = \frac{1}{(sp - sw)}\,\frac{I}{Y} - \frac{sw}{(sp - sw)} \quad \text{------------------ (V)}$$

Since, by assumption, s_W and s_P are constants, Eq. (V) shows that the share of profits in the national income is a direct function of the share of investment in the national income. To understand the mechanism through which equilibrium is achieved, we note the important assumption that $s_P > s_W$. In equilibrium, investment equals savings. Now, if the ratio of investment to national income increases, this equality is disturbed. Prices will rises and the share of profits in the national income will increase relatively to share of profits in the national income will increase relatively to share of profits in the national income will increase relatively to share of wages. Since the propensity to save out of profits exceeds that out of wages, savings out of profits and, therefore aggregate savings rise and continue to do so until they are equal to the aggregate exogenous investment. Note that in this model, an increase in the rate of investment con not lead to an increase in income in view of the assumption of full employment. The only way, therefore, that savings can be brought into equality with the increased investment is through a change in the distribution of income away from wages and in favour of profits. The mechanism explained above can also be seen in **fig. 5.2**

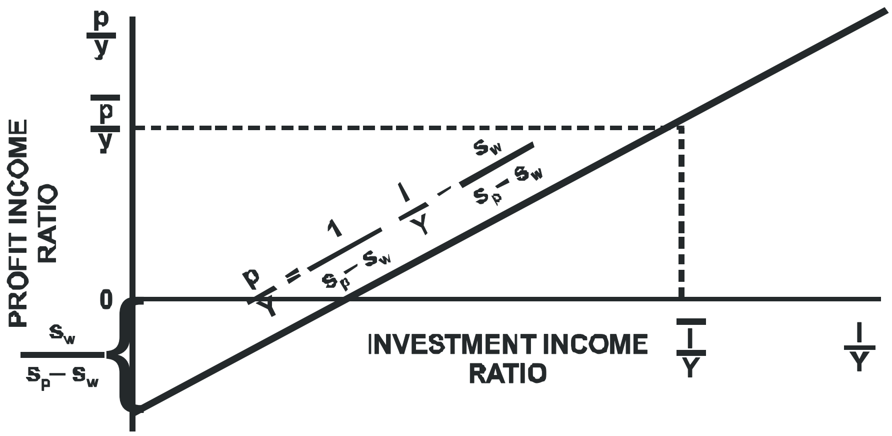

Fig : 5.2

As the propensities to save are assumed to be constant, Eq. (V) depicts a linear relation between the rate of profits and the rate of investment. Any change in the rate of investment leads to a corresponding change in the rate of profit and therefore, in the rate of savings until this rate is equal to the rate of investment. Note that since the investment is exogenous, it is independent of the saving propensities. The assumption of full-employment stresses the importance of demand in the model, that is, if the rate of investment rises, aggregated demand increases, prices and profits margins increase and real consumption falls. Implicit in this is the assumption that money wages do not increase in proportion to the increase in prices. Again, if the rate of investment falls, aggregate demand decreases, price and profit margins fall and real consumption rises. Wages, being less flexible downwards, any decrease in wages will be to a smaller extent as compared with the fall in general prices.

The model may be presented in a slightly different way. Since, investment is exogenously given and is independent of p/Y, the ratio I/Y is constant and is depicted in **fig. 5.3** by the straight line parallel to the x-axis at the level of OE. On the other had, as $s_P > s_W$, S/Y increases with P/Y. The S/Y function has a positive slope. As explained before, savings are brought into equality with investment through changes in the distribution of income consequent on the changes in prices. In the figure, a ratio of investment to income of OE results in a profit to income ratio of OE. An increase in the ratio of investment to income will increase the share of profits and lower the share of wages. On the other hand, a net increase in the aggregate propensity to save of capitalists and workers will increase the wage share.

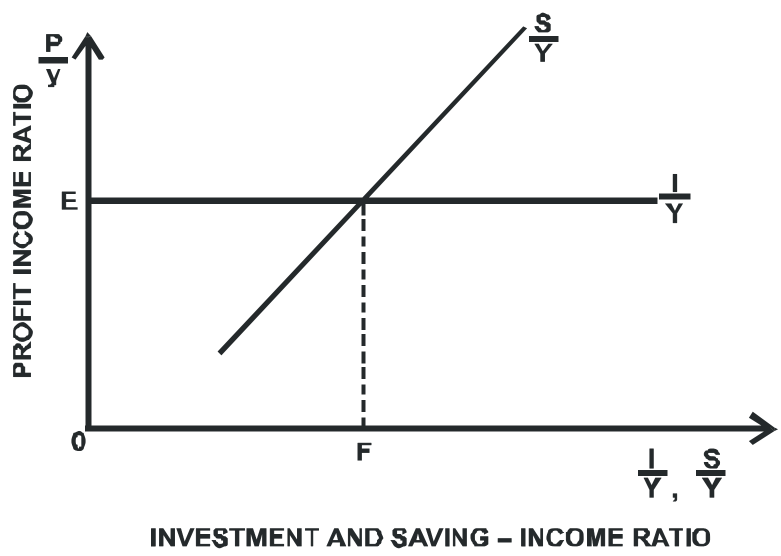

Fig : 5.3

The stability of the system depends on the relative saving propensities of workers and capitalists. Thus, if $s_P > s_W$ is positive but very small, infinitesimal changes in I/Y will tend to produce large changes in P/Y. True, equilibrium will still be stable, but for any given change in investment oscillations around equilibrium will be large. If $s_P = s_W$ the situation becomes explosive. Therefore, it is very necessary that firstly $s_P \neq s_W$ and secondly, s_P should be substantially greater than s_W. If $s_W = 0$ then Eq. V becomes,

$$\frac{p}{Y} = \frac{1}{sp} \cdot \frac{I}{Y}$$

i.e. $p = \frac{1}{sp} \cdot I$ -- (VI)

From the above, two interesting conclusions follows. Firstly, since $s_W = 0$, workers spend all that they earn. This may well be true at very low levels of wage income. Secondly, since $P = \frac{1}{sp} \cdot I$, capitalists earn what they spend. This is because the income of capitalists is directly a function of their expenditure. As increase in the expenditure of capitalists raises total profits by the extent of expenditure adjusted for the propensity to save. The share of profits will be greater the smaller is s_P. There are thus two basic factor which explain relative shares : (a) the rate of investment, and (b) the propensity to save out of profits. If, however, capitalists were to the save all their income, so that $s_P = 1$, then the increase in profits would be to the same extent as the increase in their expenditures. This is the "window's cruse" aspect of the model. Keynes in the Treatise had a model which displayed, the window's cruse effect. These assumptions $s_w = 0$ and $s_P = 1$ are essentially those of classical economics. Society is divided into two classes : one, workers whose income comes only from wages and salaries and who spend it entirely on consumption, the other, capitalists, who derived their entire income from profits on the means of production owned by them. All investment is carried out by the capitalists from these profits which constitute the only source of Finance for capital expenditures. This is why, in this model, the rate of investment is controlled by the capitalist class.

An important implication of the theory the following:
If P/y is stable as also the rate of return on capital P/K, then since –

$$\frac{P}{Y} = \frac{P}{K} = \frac{K}{Y}$$

Therefore, K/Y, that is that capital output ratio will also be stable. The importance of Kaldor's model is that by the emphasis on savings and capital stock, he takes the analysis back to the fundamental determinants of the economic system.

6.5 SUMMARY

Widespread income inequality is the concern of most of the governments worldwide. Hence, the income distribution theories given by economists like Kalecki and Kaldor have greater importance.

Karl Marx spoke a lot about the equitable distribution of income (national income) amongst the various factors of production. But, that was possible only under command economy type of economic system. Therefore, the theory given by Karl Marx was inapplicable in the capitalist type of economic system. Hence, Kalecki and Kaldor developed the theories of income distribution for the capitalist world. These theories are based on the principle of marginal productivity of factors of production.

6.6 QUESTIONS

1. Explain the concept of technical progress.
2. Elaborate upon the Degree of Monopoly theory.
3. Explain the Kalecki’s model of distribution
4. Explain Neo-Kenesian model.
5. Discuss the Kaldor’s model of distribution

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7

ELEMENTS OF GAME THEORY

UNIT STRUCTURE

7.0 OBJECTIVES

After going through this unit, you will be able to :

- Explain the elements of game theory.
- Define - the game
    - players
    - actions
    - payoffs
    - information and information set
    - the outcome
    - equilibrium
- Explain the decision trees
- Describe the games in normal forms
- Describe the games in extensive forms

7.1 INTRODUCTION

Game theory is concerned with the actions of decision makers who are conscious that their actions affect each other.

When the only two publishers in a city choose prices for their newspapers, aware that their sales are determined jointly, they are players in a game with each other. They are not in a game with the readers who buy the newspapers, because each reader ignores his or her effect on the publisher. Game theory is not useful when decisions are made that ignore the reactions of others or treat them as impersonal market farces.

Game theory as it will be presented in this book is modelling tool, not an axiomatic system. The presentation in this chapter is unconventional. Rather presentation in this starting with mathematical definitions or simple little games of the kind used later in the chapter, we will start with a situation to be modelling, and build a game from it step by step.

7.2 ELEMENTS OF GAME THEORY

The essential elements of a game are players, actions, payoffs, and information – PAPI, for short. These are collectively known as the rules of the game, and the modeller's objective is to describe a situation in terms of the rules of a game so as to explain what will happen in that situation. Trying to maximize their payoffs, the players will devise plans known as strategies that pick actions depending on the information that has arrived at each moment. The combination of strategies chosen by each player is known as the equilibrium. Given an equilibrium, the modeller can see what actions come out of the conjunction of all the player's plans, and this tells him the outcome of the game.

To define these terms let us use the example of an entrepreneur trying to decide whether to start a dry cleaning store in a town that already is served by one dry cleaner. We will call the two firms "New Cleaner" and "Old Cleaner". New Cleaner is uncertain about whether the economy will be in a recession or not, which will affect how much consumers pay for dry cleaning, and must also worry about whether Old Cleaner will respond to entry with a price war or by keeping its initial high prices. Old Cleaner is a well-established firm, and it would survive any price war, though its profits would fall. New Cleaner must itself decide whether to initiate a price war or to charge high pries, and must also decide what kind of equipment to buy, how many workers to hire, and so forth.

Players are the individuals who make decisions. Each player's goal is to maximize his utility by choice of actions.

In the Dry Cleaners Game, let us specify the players to be New Cleaner and Old Cleaner. Passive individuals like the customers, who react predictably to price changes without any thought of trying to change anyone's behaviour, are not players, but environmental parameters.

Sometimes it is useful to explicitly include individuals in the model called pseudo players whose actions are taken in a purely mechanical way.

Nature is a pseudo player who takes random actions at specified points in the game with specified probabilities.

In the Dry Cleaners Game, we will model the possibility of recession as a move by Nature. With probability 0.3, Natures decides that there will be a recession, and with probability 0.7 there will not. Even if the players always took the same actions, this random move means that the model would yield more than just one prediction. We say that there are different realizations of a game depending on the results of random moves.

An action or move by player i, denoted a_i, is a choice be can make. Player i's action set, $A_i = \{a_i\}$, is the entire set of actions available to him.
An action combination is an ordered set $a = \{a_i\}$, (I = 1, ……, n) of one action for each of the n players in the game.

We are trying to determine whether New Cleaner will enter or not, and for this it is not important for us to go into the technicalities of dry cleaning equipment and labor practices. Also, it will not be in New Cleaner's interest to start a price war, since it cannot possibly drive out Old Cleaners, so we can exclude that decision from our model. New Cleaner's action set can be modeled very simply as {enter, stay out}. We will also specify Old Cleaner's action set to be simple : it is to choose price from {Low, High},

By player i's payoff π_i, (S_1, …….. Sn), we mean either :

1. The utility player i receives after all players and Nature have picked their strategies and the game has been played out; or
2. The expected utility he receives as a function of the strategies chosen by himself and the other players.

For the moment, think of "strategy" as a synonym for "action". Definitions (1) and (2) are distinct and different, but in the literature and this book the term "payoff" is used for both the actual payoff and the expected payoff. The context will make clear which is meant. If one is modelling a particular real world situation, figuring out the payoffs is often the hardest part of constructing a model. For this pair of dry cleaners, we will pretend we have looked over all the data and figured out that the payoffs are as given by table 6.1 if the economy is normal, and that if there is a recession the payoff of each player who operates in the market is 60 thousand dollars lower.

Table 7.1 The Dry Cleaners Game

		Old Cleaner	
		Low Price	**High Price**
New Cleaner	Enter	100, -50	100, 100
	Stay out	0, 50	0, 300

Payoffs to : (New Cleaner, Old Cleaner) in thousand of dollars (normal economy)

Information is modeled using the concept of the information set. For now, think of a player's information set as his knowledge at a particular time of the values of different variables. The elements of the information set are the different values that the player think are possible. If the information set has many elements, there are many values the player cannot rule out; if it has one element, he knows the value precisely. A player's information set includes not only distinctions between the values of variables such as the strength of oil demand, but also knowledge of what action have previously been taken, so his information set changes over the course of the game.

Here, at the time that is chooses its price, Old Cleaner will know New Cleaner's decision about entry. But what do the firms know about the recession? If both firms know about the recession we model thus as Nature moving before New Cleaner; if only Old Cleaner, we put Nature's mane after New Cleaner; if neither firm knows whether there is a recession at the time they must make their decisions, we put Nature's mane at the end of the game. Let us do this last.

It is convenient to lay out information and actions together in an order of play. Here is the order of play we have specified for the Dry Cleaners Game.

1. New Cleaner chooses it's entry decision from {Enter, Stay out}.
2. Old Cleaner chooses its price from {Low, High}
3. Nature picks demand, D, to be Recession with probability 0.3 or Normal with probability 0.7

The purpose of modelling is to explain how a given set of circumstances leads to a particular result. The result of interest is known as the outcome.

The outcome of the game is a set of interesting elements that the modeller picks from the values of actions, payoffs, and other variables after the game is played out.

The definition of the outcome far any particular model depends on what variables the modeller finds interesting. One way to define the outcome of Dry Cleaners Game would be as either Enter or Stay out. Another way, appropriate if the model is being constructed to help plan New Cleaner's finances, is as the payoff that New Cleaner realizes, which is, from table 6.1, one element of the set {0, 100, -100, 40, -160}.

7.2.1 Equilibrium

To predict the outcome of a game, the modeller focuses on the possible strategy combinations, since it is the interaction of the different player's strategies that determines what happens. The distinction between strategy combinations, which are sets of strategies, and outcomes, which are sets of values of whichever variables are considered interesting, is a common source of confusion. Often different strategy combinations lead to the same outcome. In the Dry Cleaners Game, the single outcome of New Cleaner Enters would result from either of the following two strategy combinations.

$$\left\{\begin{array}{l}\text{High price is New Cleaner Enters, Low Price if New Cleaner stays} \\ \text{out Enter}\end{array}\right\}$$

$$\left\{\begin{array}{l}\text{Low price is New Cleaner Enters, High Price if New Cleaner stays} \\ \text{out Enter}\end{array}\right\}$$

Predicting what happens consists of selecting one or more strategy combination as being the most rational behavior by the players acting to maximize their payoffs.

An equilibrium $S^* = \left(S^*_1, \ldots\ldots S^*_n\right)$ is a strategy combination consisting of a best strategy for each of the n players in the game.

The equilibrium strategies are the strategies players pick in trying to maximize their individual payoffs, as distinct from the many possible strategy combinations obtainable by arbitrarily choosing one strategy per player. Equilibrium is used differently in game theory than in other areas of economics. In a general equilibrium model, for example an equilibrium is a set of prices resulting from optimal behavior by the individuals in the economy. In game theory, that set of prices would be the equilibrium outcome, but the equilibrium itself would be the strategy combination – the individuals' rules for buying and selling – that generated the outcome.

To find the equilibrium, it is not enough to specify the palyers, strategies, and payoffs, because the modeller must also decide what "best strategy" means. He does this by defining an equilibrium concept.

An equilibrium concept or solution concept F : $\{S_1,....S_n,\ \pi_1,....\pi_n\} \rightarrow S^*$ is a rule that defines an equilibrium based on the possible strategy combinations and the payoff functions.

We have implicitly already used an equilibrium concept in the analysis above, which picked one strategy for each of the two players as our prediction for the game we implicitly used is the concept of sub-game perfectness.

7.3 GAMES IN EXTENSIVE FORM :

Consider the following story. As owner manager of Jokx Joys and Games, you are thinking about introducing a new game called oligopoly, which will teach children from ages 8 to 12 the basic principles of imperfect competition. You must decide very soon whether to introduce Oligopoly and if you do decide to go ahead, you will have to spend $40,000 to complete design of the game, advertise it, and set up production.

The market for a game like Oligopoly is highly uncertain. You decide to consider two possibilities : The market will either be large, yielding total sales of 20,000 units, or it will be small, with sales totaling 6,000 units. You assess probabilities 0.4 and 0.6 far these two possibilities. These figures suppose a wholesale price per unit of $12; raising the price will cause sales to plummet, whereas lowering it will not increase demand appreciably.

Another source of uncertainty is that a competitive firm, Beljeau Games and Toys, is considering the introduction of a game called Reaganomics, which will complete directly with Oligopoly. In fact if you introduce Oligopoly and Beljeau introduces Reaganomics, the competition that ensure will force each of you to charge only $10 per unit (wholesale) – the overall market, which will be either 20,000 or 6,000 units, will not be enlarged by the fall in price – and you will each get a one half share of the market.

It will cost you $5 per unit to produce Oligopoly in addition to the $40,000 fixed costs mentioned above. You will be able to produce exactly as many units as you sell. (produce to demand)

The situation for Beljeau is somewhat similar. As you sit debating whether to introduce Oligopoly, the managers at Beljeau are debating whether to introduce Reaganomics. The press of getting these games out in time for Christmas means that you cannot wait to see what they do before deciding whether to go ahead with Oligopoly; similarly, they must decide about Reaganomics before learning of your decision on Oligopoly. But they have one advantage – they have commissioned a market survey whose results they will learn before deciding whether to proceed with Reaganomics, a survey that will tell them without error whether the market will be large or small.

Beljeau will incur a fixed cost of $60,000 if it develops Reaganomics, and a unit cost of $3 per unit (produced to demand). Reaganomics will sell for precisely the same price as Oligopoly - $12 wholesale if only one product is in the market and $10 wholesale if both firms are marketing their (respective) games.

7.3.1 Decision Trees

Looking at this problem from you point of view, we can build a decision tree that represents, the problem you face. The tree is given in figure 6.1. For those of you who have never seen a decision tree before, we after some words of translation. We start at the left – hand side; the box there with two branches coming out represents the decision that you must make right now whether or not to introduce Oligopoly. Boxes are called decision or choice nodes. If you choose to introduce Oligopoly, then things will begin to happen to you : you learn the size of the market (large or small), and you learn what Beljeau decided to do (introduce Reaganomics or not) These are things outside your contract; from you point of view they are "chance events" (although you may have a pretty good idea what Beljeau will do). So we depict them as chance nodes, or branching points in the tree marked by circles. On the other hand, if you don't introduce Oligopoly, from your point of view the problem is over.

This gives us five "branches" in the tree, each one representing a unique sequence of choices by you (Jokx) and outcomes of events that are outside your control. Note in this regard that we put in two chance nodes far Beljeu's choice, so that we have four outcomes following a decision by you to enter the market. Then for each of the five branches, we can evaluate have must contribution to profit you will receive.

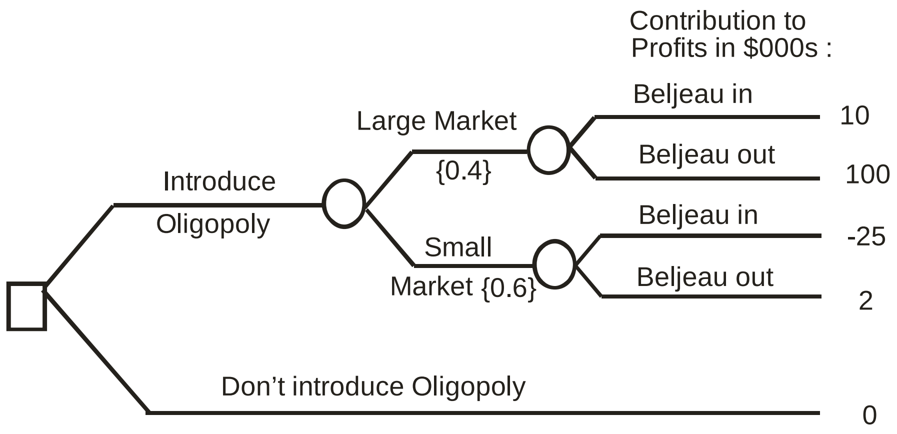
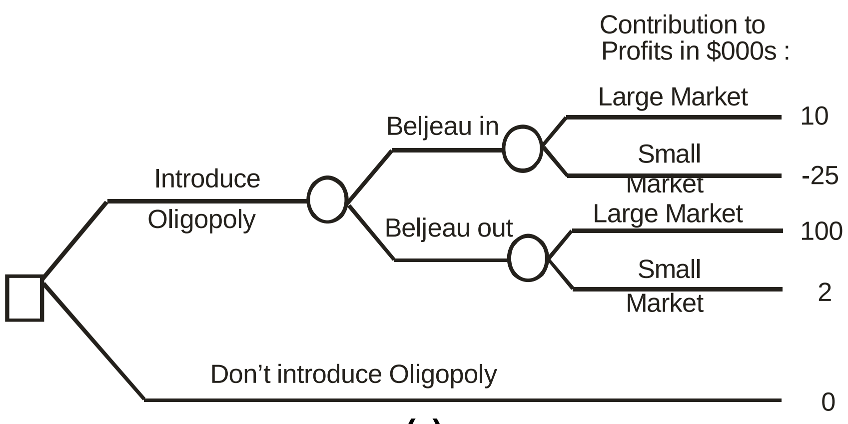


Figure 7.1: The decision tree of Jokx

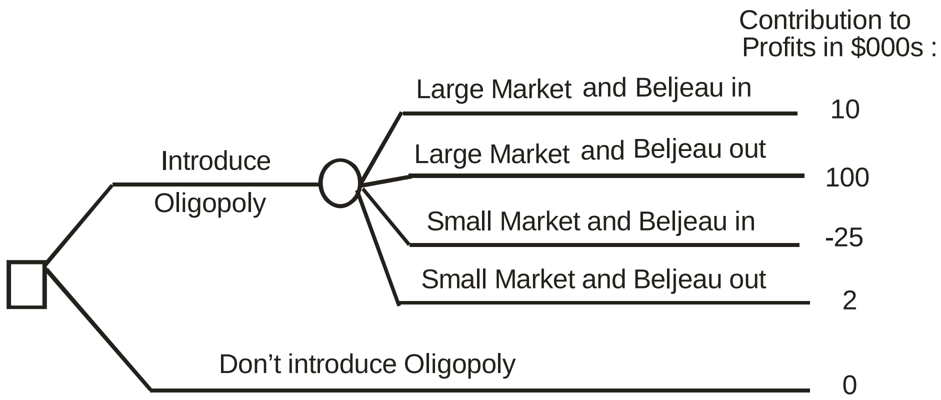
For example if you introduce Oligopoly, the market is large, and Beljeau introduces Reaganomics (the topmost branch), you will sell 10,000 units for $10 wholesale, or $1,00,000 in revenues, less fixed costs of $40,000 and manufacturing costs of (10,000) ($5) or total costs of $90,000, for a $10,000 contribution to profits, and so on. These numbers are written in figure 6.1 at the end of each branch. Finally, we know the probabilities that the market is large or small, so we put these in on the appropriate branches in the tree.

Could we depict your decision tree in other ways? Figure 6.2 gives two possibilities. In figure 6.2(9) we interchange the two chance nodes putting Beljeu's decision first and then the size of the market. And in figure 6.2(b) we combine the two chance nodes into one having four possible outcomes. The rule that we must follow in constructing decisions trees is.

The fundamental rule of decision trees. A chance node precedes a choice node in the tree if and only if the uncertainty represented by that chance node resolves in the mind of the decision maker prior to the time at which the choice must be made.



(a)



(b)

Figure 7.2 : Two different represents of Jokx tree

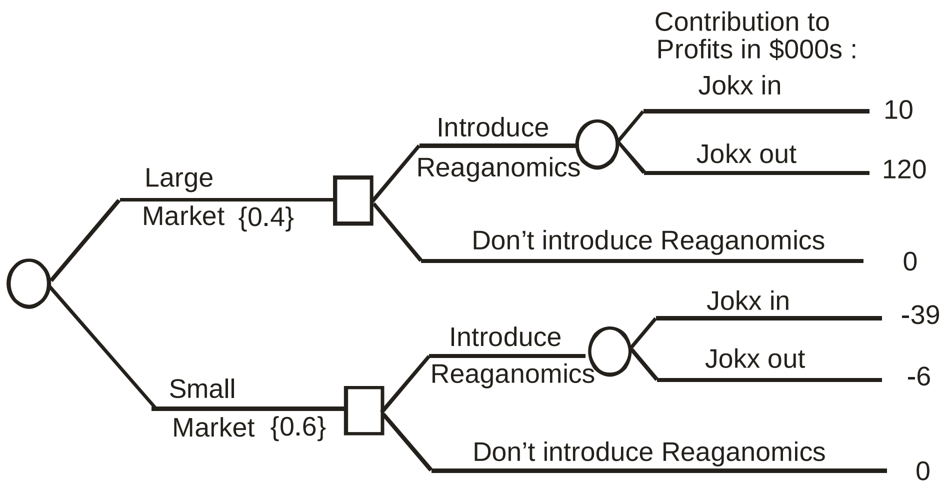


Figure 7.3 : The decision tree of Beljeau

Since you must decide on Oligopoly before any uncertainty resolves, as long as your choice node comes first, you are living within the strictures of that rule.

As for Beljeaus tree, they will learn the size of the market prior to making their decision, so the chance node for size of market must come first in their tree. And they don’t learn about your decision regarding Oligopoly until after their decision, so the chance node for our decision must come after their choice node. This gives the decision tree in figure 6.3 with contribution to profit and probabilities for the market size supplied.

This is a relatively simple example, and can see the “answer” just by inspecting the decision trees. Look at figure 6.3

first. If the market is large, then Beljeau will make a positive contribution no matter what Jokx does if they go ahead with Reaganomics, so they are pretty sure to do so. Whereas if the market is small, Beljeau is sure to lose money with Reaganomics what ever Jokx does, so they will probably decide not to proceed in this case. This means that in figure 6.1, if the market is large, you (Jokx) can be sure that Beljeau will be in the market, netting you $10,000 if you introduce Oligopoly. Whereas if the market is small, you can be pretty sure that Beljeau will be out of the market, and you will net $2,000 if you introduce Oligopoly. So since you are fairly sure that Beljeau will be out if the market is small, you can safely enter the market; you will make a positive contribution no matter what.

Despite this simplicity, we will carry forward with this example. We already know the answer to this problem, the techniques we will use will help us to see through more complex games.

7.3.2 An Extensive form representation

In a game tree, we use one tree structure to represent both players decision problems. How could we ever represent both decision problems in a single tree? The obvious problem is that the order of nodes depends on whose perspective we take. Even so, it can be done. Have a look at figure 6.4. This is rather a mess, but if you bear with me, it will become clear.

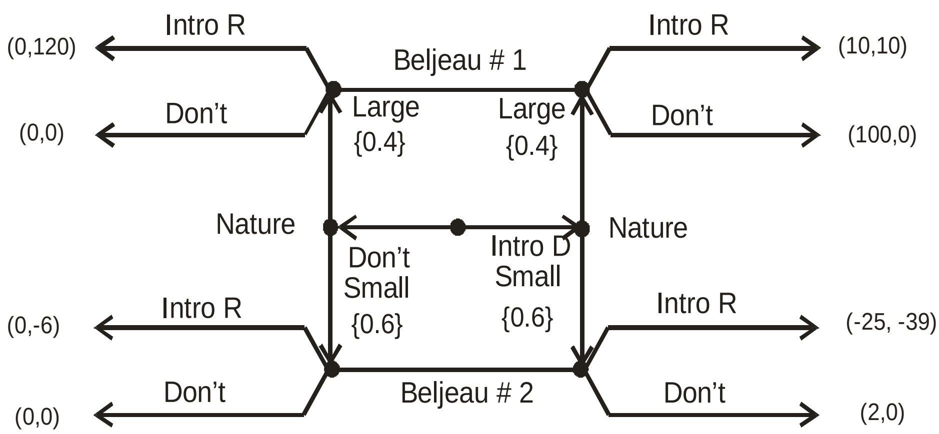


Figure 7.4 : The Game in extensive form

The game starts with the open circle or node near the middle of the page. (The game will always start at the open circle in this sort of picture) Note that Jokx appears besides this open circle, meaning that at this point it is Jokx who must choose what to do.

Jokx is limited to two choices that correspond to the two arrows pointing out of this initial node labeled intro O (for introduce Oligopoly) and don't.

Follow the choice intro O by Jokx to a second node depicted by a closed circle. (Open circles will be used to denote only the initial node). This node in labeled nature, meaning that at this point we imagine the choice of which path to follow is determined by events under the control of neither player. Nature chooses between a large market and a small one with the probabilities given on the branches. You next come to nodes at which Beljeau must choose between intro R (introduce Reaganomics) or don't, and at the end of each path through the tree you find a vector of payoffs, in this case measured by contribution to profit. Jokx payoff is first and Beijeu's second. So, far example, if jokx introduce Oligopoly (move right), the market is small (move down), and Beljeau introduces Reaganomic Cs, you reach the vector (-25, -39), meaning that Jokx loses \$25,000 and Beljeau loses \$39,000, similarly, if Jokx introduces Oligopoly the market is large, and Beljeau doesn't introduce Reaganomics, you reach the vector (100, 0) meaning Jokx has made \$1,00,000 and Beljeau has netted zero.

The one "problem" is that we've put Beljeau's decision after Jokx' decision in the tree. Beljeau, remember, doesn't know what Jokx has done when it must make a decision. We depict this in the picture by the two dashed lines. Take the upper one. This connects Beljeau's two decision nodes in the circumstances (a) Jokx doesn't introduce 0 and the market is large, and (b) Jokx doesn't introduce 0 and the market is large. Beljeau cannot distinguish between these two circumstances, and so we join these with the interpretation : Whatever Beljeau does at one of the two nodes joined by the dashed line, it must do precisely the same thing at the other. This is called an information set, in general, all the nodes in an information set represent circumstances at which some player is called upon to move without knowing which of those circumstances pertain.

Recall that we labeled the initial node Jokx, since Jokx moved there, and the second two nodes nature, since the moves there were taken by forces outside of player's control. But we didn't label the next set of nodes directly with Beljeau. Instead, we labeled the two dashed lines or information sets, and these labels on dashed lines are meant to imply that at all four of the nodes in the two information sets Beljeau chooses what to do.

Most importantly, note that Beljeau has two information sets labeled Beljeau # 1 and Beljeau # 2. These correspond to the two different decisions that Beljeau must make : Whether to introduce

Reaganomics if the market is large (# 1); and whether to introduce Reaganomics if the market is small (# 2).

There is, of course, a strong connection between the extensive form game tree in figure 6.4 and the two decision trees in figure 6.1 and 6.3. In particular, every choice node in the decision tree of a player will correspond to one information set for the player in the game tree. So, just as there are two decisions nodes for Beljeau in 6.3, so there are two information sets in 6.4 And just as there is one decision node for Jokx in 6.1, so there is one information set for Jokx in 6.4, where a single node belonging to a player that is not joined to any other by a dashed line is thought of as an information set in its own right.

Why did we put the nodes in the order we did? In many cases, we have a lot of freedom in choosing the order. The only rule that we must follow is that.

If someone learns something prior to making a particular decision, where the something learned could be either a move by nature or a move by another player then the node representing what is learned must precede the node where the decision is taken.

Hence the node (s) for nature must precede Beljeau's nodes. Compare with the rule for decision trees, where the rule is more definitive; there we say that something outside the control of a player, depicted by a chance node, comes before a choice node if and only if the players learns the resolution of that chance event before making the decision. For game trees the implication runs only one way. We are allowed to put a "chance node" before a "choice node" if the player who is choosing doesn't learn what happened at the chance node until after the choice must be made; to repair this misordering, we use information sets.

In this particular example, we have three generations of nodes : one for Jokx' decisions, one far Beljeau's and one for nature's. The rule say that in a game tree natures nodes must precede Beljeau's. but that is the only implication of the rule. Hance besides the order $I \rightarrow n \rightarrow B$ that we used in figure 6.4, it is possible to depict this situation in a game tree with the order $n \rightarrow B \rightarrow J$ and in the order $n \rightarrow J \rightarrow B$. It is probably a good exercise to draw these two "other" orders far the game tree. It hardly needs saying, but the point of this exercise is entirely in getting the information set; drawn correctly.

There is an important point here. In representing the game on a single tree, what we want to do is convey what each party knows whenever it is that party's move, and we want to include all the complete "sequences" of steps that might get us from start to finish. The word "sequence" is in quotes because there is no particular temporal sequence to things sets" of events might be better. Actual calendar time matters only insofar as if determines (to some, but not complete, extent) what one party might know when it is time for that party to move. That is, barring prescience which we will do, one can only knows things that happened before. But there is no reason to suppose that one knows everything that happened before – that is hardly descriptive of reality – and it is knowledge and not time that rules what orders of moves can / cannot be used in this sort of tree.

7.4 GAMES IN NORMAL OR STRATEGIC FORM :

From the example of section 6.1 it should be obvious that some game situation can be represented by a number of different extensive forms. Since all these extensive forms represent the same "game", we might suspect there is another way to represent the game that is a bit clearer about the essence of the situation.

What is common to all the different extensive form representations hinted at for Jokx versus Beljeau is that they all represent the same strategic problem for the two sides. Suppose that as owner of Jokx you have debited to take a vacation in Florida. All decisions concerning Oligopoly must be made before you return, and you refuse to ruin your vacation by talking on the phone with the home office. So you decide to leave complete and unambiguous instructions as to what decisions you want taken regarding Oligopoly.

That's pretty easy – you have to decide whether to proceed with Oligopoly or not. You will receive no useful information, so there are really only two possible set of instructions you might leave:
s1 : Proceed to market Oligopoly
s2 : Don't do this

What is more interesting is the similar problem for the management of Beljeau. If they are headed off for vacation, the instructions they must leave will concern whether to market Reaganomics or not, contingent on the results of the market survey. There are four sets of instruction they could leave.

t1 : No matter what the survey says, proceed with Reaganomics.
t2 : Proceed with Reaganomics if the market will be large, but not if the market will be small.
t3 : Proceed with Reaganomics if the market will be small, but not if the market will be large.
t4 : No matter what the survey says, don't proceed with Reaganomics.

Now you may find strategy t3 to be fairly silly. (In fact, at the end of section 6.1 we already indicated that t2 seems the only sensible thing to do.)

Look at the extensive form depicted in figure 6.5 there runs as follows. You (Jokx) are heading for Florida, so you select one of your two strategies. Without knowing what you chose (note the information set), Beljeau picks one of their four strategies. And then nature acts to select one of the two market sizes, with payoffs made to each side accordingly. For example, if you pick s2, Beljeau picks t2, and the market and Beljeau learns that the market is large and (according to t2) markets Reaganomics. Thus you make $0 and Beljeau makes $120 K. Note than we could just as well put Beljeau first and you second, using an information set for you to model the notion that you don't know what strategy Beljeau has selected when you select your own strategy.

Nodes belonging to nature

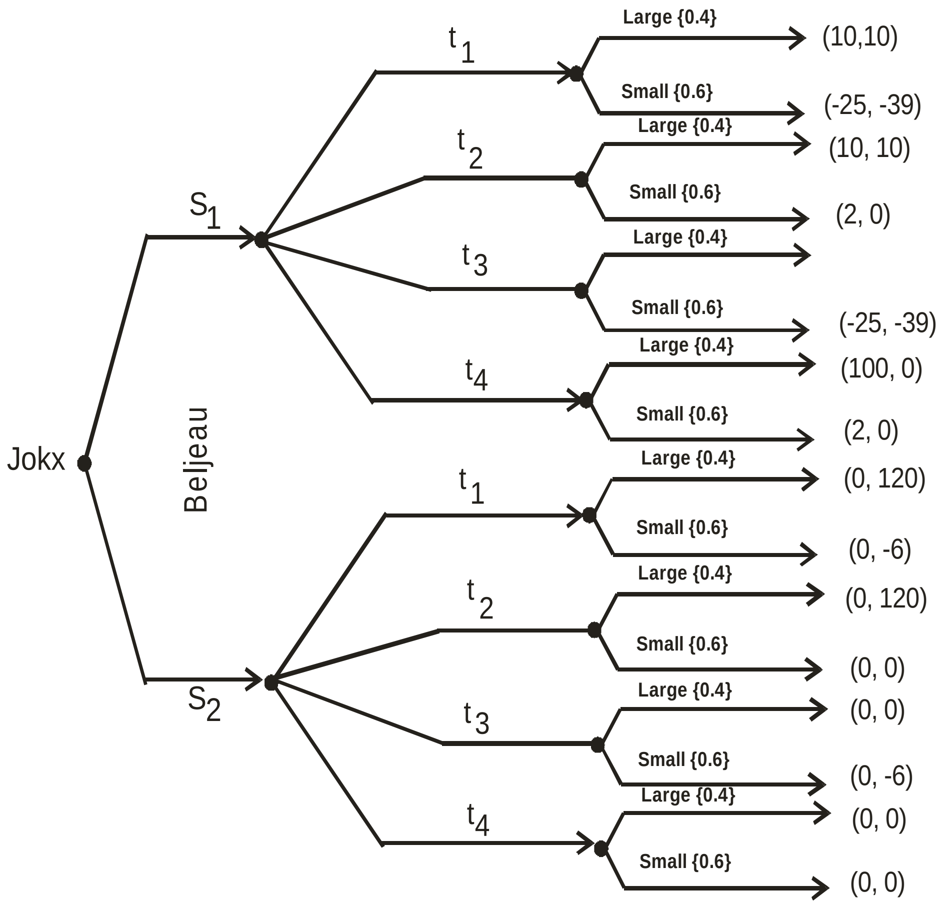


Figure 7.5 : Another extensive form representation of Jokx vs. Beljeau

One final step, and we have what is known as the normal or strategic form of the game. Let's assume that both you and Beljeau are risk neutral – that you evaluate risky prospects according to their expected value. (In general, since endpoints in games are evaluated in terms of the players von Neumann – Morgenstern utility functions, expected payoffs become expected utilities.) Then if, say, you pick strategy s1 and Beljeau picks t2, you stand a.4 probability of getting a contribution of $10,000 and a.6 probability of a contribution of $2,000. This has an expected value of $5,200. Similarly, for this pair of strategies, Beljeau has an expected value of $4,000.

In figure 6.6 we give what is known as the normal or strategic form of the game.

Figure 7.6 : The normal form represented of Jokx vs. Beljeau

Jokx' Strategy		Beljeau's Strategy			
		t1	**t2**	**t3**	**t4**
	s1	-11, -19.4	5.2, 4	35, -23.4	41.2, 0
	s2	0, 44.4	0, 48	0, -3.6	0, 0

In this case, because there are two players, this is also sometimes called a bimatrix form game. Each row in the table corresponds to one of your two strategies. Each column corresponds to one of Beljeau's four. And in the cells of the matrix, we list you expected contribution and then Beljeav's for the strategy pair corresponding to the cell.

In general, a normal or strategic form game is given by : a list of players i = 1, …….. I; for i = 1, 2, ……I, a list of strategies Si that player i might employ; and for each I tuple of strategies (S_1,…….S_I), one for each player, the payoff to each player of that combination of strategies, given by functions $u_i : \pi_{j=1}^{I} S_j \to R$. Just the list of players and the lists of their strategies is sometimes called a normal form game form.

In moving from an extensive form to the resulting normal form, we undertake the following two – step procedure. First, for each player i = 1, 2, ……, I, the set of strategies for player i is given by –

$$Si = \pi A(h)$$
$$\{h \in Hu(h) = i\}$$

That is, a strategy S_i for player i specifies precisely which action the player will take in every information set assigned to that player.

Second, for every combination of strategies for the various players, which we will call hereafter a strategy profile, one evaluates the expected utility for each player, where one is taking expectation over any randomness in the initial node or in subsequent moves by nature, using the probability distributions p and p_t that are given as part of the extensive form game.

On the other hand, there is no single way to proceed in general from a normal form game to a corresponding extensive form game. In one obvious extensive form the players all choose complete strategies simultaneously, but often other extensive forms could be constructed from a given normal form.

7.5 SUMMARY :

In short Game theory is concerned with the actions of the decision makers who are conscious that their actions will affect each other. We have seen that the game theory is a modelling tool and not an axiomatic system. Players, actions, natures, outcome, etc. are the essential elements of Game theory.

7.6 FURTHER READINGS :

(1) Silberberg E.: The structure of Economics: A Mathematical Analysis McGraw Hill, 1990.
(2) Rasmusen E.: Games and Information, Blackwell, 1994.

7.7 QUESTIONS FOR REVIEW :

(1) Explain various elements of game theory.

(2) Discuss the games in normal or strategic forms.

(3) Describe the games in extensive forms.

(4) Write a note on essential elements of game theory.

❖❖❖❖

8

NASH EQUILIBRIUM

UNIT STRUCTURE

8.0 OBJECTIVES

After going through this unit you will be able to –

- Explain the Nash Equilibrium,
- Define and explain the Battle of the sexes,
- Describe the co-ordination game,
- Define and explain sub-game perfect Nash Equilibrium,
- Discuss the Infinitely repeated games,
- Explain the Folk Theorem

8.1 INTRODUCTION

Nash equilibrium is a strategy profile in which each player's part is as good a response to what the others are meant to do as any other strategy available to that player. Along with the Nash equilibrium, the sub-game Nash Equilibrium and the Folk Theorem are the topics that will be discussed in this unit.

8.2 NASH EQUILIBRIUM:

For the vast majority of games, which lack even iterated dominance equilibrium, modellers use Nash equilibrium, the most important and widespread equilibrium concept. To introduce Nash equilibrium we will use the game Boxed pigs (Baldwin & Meese [1979]) Two pigs are put in a skinner box with a special panel at one end and a food dispenser at the other, when a pig presses the panel at a utility cost of 2 units, 10 units of food are dispensed. One pig is “dominant” (let us assume he is bigger), and if he gets to the dispenser first, the other pig will only get his leavings, worth 1 unit. If, instead, the small pigs arrives first, he eats 4 units, and even if they arrive. Table 7.1 summarizes the payoffs for the strategies press the panel and wait by the dispenser.

Table 8.1 Boxed Pigs

		Small Pigs	
		Press	Wait
Big pig	Press	5, 1	$\boxed{4}$, $\boxed{4}$
	Wait	$\boxed{9}$, -1	0 , $\boxed{0}$

Payoffs to : (Big Pig, Small Pig)

Boxed pigs has no dominant strategy equilibrium, because what the big pig chooses depends on what he thinks the small pig will choose. If he believed that the small pig would press the panel, the big pig would wait by the dispenser, but if he believed that the small pig would wait, the big pig would press. There does exist an iterated dominance equilibrium, (Press, Wait), but we will use a different line of reasoning to justify that outcomes Nash equilibrium.

Nash equilibrium is the standard equilibrium concept in economics. It is less obviously correct than dominant strategy equilibrium but more often applicable, Nash equilibrium is so widely accepted that the reader can assume that if a model does not specify which equilibrium concept is being used it is Nash or some refinement of Nash.

The strategy combinations s* is a Nash equilibrium if no player has incentive to deviate from his strategy given that the other players do not deviate. Formally,

$$\forall_i, \pi_i(s_i^*, s_{-i}^*) \geq \pi_i(s_i^{'}, s_{-i}), \forall s_i^{'}$$

The strategy combination (press, wait) is a Nash equilibrium. The way to approach Nash equilibrium is to propose a strategy combination and test whether each player's strategy is a best response to the others' strategies. If the big pig picks press, the small pig, who faces a choice between a payoff of 1 From pressing and 4 From waiting, is willing to wait.

If the small pig picks wait, the big pig, who has a choice between a payoff of 4 from pressing and O From waiting, is willing to press. This confirms that (Press, Wait) is a Nash equilibrium, and in fact it is the unique Nash equilibrium.

It is useful to draw arrows in the tables when trying to solve for the equilibrium, since the number of calculation is great enough to soak up quite a bit of mental RAM. Another solution tip, illustrated in Boxed Pigs, is to circle payoffs that dominate other payoffs (or box, them, as is especially suitable here) Double arrows or dotted circles indicate weakly dominant payoffs. Any payoff combination in which every payoff is circled, or which has arrows pointing towards it from every direction, is a Nash equilibrium. I like using arrows better in 2-by-2 games, but circles are better for bigger games, since arrows become confusing when payoffs are not lined up in order of magnitude in the table.

The pigs in this game have to smarter that the players in the Prisoner's Dilemma. They have to realize that the only set of strategies supported by self-consistent beliefs is (Press, Wait) The definition of Nash equilibrium lacks the "$\forall s - i$" of dominant strategy equilibrium, so a Nash strategy need only be a best response to the other Nash strategies, not to all possible strategies. And although we talk of "best responses", the moves are actually simultaneous, so the players are predicting each other moves. If the fame were repeated or the players communicated, Nash equilibrium would be especially attractive, because it is even more compelling that beliefs should be consistent.

Like a dominant strategy equilibrium, a Nash equilibrium can be either weak or strong. The definition above is for weak Nash equilibrium. To define strong Nash equilibrium, make the inequality strict; that is, require that no player be indifferent between his equilibrium strategy and some other strategy.

Every dominant strategy equilibrium is a Nash equilibrium, but not every Nash equilibrium is a dominant strategy equilibrium. If a strategy is dominant it is a best response to any strategies the other players pick, including their equilibrium strategies. If a strategy is part of a Nash equilibrium, it need only be a best response to the other player's equilibrium strategies.

The Modeller's Dilemma of table 7.2 illustrates this feature – of Nash equilibrium.

Table 8.2 The Modeller's Dilemma

		Column	
		Deny	Confess
Row	Deny	0 , 0	-10 , 0
	Confess	0 , -10	$\boxed{-8}$, $\boxed{-8}$

Payoffs to : (Row, Column)

The situation it models is the same as the Prisoner's Dilemma, with one major exception: although the police have enough evidence to arrest the prisoners as the "probable cause" of the crime, they will not have enough evidence to convict them of even a minor offense if neither prisoner confesses. The northwest payoff combination becomes (0, 0) instead of (-1, -1).

The Modeller's Dilemma does not have a dominant strategy equilibrium. It does have what might be called a weak dominant strategy equilibrium, because confess is still a weakly dominant strategy for each player. Moreover, using this fact, it can be seen that (Confess, Confess) is an iterated dominance equilibrium, and it is a strong Nash equilibrium as well. So the case For (Confess, Confess) still being the equilibrium outcome seems very strong.

There is, however, another Nash equilibrium in the Modeller's Dilemma: (Deny, Deny), which is a weak Nash equilibrium. This equilibrium is weak and the other Nash equilibrium is strong, but (Deny, Deny) has the advantage that its outcome is Pareto-superior : (0, 0) is uniformly greater than (-8, -8). This makes it difficult to know which behaviour to predict.

The Modeller's Dilemma illustrates a common difficulty for modellers: what to predict when two Nash equilibrium exist. The modeller could add more details to the rules of the game, or he could use an equilibrium refinement, adding conditions to the basic equilibrium concept until only one strategy combination satisfies the refined equilibrium concept. There is no single way to refine Nash equilibrium.

The modeller might insist on a strong equilibrium, or rule out weakly dominated strategies, or use iterated dominance. All of these lead to (confess, confess) in the Modeller's Dilemma or he might rule out Nash equilibrium that are Pareto-dominated by other Nash equilibria and end up with (Deny, Deny). Neither approach is completely satisfactory.

8.2.1 The Battle of the Sexes

The third game we will use to illustrate Nash equilibrium is the Battle of the Sexed, a conflict between a man who wants to go to a prize fight and a women who wants to go to a ballet. While selfish, they are deeply in love, and would, if necessary, sacrifice their preferences to be with each other. Less romantically, their payoffs are given by table 7.3.

Table 8.3 The Battle of the Sexes

		Woman	
		Prize Fight	Ballet
Man	Prize Fight	2, 1	0 , 0
	Ballet	0 , 0	1 , 2

Payoffs to : (Man, Woman)

The Battle of the Sexes does not have an iterated dominant strategy equilibrium. It has two Nash equilibria, one of which is the strategy combination (Prize Fight, Prize Fight). Given that the man chooses Prize Fight, so does the woman; given that the women chooses Prize Fight, so does the man. The strategy combination (Ballet, Ballet) is another Nash equilibrium by the same line of reasoning.

How do the players know which Nash equilibrium to choose? Going to the fight and going to the ballet are both Nash strategies, but for different equilibria. Nash equilibrium assumes correct and consistent beliefs. If they do not talk before hand, the man might go to the ballet and the woman to the fight, each mistaken about the other's beliefs. But even if the players do not communicate, Nash equilibrium is sometimes justified by repetition of the game. If the couple do not talk, but repeat the game. If the couple do not talk, but repeat the game night after night, one may suppose the eventually they settle on one of the Nash equilibria.

Each of the Nash equilibria in the Battle of the Sexes is Pareto-efficient; no other strategy combination increases the payoff of one player without decreasing that of the other. In many games the Nash equilibrium is not Pareto-efficient: (confess, confess) for example, is the unique Nash equilibrium of the Prisoner's Dilemma, although its payoffs of (-8, -8) are Pareto-inferior to the (-1, -1) generated by (Deny, Deny).

Who moves first is important in the Battle of the Sexes, unlike any of the three previous games we have looked at. IF the man could buy the fight ticket in advance, his commitment would induce, the woman to go to the fight. In many games, but not all, the player who moves first (which is equivalent to commitment) has a first mover advantage.

The Battle of the Sexes has many economic applications. One is the choice of an industry wide standard when two firms have different preferences but both want a common standard to encourage consumers to buy the product. A second is to the choice of language used in a contract when two firms want to formalize a sales agreement but they prefer different terms.

8.2.2 Coordination Games

Sometimes one can use the size of the payoffs to choose between Nash equilibria. In the following game, players Smith and Jones are trying to decide whether to design the computers they sell to use large or small floppy disks. Both players will sell more computers if their disk drives are compatible.
The payoffs are given by table 7.4.

Table 8.4 Ranked Coordination

		Jones	
		Large	Small
Smith	Large	2 , 2	-1 , -1
	Small	-1 , -1	1 , 1

Payoffs to: (Smith , Jones)

The strategy combinations (Large, Large) and (Small, Small) are both Nash equilibria, but (Large, Large) Pareto – dominates (Small, Small). Both players prefer (Large, Large), and most modellers would use the Pareto-efficient equilibrium to predict the actual outcome. We could imagine that it arise from pre-game communication between Smith and Jones taking place outside of the specification of the model, but the interesting question is what happens if communication is impossible. Is the Pareto efficient equilibrium still more plausible? The question is really one of psychology rather than economics.

Ranked coordination games, which share of games called coordination games, which share the common feature that the players need to coordinate on one of multiple Nash equilibria. Ranked coordination has the additional feature that the equilibria can be Pareto ranked. Table 7.5 show another coordination game, Dangerous coordination, which has the same equilibria as Ranked coordination, but differs in the off-equilibrium payoffs. If an experiment were conducted in which students played Dangerous coordination against each other, I would not be surprised if (Small, Small), the Pareto-dominated equilibrium, were the one that was played out. This is true even though (Large, Large).

Table 8.5 Dangerous Coordination

		Jones	
		Large	Small
Smith	Large	2 , 2	-1000 , -1
	Small	-1 , -1	1 , 1

Payoffs to : (Smith, Jones)

Is still a Nah equilibrium ; if smith thinks that Jones will pick Large, Smith is quite willing to pick large himself. The problem is that if the assumptions of the model are weakened, and Smith cannot trust Jones to be rational, well informed about the payoffs of the game, and unconfused, then Smith will be reluctant to pick Large because his payoff if Jones picks Small is then -1,000. He would play if safe instead, picking Small and ensuring a payoff of at least 1. In reality, people do make mistakes, and with such an extreme difference in payoffs, even a small probability of a mistake is important, so (Large, Large) would be a bad prediction.

Games like Dangerous coordination are a major concern in the 1988 book by Harsany and selten, two of the giants in the field of fame theory. I will not try to describe their approach here, except to say that it is different from my own. I do not consider the fact that one of the Nash equilibria of Dangerous coordination is a bad prediction as a heavy blow against Nash equilibrium. The bad prediction is based on two things: using the Nash equilibrium concept, and using the game Dangerous coordination. If Jones might be confused about the payoffs of the game, then the game actually being played out is not Dangerous coordination, so it is not surprising that it gives poor predictions. The rules of the game ought to describe the probabilities that the players are confused, as well as the payoffs if they take particular actions. If confusion is an important feature of the situation, then the 2-by-2 game of table 7.5 is the wrong model to use, and a more complicated game of incomplete information of the kind described in chapter. Again, as with the prisoner's Dilemma, the modeller's first thought on finding that the model predicts an odd result should not be "Game theory is bunk," but the more modest "Maybe I'm not describing the situation correctly" (or even "Maybe I should not trust my 'common sense' about what will happen").

Nash equilibrium is more complicated but also more useful than it looks. Jumping ahead a bit, consider a game slightly more complex than the ones we have seen so far. Two firms are choosing outputs Q_1 and Q_2 simultaneously. The Nash equilibrium is a pair of numbers (Q_1, Q_2) such that neither firm would deviate unilaterally. This troubles beginners. They say, "Sure, Firm I will pick Q_1 if it thinks Firm 2 will pick Q_2. But Firm I will realize that if it makes Q_1 bigger, then Firm 2 will react by making Q_2 smaller. So the situation is much more complicated, and (Q_1, Q_2) is not a Nash equilibrium or, if it is , Nash equilibrium is a bad equilibrium concept" If there is a problem in this model, it is not Nash equilibrium, but the model it self.

Nash equilibrium makes perfect sense as a stable outcome in this model. The student's hypothetical is false because if firm 1 chooses something other than Q_1, firm 2 would not observe Ehe deviation till it was too late to change Q_2 - remember, this is a simultaneous move game. The student's worry is really about the rules of the game, not the equilibrium concept. He seems to prefer a game in which the firms move sequentially, or maybe a repeated version of the game. If firm 1 moved first, and then firm 2, then firm 1's strategy would still be a single number, Q_1, but firm. 2's strategy its action rule would have to be a function, Q_2 (Q_1). A Nash equilibrium would then consist of an equilibrium number, Q_1, and an equilibrium function, Q_2 (Q_1). The two outputs actually chosen, Q_1 and Q_2 (Q_1) will be different from the Q_1 and Q_2 in the original game. And they should be different the new model is of a very different real word situation.

One lesson to draw from this is that it is essential to figure out the mathematical form the strategies take before trying to figure out the equilibrium. In this simultaneous move game, the strategy combination is a pair of nonnegative numbers. In the sequential game, the strategy combination is one nonnegative number and one function defined over the nonnegative numbers. Students invariably make the mistake of specifying firm 2's strategy as a number, not a function. This is a far more important point than any beginner realized. Trust me – you're going to make this mistake sooner or later, so it's worth worrying about.

8.3 SUB-GAME PERFECT NASH EQUILIBRIA

Sub game perfectness is an equilibrium concept based on the ordering moves and the distinction between an equilibrium path and an equilibrium. The equilibrium path is the path through the game tree that is followed in equilibrium, but the equilibrium itself is a strategy combination, which includes the player's responses to other player's deviations from the equilibrium path. These off-equilibrium responses are crucial to decisions on the equilibrium path. A threat, For example, is a promise to carry out a certain action if another player deviated from his equilibrium actions, an it has an influence even if it is never used.

Perfectness is best introduced with an example, which has three pure-strategy Nash equilibria of which only one is reasonable. The players are Smith and Jones, who choose disk sizes. Both their payoffs are greater if they choose the same size and greatest if they coordinate on large. Smith moves first, so his strategy set is {Small, Large}. Jones strategy is more complicated, because it must specify an action for each information set, and Jones information set depends on what Smith chose. A typical element of Jones strategy set is (Large, Small), which specifies that the chooses Large if Smith chose Large, and Small if Smith chose Small. From the strategic form we found the following three Nash equilibria.

$$\begin{bmatrix} \text{Equilibrium} & \text{Strategies} & \text{Outcome} \\ E_1 & \{\text{Large}, (\text{Large}, \text{Large}\} & \text{Both pick Large} \\ E_2 & \{\text{Large}, (\text{Large}, \text{Small}\} & \text{Both pick Large} \\ E_3 & \{\text{Small}, (\text{Small}, \text{Small}\} & \text{Both pick Small} \end{bmatrix}$$

Only Equilibrium E_2 is reasonable, because the order of the moves should matter to decisions players make. The problem with the strategic form, and thus with simple Nash equilibrium, is that it ignores who moves first. Smith moves first, and it seems reasonable that Jones should be allowed in fact should be required – to rethink his strategy after Smith moves.

Consider Jones's Strategy of (Small, Small) in equilibrium E_3. If Smith deviated from equilibrium by choosing Large, it would be unreasonable for Jones to stick to the response Small. Instead, he should also choose Large. But if Smith expected a response of Large, he would have chosen Large in the first place, and E_3 would not be an equilibrium. A similar argument shows that it would be irrational for Jones to choose (Large, Large) and we are left with E_2 as the unique equilibrium.

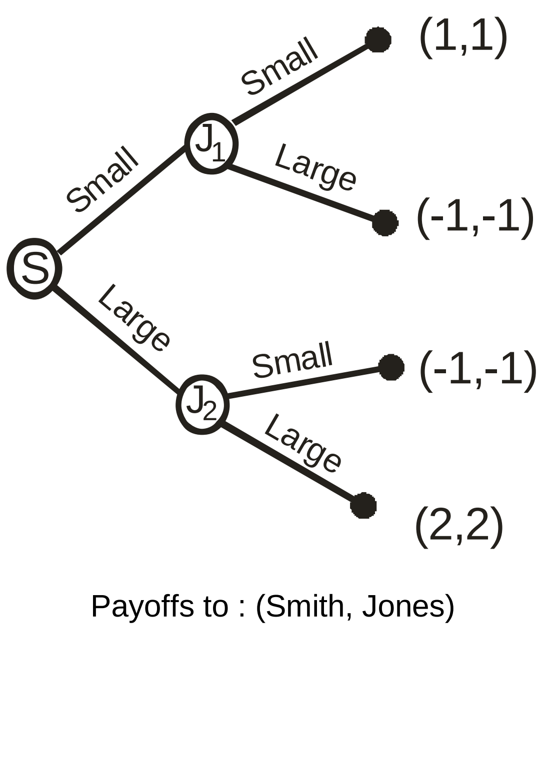
We say that equilibria E_1 and E_3 are Nash equilibria but not "perfect" Nash equilibria. A strategy combination is a perfect equilibrium if it remains an equilibrium on all possible paths, including not only the equilibrium path but all the other paths, which branch off into different "Subgames."

A subgame is a game consisting of a node which is a singleton in every player's information partition, that node's successors, and the payoffs at the associated end nodes.

A strategy combination is a subgame perfect Nash equilibrium if (a) it is a Nash equilibrium for the entire game; and (b) its relevant action rules are a Nash equilibrium for every subgame.

The extensive form of follow – the – Leader-I in figure 7.1 has three subgames: (1) the entire game, (2) the subgame starting at node J_1 and (3) the subgame starting at node J_2. Strategy combination E_1 is not a subgame perfect equilibrium, because it is only Nash in subgame (1) and (3), not in subgame (2) strategy combination E_3 is not a subgame perfect equilibrium, because it is only Nash in subgames (1) and (2), not in subgame (3). Strategy combination E_2 is perfect, because it is Nash in all three subgames.

Figure 8.1 Follow – the – Leader - I



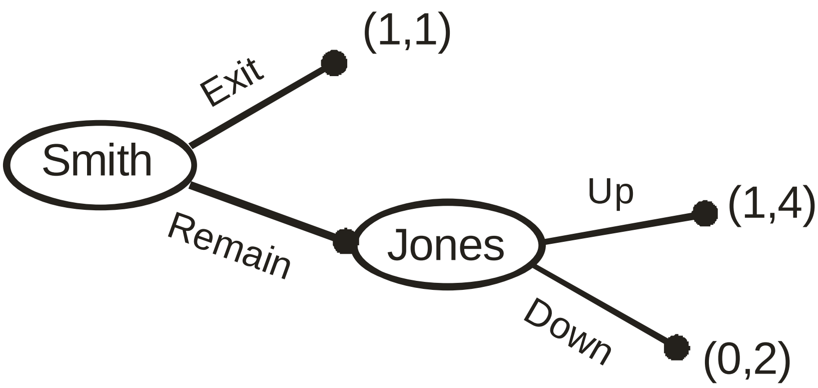
Payoffs to : (Smith, Jones)

The term sequential rationality is often used to denote the idea that a player should maximize his payoffs at each point in the game, re-optimizing his decisions at each point and taking into account the fact that he will re-optimize in the future. This is a blend of the economic ideas of ignoring sunk costs and rational expectations. Sequential rationality is so standard a criterion for equilibrium now that often I will speak of “equilibrium” without the qualifier when I wish to refer to an equilibrium that satisfies sequential rationality in the sense of being a “subgame perfect equilibrium” or in a game of asymmetric information, a “perfect Bayesian equilibrium”.

One reason why perfectness (the word “subgame” is usually left off) is a good equilibrium concept is because it represents the idea of sequential rationality. A second reason is that a weak Nash equilibrium is not robust to small changes in the game. So long as he is certain that Smith will not choose Large, Jones is indifferent between the never-to-be-used responses (Small if Large) and (Large if Large). Equilibria E_1,E_2 and E_3 are all weak Nash equilibria because of this. But if there is even a small probability that Smith will choose Large – perhaps by mistake then Jones would prefer the response (Large if Large), and equilibria E_1 and E_3 are no longer valid. Perfectness is a way to eliminate some of these less robust weak equilibria. The small probability of mistake is called a tremble.

For the moment, however, the reader should note that the tremble approach is distinct from sequential rationality consider the Tremble Game in figure 7.2. This game has three Nash equilibria all weak. (Out, Down), (Out, Up) and (In, Up). Only (Out, Up) and (In, Up) are subgame perfect, because although Down is weakly Jones’s best response to Smith’s out, it is inferior if Smith chooses in. In the subgame starting with Jones’s move, the only subgame perfect equilibrium is for Jones to choose up. The possibility of trembles, however, rules out (In, Up) as an equilibrium. If Jones has even an infinitesimal chance of trembling and choosing Down, Smith will choose out instead of In. Also, Jones will choose Up, not Down, because if Smith trembles and chooses In, Jones prefers Up to Down. This leaves only (Out, Up) as an equilibrium, despite the fact that it is weakly Pareto dominated by (In, Up).

Figure 8.2 The Tremble Game: trembling hand versus subgame perfectness.



8.4 INFINITELY REPEATED GAMES

The Prisoner’s Dilemma can be repeated an infinite number of times instead of finite number (after all, few economies have a known end date)

In fact, we can find a simple perfect equilibrium for the infinitely repeated Prisoner’s Dilemma in which both players co-operate a game in which both players adopt the Grim strategy.

Grim Strategy

1. Start by choosing Deny.
2. Continue to choose Deny unless some player has chosen confess, in which case choose confess forever.

Notice, that the Grim strategy says that even if a player is the first to deviate and choose confers, he continues to choose confess thereafter.

If column uses the Grim Strategy, the Grim Strategy is weakly Row’s best response. It Row cooperates, he will receive the higher (Confess, Deny) payoff once, but the best he can hope for thereafter is the (confess, confess) payoff.

Even in the infinitely repeated game, cooperation is not immediate, and not every strategy that punishes confessing is perfect. A notable example is the strategy of Tit-for Tat.

Tit - for - Tat

1. Start by choosing Deny
2. Thereafter, in period n choose the action that the other player chose in period (n-1)

If column uses Tit-for-Tat, Row does not have an incentive to confess first, because if Row cooperates he will continue to receive the high (Deny, Deny) payoff, but if he confesses and then return to Tit-for-Tat, the players alternate (confess, Deny) with (Deny, confess) forever, Row's average payoff from this alternation would be lower than if he had stuck to (Deny, Deny) and would swamp the one-time gain. But Tit-for-Tat is almost never perfect in the infinitely repeated Prisoner's Dilemma without discounting, because it is not rational for column to punish Row's initial confess. Adhering to Tit-for-Tat's punishments results in a miserable alternation of confess and Deny, so column would rather ignore Row's first confess. The deviation is not from the equilibrium path action of Deny, but from the off-equilibrium action rule of confess in response to confess. Tit-for-Tat, unlike the Grim strategy, cannot enforce cooperation.

Unfortunately, although eternal cooperation is a perfect equilibrium outcome in the infinite game under at least one strategy, so is practically anything else, including eternal confessing. The multiplicity of equilibria is summarized by the Folk Theorem, so called because its origin are hazy.

8.5 THE FOLK THEOREM

In an infinitely repeated n-person game with finite action sets at each repetition, any combination of actions observed in any finite number of repetitions is the unique outcome of some subgame perfect equilibrium given.

Condition 1 : The rate of time preference is zero, or positive and sufficiently small.

Condition 2 : The probability that the game ends at any repetition is zero, or positive and sufficiently small and

Condition 3 : The set of payoff combination that strictly Pareto dominate the minimax payoff combinations in the mixed extension of the one-shot game is n-dimensional.

The Folk Theorem tells us that claiming that particular behaviour arises in a perfect equilibrium is meaningless in an infinitely repeated game. This applies to any game that meets conditions, 1 to 3, not just to the Prisoner's Dilemma. If an infinite amount of time always remains in the game, a way can always be found to make one player welling to punish some other player for the sake of a better future, even if the punishment currently hurts the punisher as well as the punished. Any finite interval of time is insignificant compound to eternity, so the threat of future reprisal makes the players willing to carry out the punishments needed.

We will discuss conditions 1 to 3.

Condition 1: Discounting

The Folk Theorem helps answer the question of whether discounting future payments lessons the influence of the troublesome Last Period. To the contrary, with discounting, the present gain from confessing is weighted more heavily and future gains from cooperation more lightly. If the discount rate is very high the game almost returns to being one-shot. When the real interest rate is 1,000 percent, a payment next year is little better than a payment a hundred years hence, so next year is practically irrelevant. Any model that relies on a large number of repetitions also assumes that the discount rate is not too high.

Allowing a little discounting is none the less important to show there is no discontinuity at the discount rate of zero. If we come across an undiscounted, infinitely repeated game with many equilibria, the Folk Theorem tells us that adding a low discount rate will not reduce the number of equilibria. This contracts with the effect of changing the model by having a large but finite number of repetitions, a change which of ten eliminates all but one outcome by inducing the chainstore Paradox.

A discount rate of zero-supports many perfect equilibria, but if the rate is high enough, the only equilibrium outcome is eternal confessing. We can calculate the critical value for given parameters. The Grim strategy imposes the heaviest possible punishment for deviant behaviour. Using the payoffs for the Prisoner's Dilemma from the following table, the equilibrium payoff from the Grim strategy is the current payoff of 5 plus the value of the rest of the game.

Condition 2: A probability of the game ending

Time preference is fairly straight forward, but what is surprising is that assuming that the game ends in each period with probability θ does not make a drastic difference. In fact, we could even allow θ to vary over time, so long as it never became too large. If $\theta > 0$, the game ends infinite with probability one; or ; put less dramatically, the expected number of repetitions is finite, but it still behaves like a discounted infinite game, because the expected number of future repetitions is always large, no matter how may have already occurred. The game still has no Last Period, and it is still true that imposing one, no matter how far beyond the expected number of repetitions, would radically change the results.

The following two situations are different from each other.

1. The game will end at some uncertain date before T.
2. There is a constant probability of the game ending.

In situation (1), the game is like a finite game, because, as time passes, the maximum length of time still to run shrinks to zero. In situation (2), even if the game will end by T with high probability. If it actually lasts until T the game looks exactly the same as at time zero.

Condition 3: Dimensionality

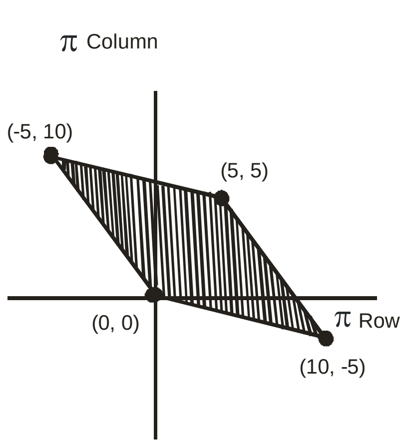
The "Minimax payoff" is the payoff that results if all the other players pick strategies solely to punish player i, and he protects himself as best he can.

The set of strategies S_{-i}^{i*} is a set of (n-1) minimax strategies chosen by all the players except i to keep i's payoff as low as possible, no mater how he responds. S_{-i}^{i*} solves.

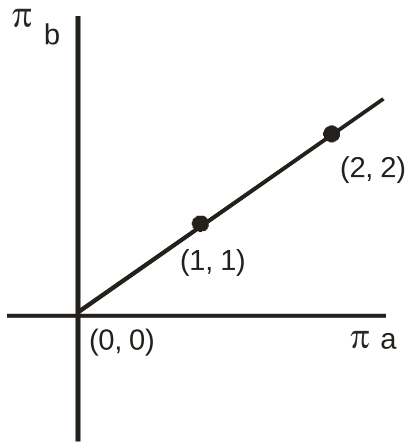
$$\underset{S_{-i}}{\text{Minimize}}\ \underset{S_i}{\text{Maximum}}\ JL_i\ (S_i, S_i), \text{.............}(1)$$

Player i's minimax payoff, minimax value or security value is his payoff from the solution (1).

The dimensionality condition is needed only for games with three or more players. It is satisfied if there is some payoff combination for each player in which his payoff is greater than his minimax payoff but still different from the payoff of every other player. Figure 1 shows how this condition is satisfied for the two-person Prisoner's Dilemma of table, but not for the two-person Ranked coordination game. It is also satisfied by the n-person Prisoner's Dilemma in which a solitary confessor gets a higher payoff than his cooperating fellow- prisoners, but not by the n-person Ranked coordination game, in which all the players have the same payoff. The condition is necessary because establishing the desired behaviour requires some way for the other players to punish a deviator without punishing themselves.



Prisoner's Dilemma



Ranked Coordination

Minimax and Maximin

In discussions of strategies which enforce cooperation, the question of deciding on the maximum severity of punishment strategies frequently arises. The idea of the minimax strategy is defined as the most severe sanction possible if the offender does not cooperate in his own punishment. The corresponding strategy for the offender, trying to protect himself from punishment, is the maximin strategy.

The strategy S_i^* is a maximin strategy for i if, given that the other players pick strategies to make i's payoff as low as possible, S_i^* gives i the highest possible payoff. In our rotation, S_i^* solves.

$$\underset{S_{-i}}{\text{Maximize}} \ \underset{S_i}{\text{Minimum}} \ \pi_i (S_i, S_{-i})$$

The following formulae show how to calculate the minimax and maximin strategies for a two-player game with player 1 as i.

$$\text{Maximin :} \quad \underset{S_1}{\text{Maximum}} \quad \underset{S_2}{\text{Minimum}} \ \pi_1$$

$$\text{Minimax :} \quad \underset{S_1}{\text{Minimum}} \quad \underset{S_2}{\text{Maximum}} \ \pi_1$$

In the Prisoner's Dilemma, the minimax and maximin strategies are both Confess. *Although the welfare Game (Table) has only a mixed strategy Nash equilibrium, if we restrict ourselves to the pure strategies the pauper's maximin strategy is Try to work,* which guarantees him at least 1, and his strategy for minimixing the Government is be Idle, which prevents the Government from getting more than zero.

Under minimax, player 2 is purely malicious but must move first (at least in choosing a mixing probability) in his attempt to cause player 1 the maximum pain. Under maximin, player 1 moves first, in the belief that player 2 is out to get him. In variable-sum-games, minimax is for sadists and maximin for paranoids. In zero-sum games, the players are merely neurotic: minimax is for optimists and maximin for pessimists.

The maximin strategy need not be unique, and it can be in mixed strategies, Since maximin behaviour can also be viewed as minimizing the maximum loss that might be suffered, decision theorist refer to such a policy as a minimax criterion, a catchier phrase (Luce & Raiffa [1957], p.279).

It is tempting to use maximin strategies as the basis of an equilibrium concept. A maximin equilibrium is made up of a maximin strategy for each player. Such a strategy might seen reasonable because each player then has protected himself from the worst harm possible. Maximin strategies have very little justification, however for a rational player. They are not simply the optimal strategies for risk – averse players, because risk-aversion is accounted for in the utility payoffs. The player’s implicit beliefs can be inconsistent in a maximin equilibrium and a player must believe that his opponent would choose the most harmful strategy out of spite rather than self-interest if maximin behaviour is to be rational.

The usefulness of minimax and maximin strategies is not in directly predicting the best strategies of the players, but in the bounds of how their strategies affect their payoffs as in condition 3 of the folk theorem.

it is important to remember that minimax and maximin strategies are not always pure strategies. In the Minimax Illustration Game of the following table.

Table The Minimax Illustration Game

	Column	
	Left	Right
Up	-2 , $\boxed{2}$	$\boxed{1}$, -2
Middle	$\boxed{1}$, -2	-2 , $\boxed{2}$
Down	0 , $\boxed{1}$	0 , $\boxed{1}$

Payoff to : (Row, Column)

Row can guarantee himself a payoff of 0 by choosing Down, so that is his maximin strategy. Column cannot hold Row's payoff down to 0, however, by using a pure minimax strategy. If column chooses Left, Row can choose Middle and get a payoff of 1 : if column chooses Right Row can choose Up and gen a payoff of 1. Column can however, hold Row's payoff down to 0 by choosing a mixed minimax strategy of (probability 0.5 of Left, Probability 0.5 of Right). Row would then respond with Down, for a minimax payoff of 0. Since either up, Middle or a mixture of the two would give him a payoff of – 0.5 (= 0.5 (-2) + 0.5 (1))

In two person zero sum games, minimax and maximin strategies are more directly useful, because when player 1 reduces players 2's payoff, he increases him own payoff. Punishing the other player is equivalent to rewarding yourself. This is the origin of the celebrated Minimax Theorem (Von Neumann [1928]), which says that a minimax equilibrium exists in pure or mixed strategies for every two person zero sum game and is identical to the maximin equilibrium. Unfortunately, the games that come up in applications are almost never zero sum games, so the Minimax Theorem is of limited applicability.

Precommitment

When if we use metastrategies, abandoning, the idea of perfectness by allowing players to commit at the start to a strategy for the rest of the game? We would still want to keep the game non cooperative by disallowing binding promises, but we could model it as a game with simultaneous choices by both players or with one move each in sequence.

If pre committed strategies are chosen simultaneously, the equilibrium outcome of the finitely repeated Prisoner's Dilemma calls for always confessing, because allowing commitment is the same as allowing equilibria to be non perfect, in which case, as was shown earlier, the unique Nash outcome is always confessing.

A different result is achieved if the players pre commit to strategies in sequence. The outcome depends on the particular values of the parameters, but one possible equilibrium is the following : Row moves first and chooses the strategy (Deny until Column Confesses; thereafter always Confers), and Column chooses (Deny until the last period; then Confess). The observed outcome would be for both players to deny until the last period, and then for Row to again deny, but for Column to Confess. Row would submit to this because of he chose a strategy that initiated confessing earlier, Column would choose a strategy of starting to confess earlier too. The game has a second mover advantage.

8.6 SUMMARY

Thus, Nash equilibrium is the standard equilibrium concept in economics. It is less obviously correct than dominant strategy equilibrium but more often applicable. Nash equilibrium is so widely accepted that the reader can assume that if a model doesnot specify which equilibrium concept is being used it is Nash or some refinement of Nash.

8.7 FURTHER READINGS

(1) Rasmusen E. : Games and Information, Blackwell, 1994

(2) Milgrom R. and J. Roberts, Economics, Organization and Management, Prentice Hall, 1992.

8.8 QUESTIONS FOR REVIEW

(1) Define and Explain Nas Equilibrium.

(2) What is the Battle of the Sexes?

(3) Write a note on co-ordination games.

(4) Explain the sub game perfect Nash Equilibrium.

(5) Define and explain the Folk Theorem.

❖❖❖❖

9

AN INTRODUCTION TO GAME THEORY

UNIT STRUCTURE

9.0 OBJECTIVES

After going through this unit, you will be able to :

- Understand the reasons of market failure
- Understand the market for lemons and the market for used cars.
- Know the market for credit
- Explain the concept of adverse selection
- Give examples of adverse selection, like, car-insurance, health insurance.
- Understand the concept of moral hazard.
- Know the difference between hidden action and hidden information.

9.1 INTRODUCTION :

The traditional theory of market behaviour is based on the assumption that buyers and sellers have the same access to all types of market information, including the quality of goods and services being exchanged. Thus information is a free good and has not to be acquired at a cost. In such a situation consumer welfare is likely to be maximized. In fact, one of the conditions of the existence of perfect competition is the presence of well informed buyers and sellers. This condition is satisfied due to the fact that a homogeneous product is being traded at a uniform price.

The assumption that buyers and sellers are both perfectly informed about the quality of goods being sold in the market is valid so long as the quality of an item can be easily identified and verified. If it does not cost much money to find out which goods are of high quality and which are of low quality, then the market prices of goods will simply adjust to reflect differences in product quality.

But in reality we often find that information is an economic good, not a free good as it is under perfect competition. This is due to market imperfection created by differences in product quality, so most real life markets are characterized by quality uncertainty.

If information about product quality is not only difficult to obtain buy costly as well then it is not possible for buyers and sellers to have the same information about goods involved in transaction. In many real life markets it is very difficult or virtually impossible to obtain accurate and timely information about the quality of the goods being traded.

Information problems arise in markets such as the labour market the market for consumer goods. In consumer goods markets information is costly to obtain. When an individual buys a used car it is very difficult for him to determine whether or not it is a good car (called a plum) or a bad car (called a lemon). By contrast, the seller of the used car has fairly good idea about the quality of the car. Thus asymmetric information often causes major problems for the efficient functioning of a market. George Akerlof first pointed out some of these difficulties which arise in market where second hand goods, such as used cars are sold.

9.2 MARKET FAILURE :

Asymmetric information is a source of market failure. The problem is that there is an externality between the sellers of good cars (Plums) and bad cars (lemons). As Hal Varian has rightly argued, "When an individual decides to try to sell a bad car, he affects the purchasers" perception of the quality of the average car on the market.

This lowers the price that they are ready to pay for the average car, and thus is injurious to people who are eager to sell cars in good condition. It is this perceived demand externality that creates the market failure.

Most cars that are offered for sale in the used car markets are those that people want most to get rid of such as the Fiat cars in India at present. The very act of offering to sell something sends a signal to the prospective buyers about its quality. If too many low quality products are offered for sale, the owners of high quality items will find it difficult to sell their products. At times the possibility of low quality production destroys the market for both for high and low quality products. This point may now be discussed in detail.

9.2.1 Quality Uncertainty and the Market for Lemons :

Akerlof has pointed out that used cars sell for much less than new cars because there are asymmetric information about their quality. In all likelihoods the seller of a used car knows much more about the car than the prospective buyer does. The buyer discovers the quality only after buying the car and driving it for sometime. Moreover the very fact that an individual wants to dispose of his old car indicates that it may be 'lemon'. No rational person is eager to sell his reliable car. Consequently the prospective buyer of a used car will always face quality uncertainty and will, therefore, will be suspicious of its quality.

Even markets for insurance, financial credit and even employment are characterized by what Akerlof calls asymmetric quality information. We may now analyse the implications of asymmetric information in the context of the market for used car.

9.2.2 The Market for Used Cars :

Let us suppose both high-quality and low-quality used cars are available in the market. We also assume that both sides of the market are aware of the quality of two kinds of used cars. Then there will emerge two markets as shown in Fig.8.1. In part (a) S_H is the supply of high-quality cars and D_H is the demand curve. Likewise, S_L and D_L in part (a) are the supply and demand curves for low-quality cars. For any existing price, S_H lies to the left of S_L because owners of high-quality cars are not normally eager to part with them and can be induced to do so only by offering a high or an attractive price. For the same reason D_H lies above D_L : buyers are ready to pay more in order to acquire a high-quality car. In Fig.8.1, the market price of a high quality car is Rs.60,000/-, for low quality cars Rs.30,000/- and 500 cars of each type are sold initially.

Since, in reality sellers know the quality of cars, but buyers do not, buyers might initially think that the odds are 50 – 50 that a car bought will be of high quality. The reason for this is that when both sellers and buyers know the quality, 500 cars of each type were sold. When deciding to purchase a car, a buyer would, therefore treat all cars as 'medium' quality. The demand for medium-quality cars D_M lies below D_H but above D_L. In Fig. 8.1 fewer high quality cars (250) and more low-quality cars (750) will be sold.

Sooner or later consumers will be able to realize the most cars sold (about 75%) are of low quality. Consequently their perceived (imaginary), not actual, demand curve shifts. The new perceived demand curve might be D_{LM}, which means that, on average, cars are thought to be of low to medium quality. However, the mix of cars in then tilted more heavily in favour of low-quality cars. Consequently the perceived demand curve shifts further to the left, pushing the mix of cars even further toward low quality. This shifting continues until the Gresham's law operates – that is, low-quality cars drive good quality cars out of the market. This means that only low quality cars are sold. This is known as the lemons problem. In this situation, the market price will fall to such a low level that no high-quality can will be offered for sale. And this will create the impression among the buyers that any car they buy will be of low quality and the only relevant demand curve will be D_L.

Of course, we have demonstrated a polar case in Fig. 8.1. Such an extreme situation in unlikely to exist in reality. A more likely outcome is that the market reaches equilibrium at a price that brings forth at least some high-quality cars. But the fraction of high-quality cars will be much smaller than it would be if consumers could identify quality before purchasing a car. But the truth is that a consumer learns about the true quality of a car only after using it for sometimes.

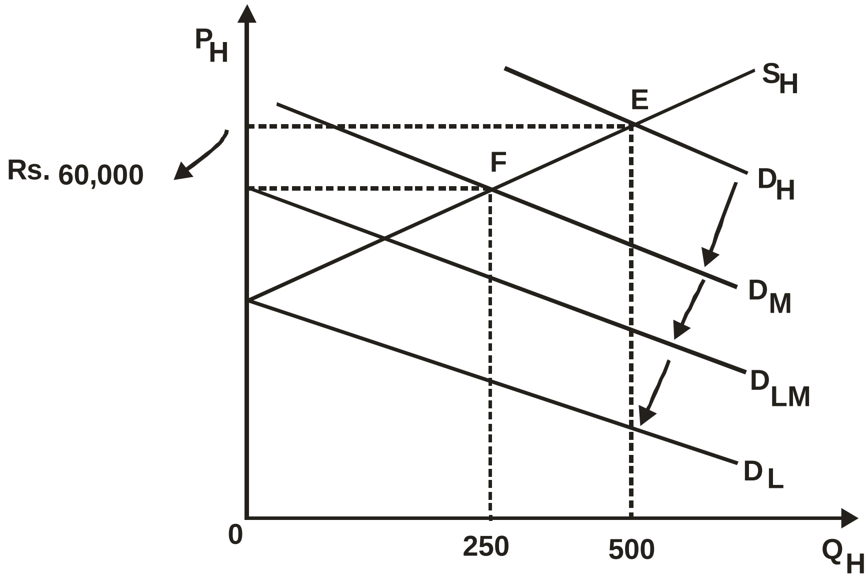


Fig. 9.1(a) High-quality used cars

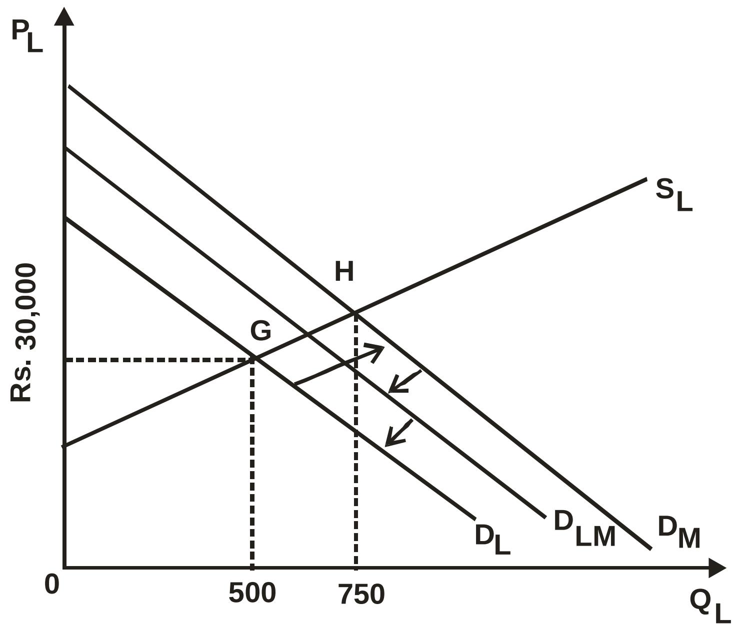


Fig. 9.1(b) Low-quality used cars

9.2.3 The Market for Credit :

By using credit cards many people borrow money without providing any collateral. Most credit cards permit the holder to run a debit balance of several thousand rupees and many people hold several credit cards. Banks issue credit cards and make money by charging interest on the debit balance. But it is not possible for a bank to distinguish between high-quality borrowers (who repay their loans) from low-quality borrowers (who do not). Clearly borrowers are more informed than lenders – i.e. they know more about whether they will pay than the lender does. So another type of lemon problem arises. Banks must charge the same interest to all borrowers. This encourages low-interest borrowers to acquire credit cards in large numbers. This pushes up the rate of interest, which increases the number of low-quality borrowers, which, in its turn, pushes the rate of interest further, and so on.

Now-a-days many banks use computerized credit histories to distinguish 'low-quality' from 'high-quality borrowers'. This practice eliminates or at least greatly reduces the problem of asymmetric information and adverse selection, which might otherwise prevent credit markets from operating. Without these histories, it would be difficult even for the most credit worthy person to borrow money in times of need.

Asymmetric information is present in various other markets. A few examples are given below.

(a) Retails Stores : Most retail stores do not allow the customers to return an unsuitable merchandise such as defective products. The store knows more about it's products than do the customers.

(b) Seller of rare stamps, coins, books and works of art : The dealers know more about the authenticity of such rare goods than the customers know. It is not possible for the customers to ascertain whether these items are real or counterfeit.

(c) Masons, plumbers and electricians : When a mason repairs or innovates the roof of an individuals house, it is not possible for him to climb up to check the quality of the work. So payment has to be made on the basis of what the plumber says.

(d) Restaurants : Near the entrance of every restaurant kitchen there is a notice, which reads 'entry is prohibited'. This means that it is not possible for a prospective customer to enter the kitchen to check whether the cook is using fresh ingredients and is obeying the health norms.

(e) Private tuition : A private tutor know more about his competence than a student does. It is not possible for a student to assess the quality of teaching in most cases. And there is no guarantee that if he quits and takes tuition from another teacher he will be better served.

9.3 ADVERSE SELECTION :

In many markets we find that the low-quality items crowd out the high-quality items due to the high cost of acquiring information.

This is an example of adverse selection. In some situation the adverse selection problem is so serious that it can completely destroyed the market. In fact, this problem leads to market failure.

Examples

We may now consider a few examples of adverse selection. Let us take the case of insurance industry first.

(a) Car Insurance :

Suppose that an insurance company wants to offer insurance for car theft. It carries out a market survey and finds out that the incidence of theft varies widely from place to place. In some places there is a high probability that a car will be stolen, and in other areas car theft is an exception rather than a regular occurrence. Suppose that the insurance company decides to offer the insurance based on the average theft rate. Why is this likely to happen? Will the insurance company's proposal evoke favourable response?

It appears that the people in the safe, communities, who do not need much insurance, are unlikely to buy the insurance at the average rate, at least in large numbers. In contrast, people who live in areas with a high incidence of car theft with will want insurance because they really need it.

But this means that the insurance claims will be made mostly by clients who live in the high-risk areas. Insurance premium rates based on the average probability of theft will be a misleading indication of the actual experience of claims filed by the company. The insurance company will not succeed in making an unbiased selection of customers; thus it will make an adverse selection.

In such a situation two outcomes are possible :

1. In order to break even, i.e. cover all costs, the insurance company must fix its rate on the basis of the worse possible outcomes.
2. In contrast, customers with a low, but not negligible, risk of car theft will be unwilling to purchase the resulting high-premium insurance policy.

(b) Health Insurance :

Adverse selection refers to the problem, encountered in the health insurance industry, that the sub-population taking out insurance is likely to have less favourable characteristics than the population in General. In fixing the rates of premium for life insurance, for example, a company may use the age – specific mortality rates for the population as a whole. Non-smokers know that their mortality rates are less than that of the population as a whole and that they are subsidizing the smokers in the premium payment. They will, therefore, be reluctant to insure themselves. Smokers have higher mortality rates than the population as a whole and are aware that they are being subsidized by non-smokers. So due to this positive demand externality, they have an incentive to insure themselves. Thus the insurance company ends up with an adverse selection of people with higher than average mortality rates.

Health insurance companies can not fix their premium rates on the average incidence of health problems in the population. They can only fix their premium rates on the average incidence of health problem in the group of potential buyers of insurance policies. But most people who want to purchase health insurance are the ones who are likely to need it most and the efficient functioning of the insurance market must reflect this divergence between cost and benefit among different individuals with the different individuals with the different health conditions. But in reality this does not happen and the adverse selection problem crops up. The problem can be solved by the introduction of policy targeted at specific sub-population with different premia for each such populations.

In such a situation everyone can be made better off by fixing the insurance premium on the basis of the average risk in the population. The people facing high risk are better off because they can purchase insurance at rates that are lower-than the actual risk they face and the people facing the low risk can purchase insurance that is favourable to them, then insurance can be offered if and only if high risk people purchase it and bear the major portion of the cost incurred by the insurance company.

This outcome is possible where the market is dominated by a compulsory purchase plan. Although in theory we say that “more choice is better”, in this case restricting choice can result in a Pareto improvement. This paradoxical result is due to the externality between the low risk and high-risk people.

This market inefficiency can be removed by employers by offering health plans to employees as part of the package of fringe benefits. This insurance company can base its sales on the average over the set of employees and is assured that all employees participate in the programme. This eliminates the adverse selection problem.

9.4 MORAL HAZARD :

The insurance industry faces another interesting problem, known as moral hazard. In the context of car insurance let us assume, for the sake of simplicity, that all the car owners live in the areas with identical probabilities of theft. So there is no problem of adverse selection. On the other hand the probability of theft may be affected by the safety measures taken by the car owners.

For example, if the car owners do not care to lock their cars or use only a flimsy lock (as is done in India by the owners of Ambassador or Fiet Cars) the car is much more likely to be stolen than if a strong and secure lock is used. Similarly in case of health insurance, the customers are less likely to need the insurance if they take sufficient health care. Actions that affect the probability that some event occurs is known as taking care.

When it fixes the premium rates the insurance company has to take into account the incentives that the consumers are likely to have in order to take adequate care. In most cases where no insurance is available consumers have an incentive to take maximum possible care. If there is no such thing as car insurance, then all car owners will use the best possible locks. Some will even use double lock system at high costs. In such a situation the individual bears the full cost of his actions and so he is eager to 'invest' in taking care until, at the margin, the cost and benefit of doing so are the same.

But if the car owner can take an insurance, then the cost imposed on him for his car being stolen is much less. In case the car is stolen then the person has just to report the incidence to the insurance company within a stipulated time period and he will get sufficient compensation to be able to buy another car. In the extreme situation, where the insurance company gives full compensation to the individual for the theft of car, he has hardly any incentive to take care of his car. This lack of incentive on the part of a car owner to take care office car is known as moral hazard.

Moral hazard refers to the effects of certain types of insurance system in causing a divergence between the private marginal cost of some action and the marginal social cost of the action, thus resulting in a sub-optimal allocation of resources. For example, a person may be insured against illness at a cost which is less than the cost of medical care to society consequently he may increase his use of medical facilities beyond the socially optimal level.

If an individual has full medical coverage, he may visit the doctor more often than he would if his coverage were limited. If the insurance company can monitor the behaviour of insured persons, it can charge higher fees from these who make more claims. But if this is not possible, it may find it's payments of claims to be larger than expected. If moral hazard problem is present insurance companies may be forced to increase premium for every client or refuse to sell insurance at all.

This, in its turn, would lead insurance companies to recalculate premiums in case of all types of customers. The problem of moral hazard leading to overuse of facilities arise both in private insurance system and in some state control agreements.

There is thus a trade-off in the insurance business. Too little insurance means that people bear a lot of risk, too much insurance means that people will take inadequate care to protect themselves from theft or health hazards.

If the amount of care is observable the problem gets solved automatically. The insurance company can fix its rates on the amount of care taken. In practice insurance companies after different rates to businesses that have fire protection system in their buildings, or to charge smokers different rates than non-smokers for health insurance. In such cases the insurance firm seeks to discriminate among users depending on the choices they have made that influences the likelihood of damage.

But insurance companies are not able to observe all the relevant actions of all the insured people so a basic trade-off is involved : full insurance means that too little care will be taken because the individuals are not required to bear the full costs of their actions.

The moral hazard problem arises when an insured person whose actions cannot be observed affect the probability of an event occurring or the magnitude of a payment associated with an event. When, for example, an individual get this insurance cover he takes less care to ensure that an accident does not occur than when he is not insured. If for example, an individuals' house is fully insured against theft, he may not lock the doors properly when he leaves the house. There is always the possibility that an individuals behaviour will change simply because he has insurance. This is indeed the moral hazard problem.

For this reason most insurance companies are reluctant to offer complete insurance to their customers. They will always try to partially shift the risk to the customers. This is why most insurance policies include a 'deductible', i.e. an amount that the insured party has to pay in any claims. By forcing the sacrifice a portion of a claim the insurance companies are able to create an incentive among the customers to take at least a minimum amount of care. No doubt no insurance company is willing to insure a customer completely if it is able to verify the amount of theft or medical care taken. Yet the company will not allow the customer to purchase as much insurance as he wants if the company cannot observe the levels of care he takes. Thus, although in theory a customer can choose any amount of care, in practice he cannot do it.

Moral hazard problem also arises in case of workers who perform below their capabilities when employers cannot monitor their behaviour. This known as job shirking.

9.4.1 Difference with a Competitive Solution :

Thus a paradoxical result is found when moral hazard problems are present. In a competitive market the amount of a good that is traded is determined by demand (which indicates the marginal willingness to pay) and supply (which indicates the marginal willingness to sell). In the presence of moral hazard, a market equilibrium has the property that each customer would like to buy more insurance. But insurance companies would be eager to provide more insurance if the customers continued to take some amount of care. But this not occur. The reason is that if the customers were able to purchase more insurance they would take less care. This is, no doubt, rational behaviour.

9.5 INEFFICIENT ALLOCATION OF RESOURCES :

The moral hazard problem is not faced only by insurance companies. It also obstructs the efficient allocation of resources by the market. Fig. 8.2 show the effects of moral hazard. Here D is the demand for automobile driving in miles per day. The demand curve, which shows the marginal benefits of driving, is downward sloping because most people develop alternative means of transport as the cost of driving increases. Let us suppose that initially the cost of driving includes the insurance cost and that insurance company can accurately measure the distance traveled. In this case the marginal cost of driving is given by MC in the absence of moral hazard. Car owners know that more driving will lead to a rise in insurance premium and so increase their total cost of driving (The cost per mile of driving is assumed to remain constant). If, for instance, the cost of driving is Rs.6/- per mile. (50% of which is insurance cost) car owners will travel 30 miles per day.

A moral hazard problem will arise if insurance companies are unable to monitor individual driving habits, in which case the insurance premium will not depend on miles driven. This means that the driers will take it for grated that any additional cost of accident that they incur will be divided – among a large group of people, with a very small fraction of such cost will have to be borne by a single individual. In other words, every driver will get a market size (or, scale) effect. Since their insurance premium does not vary with the distanced covered, an additional miles of driving will cost Rs.4/-. This point is illustrated by a downward shift of the marginal cost curve to MC'. As a result the number of miles driven will increase from 30 miles to the socially inefficient level of 40 miles per day.

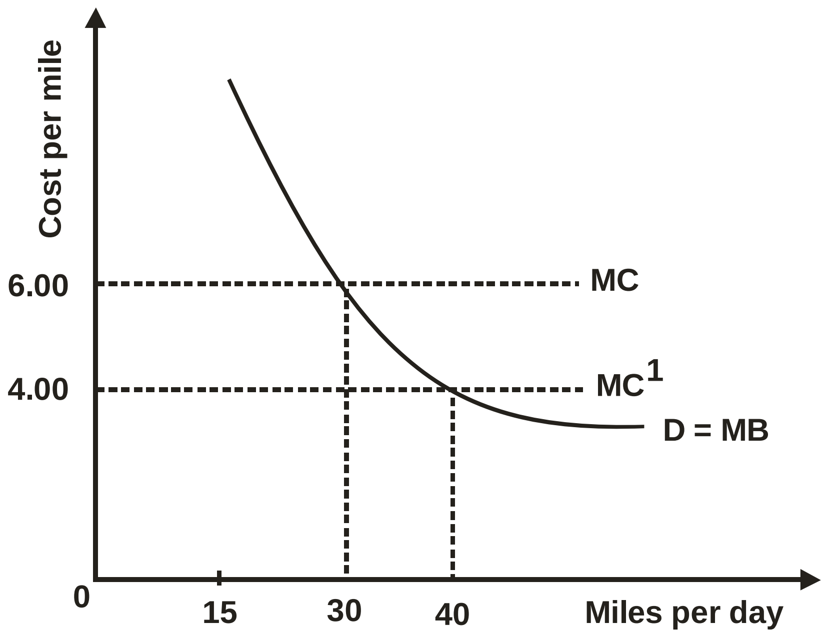


Fig. 9.2 The Effects of Moral Hazard

Moral hazard creates economic inefficiency simply because the insured individual perceives either the cost or the benefit of the activity differently from the true social cost or benefit. In Fig. 8.2 the efficient level of driving is given by the intersection of the marginal benefit (MB) and marginal cost (MC') curves. In the presence of moral hazard an individual car owner's perceived marginal cost (shown by the curve MC') is less than the actual cost (indicated by the curve MC), and the number of miles traveled per day (40) is higher than the socially optimal level (30 mile) at which marginal benefit is equal to marginal cost.

Labour Market :

Moral hazard problem also arises in the labour market. The efficiency wage theory holds that a high wage improves worker effort. The main postulate of this theory is that firm cannot perfectly monitor the work effort of their employers and that workers must themselves decide how hard to work. Workers are left with two options either they can work hard or they can choose to shirk and run the risk of losing their job when caught. This is a real life example of moral hazard. This refers to the tendency of workers to behave inappropriately when their behaviour is not properly monitored of course, by paying a high wage it is possible to solve the problem. The higher the wage, the more attractive is the job to a worker because he knows that if the loses this lucrative job he may not get another very quickly. So the higher is the potential loss in case of job loss (if the worker gets fired). By paying a higher wage, the firm can prove that prevention is better than cure – it can at least induce most of its employees not to shirk. This may lead to an increase in labour productivity.

9.5.1 Hidden Action Vs Hidden Information :

Moral hazard creates hidden action problem because one side of the market cannot observe the actions of the other. Equilibrium in a market involving hidden action requires some from of allocation or rationing – firms would like to provide more than they normally do, but they are reluctant to do so since it will change the incentive of their customers.

In contrast, adverse selection creates a hidden information problem because on side of the market cannot observe the 'type' or quality of the goods on the other side of the market. Equilibrium in a market involving this problem will lead to undertrading. Too little trading will take place due to the externality between the 'good' and 'bad' types. The equilibrium will always be inefficient relative to the equilibrium with free and full information.

9.5.2 Difference between the two

Very often, however, the probability and the extent of damage do not remain fixed. If efforts to reduce the chance and the extent of damage are costly to observe, buying insurance will reduce the incentive of insurance companies to supply such efforts. This is known as moral hazard. Methods to mitigate moral hazard include coinsurance and deductibles. Moral hazard is a principal – agent problem.

Adverse selection occurs when the insured individual has more information about the probability of a loss than the insurance company does. It is different from moral hazard in that the latter involves a change in the behaviour of insured party, while adverse selection involves only an asymmetric information between the insurer and the insured.

9.6 SUMMARY :

- Asymmetric information is the source of market failure.
- Akerlof has pointed out that used cars sell for much less than new cars because there are asymmetric informations about their quality.
- Low quality items, many a times, crowding out the high quality items due to the high cost of acquiring information. This is an example of adverse selection.

9.7 FURTHER READINGS :

1. Rasmusen E. : Games and Information, Blackwell, 1994.
2. Silberg E. : The Structure of Economics.

9.8 QUESTIONS FOR REVIEW :

1. Explain the concept of Market Failure with appropriate example.
2. When and how the adverse selection takes place?
3. Explain the concept of Moral Hazard.
4. Distinguish between Hidden Action and Hidden Information.

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10

HIDDEN ACTION & MORAL HAZARD

UNIT STRUCTURE

10.0 OBJECTIVES

After going through this unit you will come to know –

- the categories of asymmetric information models.
- the concepts of signalling & screening.
- the principal agent problem.
- production game – I: full information.
- production game – II: full information : Agent moves first.
- production game – III: a flat wage under certainty.
- production game – IV: an output based wage under certainty.
- production game – V: an output based wage under uncertainty.

10.1 INTRODUCTION

Imperfect, incomplete and asymmetric information leads to market failure and departure from the ideal world of perfect competition and Pareto optimality. Asymmetric information refers to a situation in which buyers and sellers do not have the same information regarding product quality. So it refers to qualitative uncertainty. In fact one of the main sources of market failure is asymmetric information.

Hence, we will discuss the categories of asymmetric information models and situation of market failure in this unit.

10.2 THE CATEGORIES OF ASYMMETRIC INFORMATION MODELS

It used to be the economist's generic answer to someone who brought up peculiar behaviour which seemed to contradict basic theory was "It must be some kind of price discrimination." Today, we have a new answer. "It must be some kind of asymmetric information." In a game of asymmetric information, Prayer Smith knows something that player Jones does not. This covers a broad range of models (Including price discrimination now a days), so perhaps it is not surprising that so many situations come under its rubric. We will divide games of asymmetric information into five categories.

1. **Moral hazard with hidden actions:**

   Smith and Jones begin with asymmetric information and agree to a contract, but then Smith takes an action unobserved by Jones, Information is complete.

2. **Adverse selection :**

   Nature begins the game by choosing Smith's type (his payoff and strategies), unobserved by Jones. Smith and Jones then agree to a contract, Information is incomplete.

3. **Mechanism design in adverse selection and in moral hazard with hidden information:**

   Jones is designing a contract for Smith designed to elicit Smith's private information. This may happen under adverse selection – in which case Smith knows the information prior to contracting- or moral hazard with hidden information – in which case Smith will learn it after contracting.

10.3 SIGNALLING AND SCREENING

Signalling and Screening: Nature begins the game by choosing Smith's type, unobserved by Jones. To demonstrate his type, Smith takes actions that Jones can observe. If Smith takes the action before they agree to a contract, he is signalling, if he takes it afterwards, he is being screened information is incomplete.

Signalling and screening are special cases of adverse selection, which is itself a situation of hidden knowledge. Information is complete in either kind of moral hazard, and incomplete in adverse selection, signaling and screening.

Note that some people may say that information becomes incomplete in C model of moral hazard with hidden knowledge, even though it is complete at the start of the game. That statement runs contrary to the definition of complete information however, the most important destinations to keep in mind one whether or not the players agree to a contract before an after information becomes asymmetric and whether their own actions are common knowledge.

We will make heavy use of the principal agent model to analyze asymmetric information. Usually this term is applied to moral hazard models, since the problems studied in the law of agency usually involve an employee who disobeys orders by choosing the wrong actions, but the paradigm will be useful in all of these contexts. The two players one the principal and the agent, who use usually representative individuals. The principal hires an agent to perform a task, and the agent acquires an informational advantage about his type, his actions, or the outside world at some point in the game. It is usually assumed that the players can make a binding contract at some point in the game, which is to say that the principal can commit to paying the agent an agreed sum if he observes a certain outcome. Implicity is the background of such models are courts which will punish any player who breaks a contract in a way that can be proven with public information.

The principal (or uninformed player) is the player who has the coarser information partition.

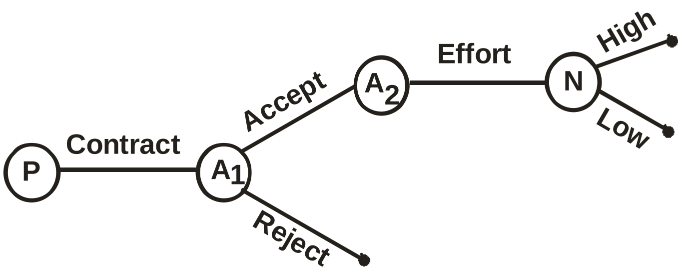
The agent (or informed player) is the player who has the finer information partition.

Figure 10.1 shows the game trees for five principal agent problems corresponding to the categories listed above. In each model the principal (P) offers the agent (A) a contract, which he accepts or rejects. In some, Nature (N) makes a move or the agent chooses an effort level, message or signal. The moral hazard models, (a) and (b), are games of complete information with uncertainty. The principal offers a contract, and after the agent accepts, Nature adds noise to the task being performed. In moral hazard with hidden actions (a) in Fig. 1, the agent moves before Nature, and in moral hazard with hidden Knowledge, (b) in Fig. 1, the agent moves after Nature and conveys a message to the principal about Natures move.

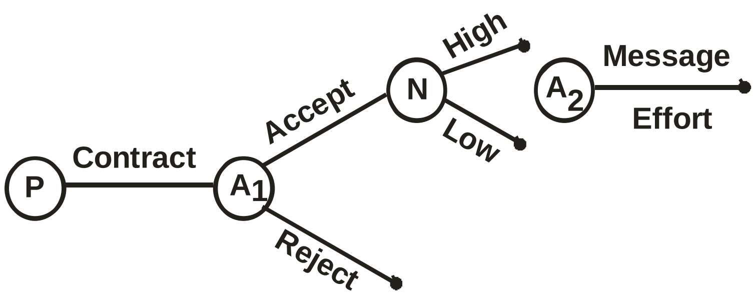
Adverse selection models have incomplete information, so Nature moves first and picks the type of the agent, generally on the basis of his ability to perform the task. In the simplest model, Fig. 1(c), the agent simply accepts or rejects the contract. If the agent can send a “signal” to the principal, as in Fig. 1 (d) and 1(e), the model is signaling if he sends the signal before the principal offers a contract, and his screening otherwise. A “Signal” is different from a “message” because it is not a costless statement, but a costly action. Some adverse selection models contain uncertainty and some do not.

A problem we will consider in detail is that of an employer (the principal) hiring a worker (the agent). If the employer knows the worker’s ability but nor his effort level.

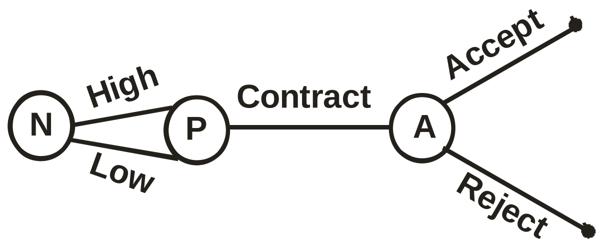
a) Moral hazard with hidden action



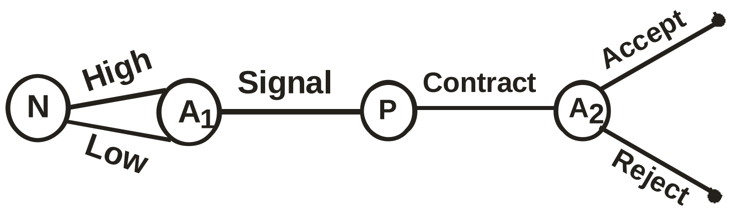
b) Moral hazard with hidden information



c) Adverse selection



d) Signalling



e) Screening

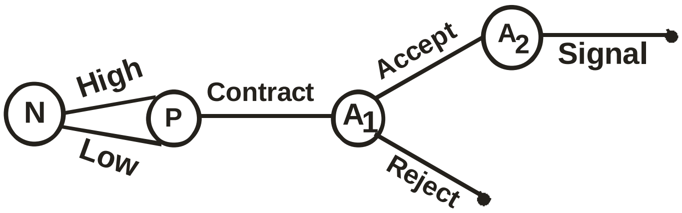


Figure 10.1 Categories of asymmetric information models.

The problem is moral hazard with hidden action. If neither player knows the worker's ability at first, but the worker discovers it once he stands working, the problem is moral hazard with hidden knowledge, if the worker knows his ability from the start, but the employer does not, the problem is adverse selection.

If in addition to the worker knowing his ability from the start, he can acquire credentials before he makes a contract with the employer, the problem is signalling. If the worker acquires his credentials in response to a wage offer made by the employer, the problem is screening.

Table 10.1 Application of the principal- agent model

	Principal	**Agent**	**Effort or type and signal**
Moral hazard with hidden actions	Insurance company Insurance company plantation owner Bond holders Tenant Landlord Society	Policy holder Policy holder Share cropper Stock holders Landlord Tenant Criminal	Care to avoid theft Drinking and smoking Farming effort Riskiness of corporate projects Upkeep of the building Upkeep of the building Number of robberies
Moral hazard with hidden knowledge	Shareholders FDIC	Company president Bank	Investment decision Safety of loans
Adverse selection	Insurance company Employer	Policy holder Worker	Infection with HIV virus Skill
Signalling and Screening	Employer Buyer Investor	Worker Seller Stock Issuer	Skill and education Durability and warranty stock value and percentage retained.

10.4 A PRINCIPAL – AGENT MODEL : THE PRODUCTION GAME

In the archetypal principal – agent model, the principal is a manager and the agent worker. In this section we will devise a series of these types of games, the last of which will be the standard principal – agent model.

Denote the monetary value of output by q(e), which is increasing in effort, e. The agent's, utility function U (e,w) is decreasing in effort and increasing in the wage, w, while the principals utility V(q-w) is increasing in the difference between output and the wage.

The Production Game

Players
The principal and the agent

The order of play.

1. The principal offers the agent a wage w.
2. The agent decides whether to accept or reject the contract.
3. If the agent accepts, he exerts effort e.
4. Output equals q(e), where q' > 0.

Payoffs.

If the agent rejects the contract, then π agent = $\bar{U}$ and π principal = 0

If the agent accepts the contract, then π agent = U (e , w) and π principal = V (q – w)

As assumption common to most principal –agent models is that either the principal or the agent is one of many perfect competition. In the background, other principals compete to employ the agent, so the principal's equilibrium profit equals zero, or many agents compete to work for the principal, so the agents equilibrium utility equals the minimum for which he will accept the job, called the reservation utility, $\bar{U}$. There is some reservation utility level even if the principal is a monopolist, however, because the agent has the option of remaining unemployed if the wage is too low.

One way of viewing the assumption in the Production Game that the principal moves first is that many agents compete for one principal. The order of moves allows the principal to make a take-it-or-leave-it offer, leaving the agent with as little bargaining room as it he had to compete with a multitude of other agents. This is really just a modelling convenience, however, since the agents reservation utility, $\bar{U}$, can be set at the level a principal would have to pay the agent in competition with other principals. This level of $\bar{U}$ can even be calculated, since it is the level at which the principal's payoff from profit maximization using the optimal contract is driven down to the principals reservation utility by competition with other principals. Here the principal's reservation utility is zero, but that too can be chosen to fit the situation being modeled. As in the game of

Nuisance Suits, the main concern in choosing who makes the offer is to avoid getting caught up in a bargaining subgame.

Refinements of the equilibrium concept will not be important here; Nash equilibrium will be sufficient for our purposes, because information is complete and the concerns of perfect Bayesian equilibrium will not arise. Subgame perfectness will be required, since otherwise the agent might commit to reject any contract that does not give him all of the gains from trade, but it will not drive the important results.

10.5 PRODUCTION GAME I : FULL INFORMATION

In the first version of the game, every more is common knowledge and the contract is a function w (e).

Finding the equilibrium involves finding the best possible contract from the point of view of the principal, given that he must make the contract acceptable to the agent and that he foresees how the agent will react to the contract's incentives. The principal must decide what he wants the agent to do and how to give him incentives to do it as cheaply as possible.

The agent must be paid some amount $\tilde{w}$ (e) to exert effort e, where $\tilde{w}$ (e) is defined to be the w that solves the participation constraint.

$$U(e, w(e)) = \bar{U} \tag{9.1}$$

Thus, the principal's problem is

$$\underset{e}{\text{Maximize}}\, V(q(e) - \tilde{w}(e)) \tag{9.2}$$

The first-order condition for this problem is

$$V'(q(e) - \tilde{w}(e))\left(\frac{\partial q}{\partial e} - \frac{\partial \tilde{w}}{\partial e}\right) = 0 \tag{9.3}$$

Which implies that

$$\frac{\partial q}{\partial e} = \frac{\partial \tilde{w}}{\partial e} \tag{9.4}$$

From the implicit function theorem and the participation constraint,

$$\frac{\partial \tilde{w}}{\partial e} = -\left(\frac{\frac{\partial U}{\partial e}}{\frac{\partial U}{\partial \tilde{w}}} \right) \tag{9.5}$$

Combining equations (7. 4) and (7.5) yields.

$$\frac{\partial U}{\partial \tilde{w}} \frac{\partial q}{\partial e} = -\frac{\partial U}{\partial e} \tag{9.6}$$

Equation (9.6) says that at the optimal effort level, e*, the marginal utility to the agent which would result if he kept all the marginal output from extra effort equals the marginal disutility to him of that effort. As usual, the outcome can be efficient even if the agent does not actually keep the extra output, since who keeps the output is a distributional question.

Figure 10.2 shows this graphically. The agent has indifference curves in effort-wage space that slope up wards, since if effort is increased the wage must be increased to keep utility the same. The principal's indifference curves also slope upwards, because although he does not care about effort directly, he does care about output, which rises with effort. The principal might be either risk averse or risk neutral; his indifference curve is concave rather than linear in either case because figure 10.2 shows a technology with diminishing returns to effort of effort starts out being higher, extra effort yields less additional output so the wage con not rise as much without reducing profits.

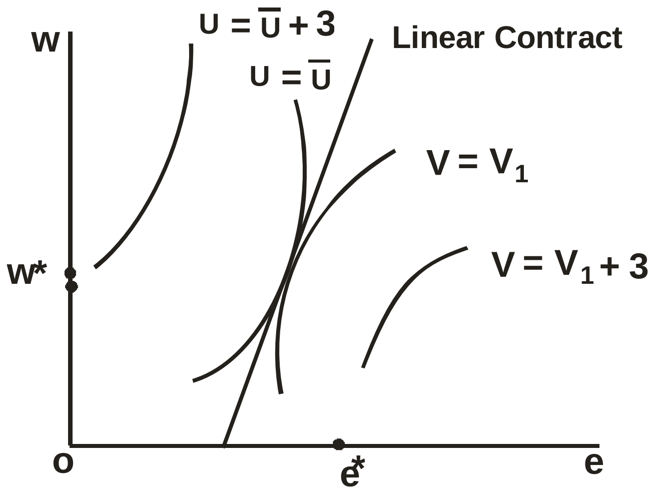


Figure 10.2 : The efficient effort level in production Game I

Under perfect competition among the principals the profits are zero, so the reservation utility $\overline{U}$ is chosen so that at the profit-maximizing effort e*, $\tilde{w}(e^*) = q(e^*)$, and $U(e^*, q(e^*)) = \overline{U}$ (9.7)

The principal selects the point on the $U = \overline{U}$ indifference curve that maximizes his profits, which is at the effort e* and w*. He must then design a contract that will induce the agent to choose this effort level. The following three contracts, shown in Fig 10.3 are equally under full information.

1. The forcing contract sets w(e*) = w* and $w(e \neq e^*) = 0$. This is certainly a strong incentive for the agent to choose exactly e = e*.

2. The threshold contract sets $w(e \geq e^*) = w^*$ and $w(e < e^*) = 0$. This can be viewed as a flat wage for low effort levels, equal to O in this contract, plus a bonus if effort reaches e*. Since the agent dislikes effort, the agent will choose exactly e = e*.

3. The linear contract shown in figure 9.2, sets w(e)= $\alpha + \beta e$, where α and β are chosen so that w* = $\alpha + \beta e^*$ and the contract line is tangent to the indifference curve $U = \overline{U}$ at e*. The most northwesterly of the agent's indifference curves that touch this contract line touches it at e*.

Let's now fit out Production Game I with specific functional forms. Suppose the agent exerts effort $e \in [o, \infty]$, and output equals q(e) = 100 * log (1 + e). If the agent rejects the contract, then π agent = $\overline{U} = 3$ and π principal = 0, whereas if the agent accepts the contract, then π agent = U (e, w) = log (w) – e^2 and π principal = q(e) – w(e).

The agent must be paid some amount $\tilde{w}(e)$ to exert effort e, where $\tilde{w}(e)$ is defined to solve the participation constraint.

$$U(e, w(e)) = \overline{U}, \text{so } \log(\tilde{w}(e)) - e^2 = 3$$

Knowing the particular functional form as we do, we can solve (7.8) for the wage function:

$$\tilde{w}(e) = \mathrm{Exp}(3 + e^2) \qquad (9.9)$$

This makes sense. As effort rises, the wage must rise to compensate, and rise more than exponentially if utility is to be kept equal to 3.

The principals problem is

$$\underset{e}{\text{Maximize}}\ V(q(e)-\tilde{w}(e)) = 100 * \log(1+e) - \text{Exp}(3+e^2) \qquad (9.10)$$

The first order condition for this problem is

$$V'(q(e)-\tilde{w}(e))\left(\frac{\partial q}{\partial e}-\frac{\partial \tilde{w}}{\partial e}\right)=0 \qquad (9.11)$$

Or, for out problem, since the firm is risk-neutral and $v'=1$

$$\frac{100}{1+e}-2e(\text{Exp}(3+e^2))=0 \qquad (9.12)$$

We can simplify the first order condition a little to get

$$(2e+2e^2)\ \text{Exp}\ (3+e^2)=100 \qquad (9.13)$$

But this cannot be solved analytically. Using the computer program mathematics, we found that $e^* \approx 0.849$ from which , using the formulas above, we get $q^* \approx 100 * \log(1+0.849)$ ≈ 61.48 and $w^* \approx 41.32$.

Here, the implicit function theorem was not needed, because specifying the functional forms allowed us to find the solution using algebra instead.

Note that if $\bar{U}$ were high enough, the principals payoff would be zero. If the market for agents were competitive, this is what would happen, since the agent's reservation payoff would be from working for another principal.

To implement the contract, a number of types of contracts could be used as shown in fig. 10.3.

1. The <u>forcing contract</u> sets w(e*) = w* and $w\left(e \neq 0.84\right)=0.$ Here, w (0.84) = 41 and $w\left(e \neq e^*\right)=0.$

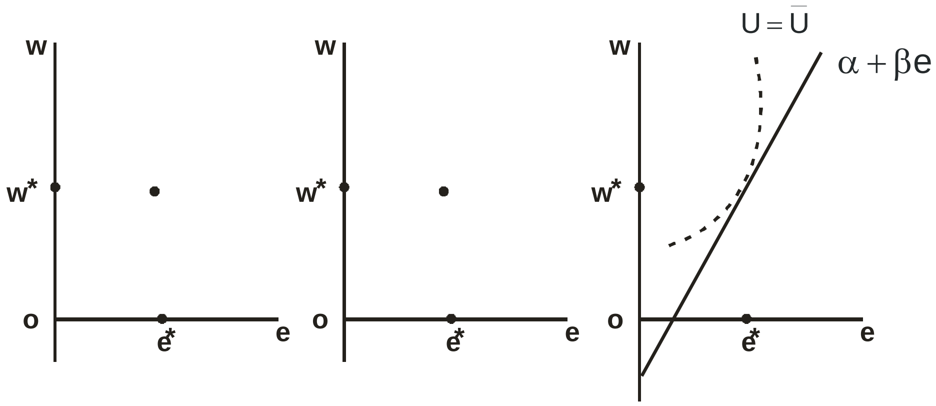
2. The <u>threshold contract</u> sets $w(e \geq e^*) = w^*$ and w(e < e*) = 0. Here, $w(e \geq 0.84)=41$ and w(e < 0.84) = 0.

3. The linear contract sets w(e) = $\alpha+\beta$e, where α and β are chosen so that $w^*=\alpha+\beta\ e^*$ and the contract line is tangent to

the indifference curve $U = \overline{U}$ at e*. The slope of that indifference curve is the derivative of the $\tilde{w}(e)$ function, which is

$$\frac{\partial \tilde{w}(e)}{\partial e} = (3+e^2) * 2e * \text{Exp}\ (3+e^2) \qquad \ldots\ldots (7.18)$$

At e*= 0.84 this takes the value 253. That is the β for the linear contract. The α must solve w(e*) = 41 $= \alpha + 253\ (0.84)$, so $\alpha = -|>|$ 52



(a) Forcing Contract   (b) Threshold Contract   (c) Linear Contract

Fig. 10.3 Three contracts that induce effort e* for wage w*

You ought to be a little concerned that the linear contract satisfy the incentive compatibility constraint. We constructed it so that it satisfied the participation constraint, because if the agent chooses e= 0-84, his utility will be 3. But might he prefer to choose some larger or smaller e and get even more utility.

He will not, because his utility is concave. That makes the indifference curve convex, so its slope is always increasing, and no preferable indifference curve touches the equilibrium contract line, as the diagram shows here.

Before going on to versions of the game with asymmetric information, it will be useful to look at one other version of the game with full information, in which the agent, not the principal, proposes the contract. This will be called Production Game II.

Equation (8.16) implies that

$$\frac{\partial U}{\partial w}\,\frac{\partial q}{\partial e} = -\frac{\partial U}{\partial e} \qquad (8.17)$$

Comparing this with equation (8.6), the equation when the principal had the bargaining power, it is clear that e* is identical in Production Game I and Production Game II. It does not matter who has the bargaining power, the efficient effort level stays the same.

Figure 10.2 can be used to illustrate this game as well. Suppose that $V_1 = 0$. The agent must choose a point on the $V_1 = 0$ indifference curve that maximizes his own utility, and then provide himself with contract incentives to choose that point. The agent's payoff is highest at effort e* given that he must make $V_1 = 0$, and all three contracts described in Production Game I provide him with the correct incentives.

The efficient effort level is independent of which side has the bargaining power because the gains from efficient production are independent of how those gains are distributed so long as each party has no incentive to abandon the relationship. This is the same lesson as that of the Coase theorem, which says that under general conditions the activities undertaken will be efficient and independent of the distribution of property rights (Coase [160]). This property of the efficient effort level means that the modeler is free to make the assumptions on bargaining power that help to focus attention on the information problems he is studying

10.6 PRODUCTION GAME II : FULL INFORMATION AGENT MOVES FIRST

In this version, every move is common knowledge and the contract is a function w(e). The order of play, however, is now as follows

The order of play

1. The agent offers the principal a contract w(e).
2. The principal decides whether to accept or reject the contract.
3. If the principal accepts, the agent exerts effort e.
4. Output equals q(e), where $q^1 > 0$.

In this game, the agent has all the bargaining power, not the principal. The participation constraint is now that the principal must earn zero profits, so $q(e) - w(e) \geq 0$. The agent will maximize his own payoff by driving the principal to exactly zero profits, so $w(e) = q(e)$.

Substituting q(e) for w(e) to accept for the participation constraint, the maximization problem for the agent in proposing an effort level e at a wage w(e) can therefore be written as

$$\text{Maximize } U(e, q(e)) \qquad (9.15)$$

The first- order condition is

$$\frac{\partial U}{\partial e} + \left(\frac{\partial U}{\partial q}\right)\left(\frac{\partial q}{\partial e}\right) = 0 \qquad (9.16)$$

Since $\frac{\partial U}{\partial q} = \frac{\partial U}{\partial w}$ when the wages equals output.

10.7 PRODUCTION GAME III : A FLAT WAGE UNDER CERTAINTY

In this version of the game, the principal can condition the wage neither on effort nor on output. This is modeled as a principal who observes neither effort nor output, so information is asymmetric.

It is easy to imagine a principal who cannot observe effort, but it seems very strange that he can not observe output, especially since he can deduce the output from the value of his payoff. It is not ridiculous that he cannot base wages on output, however, because a contract must be enforceable by some third party such as a court. Law professors complain about economists who speak of "unenforceable contracts". In law school, a contract is defined as an enforceable agreement, and most of a contracts class is devoted to discovering which agreements are contracts. If, for example, a teacher does a poor job of inspiring his students, that may be clear to his school but very costly to prove in court, so his school but very costly to prove in curt, so his wage cannot be based on his output of inspiration. For such situations, Production Game III is appropriate output is not <u>Contractible</u> or <u>verifiable</u>, which usually leads to the same outcome as when it is unobservable in a contracting model.

The outcome of Production Game III is simple and in efficient. If the wage is non-negative, the agent accepts the job and exerts zero effort, so the principal offers a wage of zero.

If there is nothing on which to condition the wage, the agency problem cannot be solved by designing the contract carefully. If it is to be solved at all, it will be by some other means such as reputation or repetition of the game. Typically, however, there is some contractible variable such as output upon which the principal can condition the wage. Such is the case in Production Game IV.

10.8 PRODUCTION GAME IV : AN OUTPUT BASED WAGE UNDER UNCERTAINTY

In this version, the principal cannot observe effort but can observe output and specify the contract to be w(q).

Now, the principal picks not a number w but a function w(q). His problem is not quite so straightforward as in Production Game I, where he picked the function w(e), but here, too, it is possible to achieve the efficient effort level e* despite the unobservability of effort. The principal starts by finding the optimal effort level e*, as in Production Game I. That effort yields the efficient output level q* = q (e*). To give the agent the proper incentives, the contract must reward him when output is q*. Again, a variety of contracts could be used. The forcing contract, for example, would be any wage function such as $U(e^*, w(q^*)) = \bar{U}$ and $U(e, w(q)) < \bar{U}$ for $e \neq e^*$.

Production Game IV shows that the observability of effort is not a problem in itself, if the contract can be conditioned on something which is observable and perfectly corrected with effort. The true agency problem occurs when that perfect correlation breaks down as in Production Game IV.

10.9 PRODUCTION GAME V : AN OUTPUT- BASED WAGE UNDER UNCERTAINTY

In this version, the principal cannot observe effort but can observe output and specify the contract to be w(q). Output, however, is a function $q(e, \theta)$ both of effort and the state of the world $\theta \in R$, which is chosen by Nature according to the probability density $f(\theta)$ as the new more (5) of the game. More (5) comes just after the agent chooses effort. So the agent cannot choose a low effort knowing that Nature will take up the slack. If the agent can observe Nature's more before his own, the game becomes moral hazard with hidden knowledge and hidden actions.

Because of the uncertainty about the state of the world, effort does not map cleanly onto the observed output in Production Game V. A given output might have been produced by any of several different effort levels, so a forcing contract will not necessarily achieve the desired effort. Unlike the case in Production Game IV, here the principal cannot deduce that $e = e^*$ from the fact that $q = q^*$. Moreover, even if the contract does induce the agent to choose e^*, if it does so by penalizing him heavily when $q = q^*$ it will be expensive for the principal. The agent's expected utility must be Kept-equal to $\bar{U}$ because of the participation constraint, and if the agent is sometimes paid a 1000 wage because output happens not to equal q^*, he must be paid more when output does equal q^* to make up for it. If the agent is risk averse, this variability in his wage requires that his expected wage be higher than the w^* found earlier, because he must be compensated for the extra risk. There is a tradeoff between incentives and insurance against risk.

Moral hazard becomes a problem when $q(e)$ is not a one-to-one function because a single value of e might result in any of a number of values of q, depending on the value of θ. In this case, the output function is not invertible; knowing q, the principal can not deduce the value of e perfectly without assuming equilibrium behaviour on the part of the agent.

The combination of unobservable effort and lack of invertibility in Production Game V means that no contract can induce the agent to put forth the efficient effort level without incurring extra costs, which usually take the form of an extra risk imposed on the agent. We will still try to find a contract that is efficient in the sense of maximizing welfare given the informational constraints. The terms "first-best" and "second-best" are used to distinguish these two kinds of optimality.

A first best contract achieves the same allocation as the contract that is optimal when the principal and the agent have the same information set and all variables are contractible.

A second best contract is Pareto – optimal given information asymmetry and constraints on writing contracts.

The difference in welfare between the first-best world and the second-best world is the cost of the agency problem.

The first four Production Games were easier because the principal could find a first best contract without searching very far. But even defining the strategy space in a game like Production Game V is tricky, because the principal may wish to choose a very complicated function w (q).

10.10 SUMMARY

Asymmetric information refers to the situation in which buyers and sellers do not have the same information regarding the product quality.

Adverse selection refers to situations in which the behaviour of different types of economic agents is not observable.

Moral hazard refers to a situation where one side of the market can not see the actions of the other side.

10.11 FURTHER READINGS

(1) Rasmusen E.: Games and Information, Blackwell, 1994.

(2) Silberberg E: The structure of Economics: A Mathematical Analysis, McGraw Hill, 1990.

10.12 QUESTIONS

(1) Explain the various categories of asymmetric information models.

(2) Explain clearly the meaning of adverse selection and moral hazard.

(3) Distinguish between adverse selection and moral hazard by giving suitable examples.

(4) What is market signalling?

❖❖❖❖

11

OLIGOPOLY AND GAME THEORY

UNIT STRUCTURE

11.0 OBJECTIVES

After going through this unit you will be able to –

- Define and describe oligopoly.
- Understand and explain Cournot's oligopoly model.
- Describe Bertrand model.
- Explain monopoly power.
- Discuss price discrimination.
- Define and explain game of entry deterence.

11.1 INTRODUCTION :

An oligopoly exists when a few sellers of a commodity or service deal with a larger number of buyers. In the case of oligopoly a small number of companies supply the major portion of an industry's output. In effect, the industry is composed of a few large firms which account for a significant share of the total production. Thus the actions of the individual firms have an appreciable effect on their competitors. However, the presence of only a few sellers does not imply that competition is absent.

The most important point about oligopoly is that an oligopolist is a price searcher. No firm under oligopoly can take decisions on price independently. Each firms has to take into account the behaviour of rival firms while taking a decision on price.

The oligopoly various diverse behaviour patterns are observed. This is why there are several models of oligopoly behaviour. Each model is based on one or more assumptions. But none is fool-proof in the sense that there is no grand model which captures many different behaviour patterns which are observed in the real world.

11.2 NON-COLLUSIVE OLIGOPOLY : THE COURNOT MODEL

The distinctive feature of the different oligopoly models is the way they attempt to capture the interdependence of firms in the market. Perhaps the best known is the cournot model. In fact, the earliest duopoly model was developed in 1838 by the French economist Augustin Cournot. It is treated as the classical solution of the duopoly problem. Although the basic model is rather simplistic, Its provides useful insights into industries with a small number of firms. Although here we consider the cournot duopoly model (with two firms), the same analysis can be extended to cover more than two firms.

In the Cournot model of duopoly it is assumed that firms produce a homogenous good and know the market demand curve. Each firm has to decide how much to produce, and the two firms take their decisions at the same time. When making its productions decision, each firm takes its competitor into account. It knows that its competitor is also taking output decision, i.e., it is deciding how much to produce. So the market price will depend on the total output of both firms.

The essence of the Cournot model is that each duopolist treats the output level of its competitors as fixed and then decides how much to produce. Cournots analysis shows that two firms would react to each other's output changes until they eventually reached a stable output position from which neither would wish to depart. In the Cournot model it is the quantity, not price which is adjusted, with one firms altering its output on the assumption that his rivals output will remain unchanged. Since both firms reason in this way, output will eventually be expanded to point where the firms share the market equally and both are able to make only normal profits.

The original model was presented in a simple was by assuming that two firms (called duopolist) have identical products and identical costs, Cournot illustrated his model with the example of two firms each owning a spring of mineral water which is produced at zero costs. The original model leaves a few questions unanswered. This is why modern economists generalise the presentation of the Cournot model by using the reaction curves approach. This approach is a more powerful method of analysing oligopolistic markets, because it allows the relaxation of the assumption of identical costs and identical demands. This approach is based on the concept of Isoprofit curves of the competitors, which are a type of indifference curves of the profit-maximising firms.

11.2.1 Isoprofit Curves

An isoprofit curve for firm 1 is the locus of points defined by different levels of output of firms 1 and its rival firm 2, which yield to firm 1 the same level of profit, as shown in figure 10.1 similarly, an isoprofit curve for firm 2 is the locus of points of different levels of output of two competitors which yield to firm the same level of profit, as shown in figure 8.2.

Isoprofit curves are lines showing those combinations of two substitute products q_1 and q_2 that yield a constant level of profit to firm 2. Profits of firm 2 will increase as it moves to isoprofit lines that are further to the left. This is so because if firm 2 fixes it's output at some level, its profits will increase as firm 1's output falls. Firm 2 will make the maximum amount of profit when it is a monopolist, i.e. when firm 1 decides to produce zero unit of output.

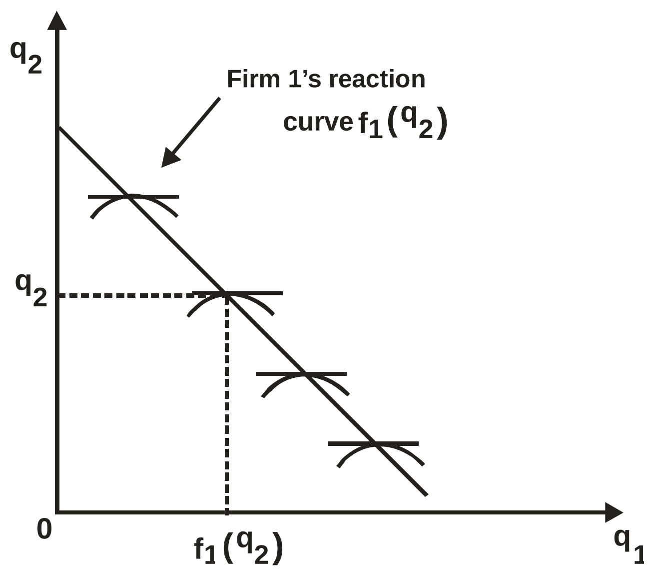


Fig. 11.1: Firm 1's isoprofit map

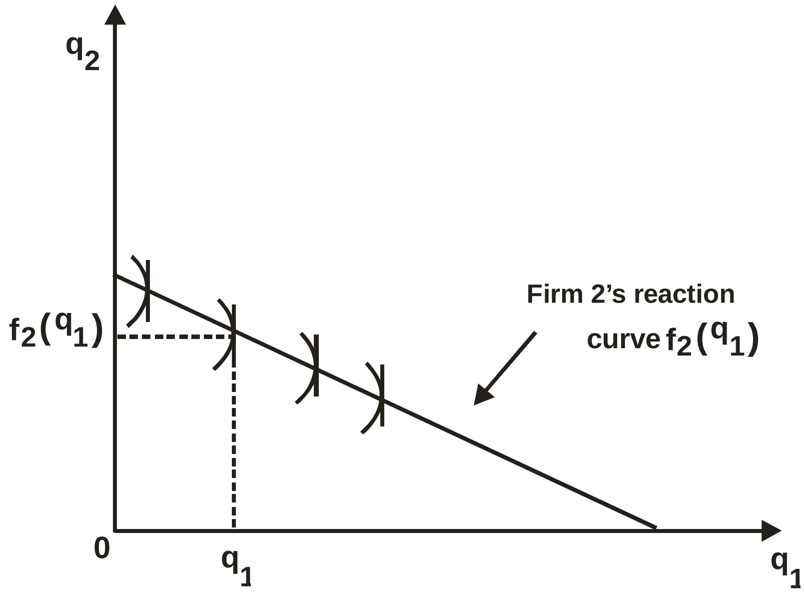


Fig. 11.2 Firm 2's isoprofit map

For each possible level of firm 1's output, firm 2 wants to choose its own output in order to make its profits as large as possible. This means that for each level of firm 1's output (q_1), firm 2's will choose the level of output (q_2) that put it on the isoprofit curve farthest to the left as illustrated in Fig. 11.2.

At this point the slope of the isoprofit line must be infinite. The locus of these tangency points is firm 2's reaction curve, $f_2(q_1)$. The reaction curves gives the profit-maximising output of firm 2, for each level of output of firm 1. The reaction curve of firm 2 is the locus of points of profit that firm 2 can attain, given the level of output of its rival. It is called reaction curve, because it shows how firm 2 will determine its output as a reaction to firm 1's decision to produce a certain level of output. Firm is reaction curve is shown in figure 11.1.

Its output is a function of firm's 2 output level so $f_1 = f_1(q_2)$ just as $f_2 = f_2(q_1)$. The reaction curve shows the relationship between a firms profit maximising output and the amount it thinks its competitor will produce. Firm 1's profit maximising output is thus a decreasing function of how much it thinks firm 2 will produce.

For each choice of output (q_1) by firm 1, firm 2 chooses the output level $f_2(q_1)$ associated with the isoprofit curve farthest to the left.

11.2.2 Cournot Equilibrium

Each firm's reaction curve tells us how much to produce, given the output of it's competitor. In equilibrium, each firm sets output according to its own reaction curve. The equilibrium output levels are, therefore found at the intersection of the two reaction curves in figure 10.3. We call the resulting set of output levels Cournot equilibrium.

Cournot's equilibrium (which indicates how much output will each firm produce) is determined by the intersection of the two reaction curve (point E). At such a point, each firm is producing a profit-maximising level of output given the output choice of the other firm. In this equilibrium, each firm correctly assume how much it's competitor will produce and it maximises its profit accordingly.

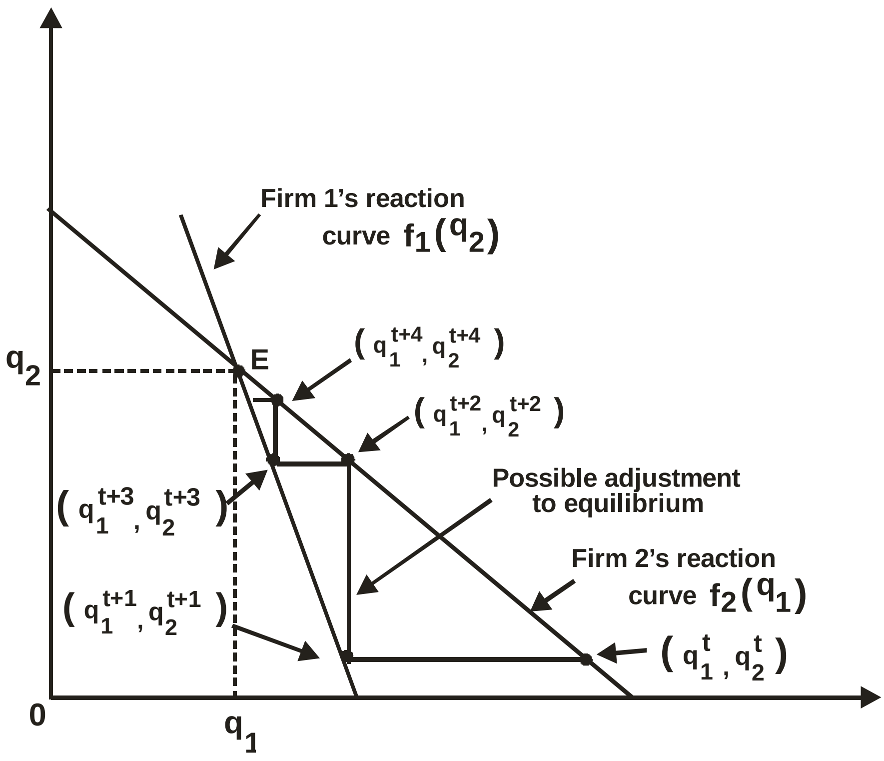


Fig. 11.3 : Cournot Equilibrium and its stability

11.2.3 Stability

The Cournot equilibrium is a stable equilibrium, provided firm 1's reaction curve is steeper than firm 2's reaction curve. In figure 11.3 we start with output $\left(q_1{}^t, q_2{}^t\right)$, Given firm 2's level of output, firm 1 optimally chooses to produce $q_1{}^{t+1}$ next period. This point is located by moving horizontally to the left until we hit firm 1's reaction curve.

If firm 2 expects firm 1 to continue to produce q_1^{t+1} its optimal response is to produce q_2^{t+1}. We find this point by moving vertically upward until we hit firm 2's reaction curve. This direction of arrows indicates the sequence of output choice of the two firm, Through such movements along the 'staircase,' we trace out an adjustment process which converges to the cournot equilibrium. Thus cournot equilibrium is table.

Let us suppose the two firms are initially producing output levels that differ from the Cournot equilibrium. The Cournot model does not say anything about the dynamics of the adjustment process, i.e. whether the firms adjust their output until the Cournot equilibrium is reached. In truth, during any adjustment process, the central assumption of the model (i.e., each firm can assume that its competitor's output remains fixed) will not hold. Since both firms would be adjusting their outputs, neither output would remain fixed, such dynamic adjustment is explained by other models.

11.3 EXTENSION OF THE COURNOT MODEL: THE CASE OF MANY FIRMS

It is possible to generalise the Cournot model by considering a situation in which there are many firms. Here we assume that each firm has an expectation about the output choices of the other firms.

Let us suppose there are n firms and industry output Q is the joint contribution of all the firms, i.e., $Q = q_1 + q_2 + ...q_n$.

For an industry with 'n' firms, the total equilibrium output for a cournot oligopoly is given by

$Qn = Qc\left[\frac{n}{n+1}\right]$, where $n \geq \ell$ and Qc is the output resulting from a perfectly competitive market.

Then the profit-maximising condition for firm i is :

$$p(Q) + \frac{\Delta p}{\Delta q}\, qi = MC\,(qi)$$

If we factor out p(Q) and multiply the second term by Q/Q, we get

$$P(Q)\left[\ell + \frac{\Delta p}{\Delta q}.\frac{Q}{P(Q)}.\frac{qi}{Q}\right] = MC(qi)$$

Here $\frac{\Delta P}{\Delta Q}.\frac{Q}{P(Q)}$ is elasticity of the market demand curve and qi/Q is the firm i's share of total market output. If we express qi/Q as s_i, we get $P(Q)\left[\ell - \frac{s_i}{|e(Q)|}\right] = MC\,(qi)$

This may also be expressed as $P(Q)\left[\ell - \frac{\ell}{|e(Q)|/s_i}\right] = MC\,(qi)$

Here the term e(Q) / s_i is the elasticity of the demand curve faced by the firm; the smaller the market share of the firm, the more elastic the demand curve it faces.

In an extreme situation in which $s_i = \ell$, the firm is a monopolist. In this case the demand curve facing the firm is the market demand curve. So the equilibrium condition is the same as that of a monopolist, i.e. MR = MC, where $MR = p(Q)\left[\ell - \frac{\ell}{|e(Q)|}\right]$

If, in another extreme situation, the firm is a very small part of a large market, its market share is virtually zero, and the demand curve facing the firms is completely elastic, in which case p= MC as is the case with a firm under pure competition.

Thus if there are a large number of firms, none can exert much influence on the market price. In this case, the cournot equilibrium is very similar to competitive equilibrium.

11.4 THE BERTRAND MODEL: SIMULTANEOUS PRICE SETTING

In the Cournot model, firms choose their quantities and the free play of the market determines the price. An alternative approach is to take the firms as price setters and allow the market to determine the quantity sold. This means that when a firm chooses its price, it has to forecast the price set by the other firm, its only rival, in a duopoly situation. This model of oligopoly behaviour is known as the Bertrand model. The Bertrand model was developed in 1883 by another French economist Joseph Bertrand Like the Cournot model, it applies to firms that produce the same homogeneous product and make their decisions at the same time. In this case, however, the firms choose prices instead of quantities. This change can dramatically affect the market outcome.

In the Bertrand duopoly model, each supplier assumes that his rival will not change price in response to his own initial price cut and this assumption will encourage him to cut his price in order to increase his sales. Since both firms reason in this way the price will eventually be driven down to the competitive level.

In homogeneous oligopoly Bertrand equilibrium is the same as competitive equilibrium where p = MC. This point may now be explained.

Price cannot be less than MC since then either firm would increase its profits by producing less. So price has to be greater than or at least equal to marginal cost. Let us suppose that both firms are selling at same price Pe which is greater than marginal cost. Now if firm 1 reduces it by a very small amount k and its rival, firm 2, keep it's price unchanged at Pe, then firm 2 will lose all of its customers. In other words, the entire market will be captured by firm 1.

If firm 1 is confident that firm 2 will charge a price Pe, which is greater than marginal cost, it will pay firm 1 to cut it's price to Pe-K. Firm 2 can also think in the same way and act accordingly. This means that no price in which is higher than marginal cost can be equilibrium price; thus Bertrand competition is the same as pure competition, in which only equilibrium is the competitive equilibrium.

The Bertrand model carries enormous good sense. If one firm 'bids' for the consumer's business by fixing a price above marginal cost, then the other firm can always make a profit by undercutting this price with a lower price. Thus if each firm quotes a price equal to marginal cost, no one can gain at the expense of the other. In other words, this is the only price that each firm cannot rationally expect to be undercut.

It may be noted that Bertrand equilibrium is comparable to Nash equilibrium, to be discussed later in this chapter in the context of game theory. At present it suffices to note that because of the incentive to cut prices, the Nash equilibrium in this case is the competitive equilibrium, i.e. both firms set price equal to marginal cost.

11.4.1 Criticisms

There are two main criticisms of the Bertrand model. First, when firms produce a homogeneous product it is more natural to compete by setting quantities rather than prices second even if firms set prices and choose the same price there is no reason why sales would be divided equally among the firms. In spite of these criticisms the Bertrand model is useful because it shows how the equilibrium outcome in an oligopoly depends crucially on the firm's choice of the strategic variable. Moreover, if firms produce a homogeneous product and compete by first setting output capacities and then setting price, the result is Cournot equilibrium once again. C of course, in terms of quantities.

11.5 MONOPOLY POWER :

Through creative genius, superior efficiency or better foresight, one firm can come to dominate an industry. The monopoly profits that result provide a powerful incentive for imitation by others. As a consequence monopoly power tends to be transitory and fleeting.

There are various sources of monopoly power. The following are the most common source:

(1) Patents, copyrights and trademarks:

Under the patent law of the country a firm may be granted the exclusive right to produce a certain commodity or to use a specific production process for a certain period of time. Patents are offered as rewards to stimulate inventive or creative efforts which advance knowledge and make information available to society. It is apprehended that a person would have little incentive to invent a better computer if he cannot reap some reward for doing so. Consequently a modern industrial society provides a legal monopoly for a limited period.

The grant of a patent is supposed to ensure that no one will copy the inventor's inventions during the life of the patent. No Doubt the patent offers the inventor legal protection against patent copying, but it does not provide absolute protection. The monopoly power that comes from patent is limited, because patents are narrowly defined; this allows other firm to imitate the patented process or product closely. The effect of copyrights and trademarks is similar, since each gives some legal protection from infringement. However, patents, copyrights and trademarks offer only limited protection against outsiders.

(2) Control of an essential raw material :

Monopolies also come into existence through ownership of a key resource. It is obvious that if one firm controls the supply of an essential input, then other firms will be unable to compete. Such monopolies are called input monopolies. The Aluminium Company of America (ALOCA) once controlled most domestic bauxite deposits from which aluminium is made. The International Nickel

Company once owned about 90% of the world's nickel. These monopolies usually break up when new sources of supply are found. Another company that has a virtual monopoly through its control of natural resources is the DeBeers Consolidated Mines Ltd. of South Africa, which handles about 80% of the world's uncut diamonds. The company has run a worldwide cartel for more than a century.

(3) Natural Monopoly:

A natural monopoly arises when only one firm can survive in the industry. Such a monopoly is a natural result of the interaction between technological conditions that requires large scale for efficient production and demand conditions that make one plant of minimum efficient size just efficient to supply the entire market at a price that covers full cost. Under these conditions it is economically most efficient and socially most desirable to have a single firm supplying the entire market at a price that covers full cost. And so it is desirable to have a single firm supplying the entire market, because the cost of meeting the market demand is at a minimum. If there were several firms in the market, competition among them would lead to losses of all the firms. As a result, exit would occur until the remaining firm would have control over price. At that point, the price would be raised and the profits would become positive. Public utilities provide the classic example of natural monopoly and are generally regulated by society because they all are charactrised by increasing returns to scale (IRS).

In short, a natural monopoly is a special type of monopoly that arises from economies of scale. Example of these are gas pipe line company, a telephone company and electric utility. In these cases the average cost of production falls over a wide range output and an single firm can supply the output at a lower price than two or more small firms. This monopolist is called 'natural' because it arises naturally from the type of product being sold. Here monopoly has an advantage over competition. So monopoly is not always injurious to society.

(4) Government controls on entry:

There are industries where entry is controlled by the government. Consequently a government agency can confer monopoly power on a firm at least locally. Simply by precluding entry. Examples are nationalised banking, Cable TV firms hospitals, etc. Usually some revenues are collected form the firms in exchange for the government protection from potential entrants. In many cases, the firm must submit to some form of direct regulation regarding the prizes charge, the quality and scope of service provided, and the rate of return earned on it's investment. These firms may still be able to make positive profits that are attractive to outsiders, but entry can be barred. This is the most insurmountable of all entry barriers, because the power of the state is exercised to prevent the entry of new firms into the industry.

11.6 PRICE DISCRIMINATION UNDER MONOPOLY

Price discrimination arises when a firm sells its (homogeneous) product at different prices at the same time. The monopolist is able to sell his product in some situations in two or more markets at different prices and thereby increases his profit.

Discrimination is possible if, and only if :

(1) The market is segmentable, that is, customers should be distinguishable on some basis so that they remain almost ignorant of discrimination. In certain cases doctors charge two types of fees for their services, high for the rich and low for the poor. Books are priced at different rates – low price for home buyers and high price for foreigners.

(2) There is no resale. Otherwise some customer-speculators would buy at the cheaper rate and resell it in the high priced area and thus would render price discrimination in effective of course, services like haircuts or consultancy cannot be resold.

Price discrimination is profitable if, and only if, the price elasticity of demand is different in different markets.

11.6.1 Equilibrium of a Discriminating Monopolist

Total profit of a price discriminating monopolist is the difference between his total revenue from both the markets and his total cost of production.

$$\pi = R_1(Q_1) + R_2(Q_2) - C(Q_1 + Q_2)$$

Where q_1 and q_2 are quantities he sells in different markets, $R_1(Q_1)$ and $R_2(Q_2)$ are his revenue functions and $C(Q_1 + Q_2)$ is his cost function. For profit maximisation we must have :

$$\frac{\partial \pi}{\partial Q_1} = \frac{\partial R}{\partial Q_1} - \frac{\partial c(Q_1 + Q_2)}{\partial Q_1} = R_1'(Q_1) - C'(Q_1 + Q_2) = 0$$

$$\frac{\partial \pi}{\partial Q_2} = \frac{\partial R}{\partial Q_2} - \frac{\partial c(Q_1 + Q_2)}{\partial Q_2} = R_2'(Q_2) - C'(Q_1 + Q_2) = 0$$

Thus the profit – maximising monopolist in this case would be in equilibrium at the point where combined marginal revenue $(\text{CMR}) = \text{MR}_1 = \text{MR}_2 = \text{MC}$

The second – order condition for profit maximisation requires

$$\frac{\partial^2 R_1}{\partial Q_1^2} < \frac{\partial^2 c}{\partial Q^2} \text{ and } \frac{\partial^2 R_2}{\partial Q_2^2} < \frac{\partial^2 c}{\partial Q^2}$$

That is the MR in each market must be increasing less rapidly than the MC for the output as whole.

The equality of the MR in the two markets does not necessarily imply the equality of prices in the two markets. It implies.

$$P_1\left[1-\frac{1}{Ep_1}\right] = P_2\left[1-\frac{1}{Ep_2}\right] \; [\because \text{MR} = P\left[1-\frac{1}{Ep}\right]\,]$$

$$\text{and } \frac{P_1}{P_2} = \frac{1-\dfrac{1}{Ep_2}}{1-\dfrac{1}{Ep_1}}$$

If $Ep_1 = Ep_2,\ p_1 = p_2$

$Ep_1 = Ep_2,\ p_1 < p_2$ and $Ep_1 < Ep_2$

implies $p_1 > p_2$

That is, price will be lower in the market with greater price elasticity of demand.

Fig 11.4. illustrates pricing under price discrimination when both the market segments have some degree of imperfect competition. At point E the firm is in equilibrium where CMR = MC in fig. 11.4 (c) Here MC = MR_1 in fig 11.4 (a) gives output in market 1 equal to OQ_1 and $\text{MC} = \text{MR}_2$ in fig 11.4 (b) gives output in market II equal to OQ_2. Corresponding to these output levels $(OQ_1 \text{ and } OQ_2)$ prices are OP_1 and OP_2 in markets I and II, respectively.

Although $\text{MR}_1 = \text{MR}_2$, $P_1 \neq P_2$. Therefore the monopolist is discriminating.

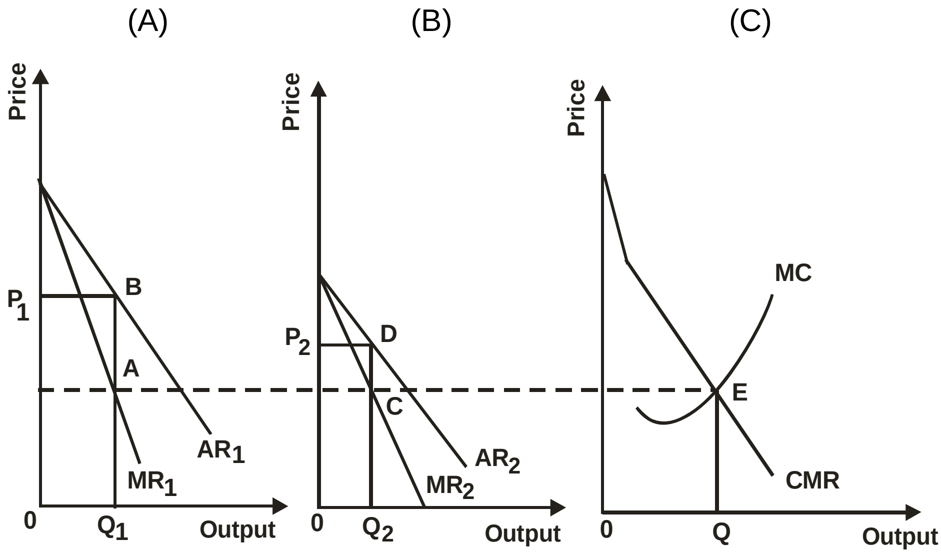


Fig. 11.4 Equilibrium of a discriminating monopolist

11.7 GAME OF ENTRY DETERENCE

An old question in industrial organization is whether an incumbent monopolist can maintain his position by threatening to wage a price war against any new firm that enters the market. This idea was heavily attached by Chicago School economists such as McGee (1958) on the grounds that a price war would hurt the incumbent more than collusion with the entrant. Game theory can present this reasoning very cleanly. Let us consider a single episode of possible entry and price warfare, which nobody expects to be repeated. We will assume that even if the incumbent chooses to collude with the entrant, maintaining a duopoly is difficult enough that market revenue drops considerably from the monopoly level.

Entry Deterrence I

PLAYERS
Two firms, the entrant and the incumbent.

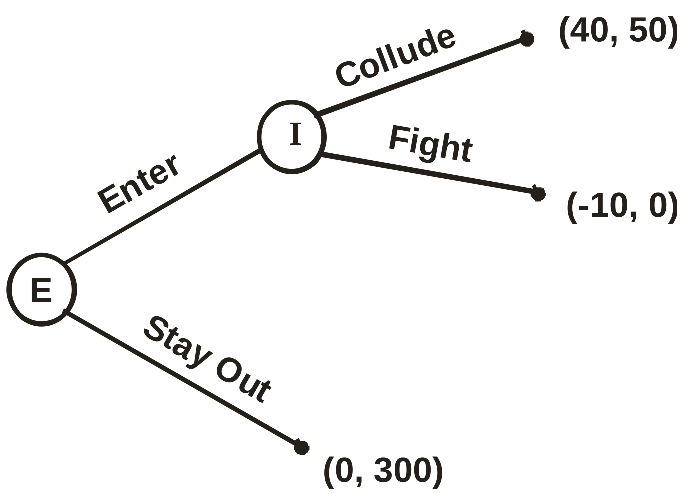
THE ORDER OF PLAY
1. The entrant decides whether to Enter or stay out.
2. If the entrant enters, the incumbent can collude with him, or fight by cutting the price drastically.

PAYOFFS
Market profits are 300 at the monopoly price and O at the fighting price. Entry costs are 10. Duopoly competition reduces market revenue to 100, which is split evenly.

Table 11.1 Entry Deterrence I

		Incumbent	
		Collude	Fight
Entrant	Enter	40,50	-10, 0
	Stay out	0, 300	0, 300

Payoffs to (Entrant, Incumbent)



Payoffs to: (Entrant, incumbent)

Figure 11.5 Entry Deterrence I

The strategy sets can be discovered from the order of play. They are (Enter, stay out) for the entrant, and {Collude if entry occurs, Fight if entry occurs} for the incumbent. The game has the two Nash equilibria indicated in boldface in table 11.1. (Enter, Collude) and (Stay Out, Fight). The equilibrium (stay out, Fight) is weak, because the incumbent would just as soon collude given that the entrant is staying out.

A piece of information has been lost by condensing from the extensive form figure to the strategic form, table 11.1 i.e. the fact that the entrant gets to move First once he has chosen Enter, the incumbents best response is collude. The threat to fight is not credible and would be employed only if the incumbent could bind himself to fight, in which case he never does fight, because the entrant chooses to stay out. The equilibrium (stay out, Fight) is Nash but no subgame perfect, because if the game is started after the entrant has already entered, the incumbent's best response is collude. This does not prove that collusion is inevitable in duopoly, but it is the equilibrium for Entry Deterrence I.

The trembling hand interpretation of perfect equilibrium can be used here. So long as it is certain that the entrant will not enter, the incumbent is indifferent between Fight and Collude, but if there were even a small probability of entry- perhaps because of a lapse of good judgement by the entrant the incumbent would prefer collude and the Nash equilibrium would be broken.

Perfectness rules out threats that are not credible. Entry Deterrence I is a good example because if a communication move were added to the game tree, the incumbent might tell the entrant that entry would be followed by fighting, but the entrant would ignore this noncredible threat. If, however, some means existed by which the incumbent could precommit himself to fight entry, the threat would become credible. The next section will look at one context, nuisance lawsuits, in which such pre-commitment might be possible.

11.8 SUMMARY

Oligopoly market is characterised by few sellers. Being few in number they are dependent on each other for their decision regarding price and output. This is where dominent and various other strategies play a vital role and game theory comes into picture.

11.9 FURTHER READINGS

(1) Rasmusen E: Games and Information, Blackwell, 1994.

(2) Mas- Colell A.M.D. Whinston and J.R. Green: Microeconimic Theory, Oxford University Press, 1995.

11.10 QUESTIONS

(1) Describe clearly the Cournot's model.

(2) Critically evaluate the Bertrand model.

(3) What is monopoly power?

(4) Explain the equilibrium of price discriminating monopolist.

(5) Discuss the game of entry deterence.

❖❖❖❖

12

ADVERSE SELECTION

UNIT STRUCTURE

12.0 OBJECTIVES

After going through this unit, you will be able to :

- Explain the concept of adverse selection,
  - Adverse selection under uncertainty
  - Adverse selection with identical taste
  - Adverse selection with heterogeneous taste
- Explain the concept of risk aversion

12.1 INTRODUCTION :

In moral hazard with hidden knowledge and adverse selection the principal tries to sort out agents of different types. In moral hazard with agent's action rather than his choice of contract, and agent's action rather than his choice of contract, and agents accept contracts before acquitting information. Under adverse selection, the agent has private information about his type or the state of the world before he agrees to a contract, which means that the emphasis is on which contract he will accept.

12.2 ADVERSE SELECTION :

For comparison with moral hazard, let us consider still another version of the production game.

Production Game VI : Adverse Selection

Players

The principal and the agent

The order of play

0. Nature chooses the agent's ability a, unobserved by the principal, according to distribution F(a)
1. The principal offers the agent one or more wage contracts W_1 (q), W_2(q),…………
2. The agent accepts one contract or rejects them all.
3. Nature chooses a value for the state of the world, θ, according to distribution $G(\theta)$. Output is then $q = q\ (a,\ \theta)$.

Payoffs

If the agent rejects all contracts, them π agent $= \bar{U}$ and π principal = 0. Otherwise, π agent = U (w) and π principal = v (q – w).

Under adverse selection, it is not the worker's effort but his ability that is non – contractible. Without uncertainty (move (3)), the principal would provide a single contract specifying high wages for high output and low wages for low output, but unlike under moral hazard, either high or low output might be observed in equilibrium if both types of agent accepted the contract. Also, in adverse selection, unlike moral hazard, offering multiple contracts can be an improvement over offering a single contract. The principal might, for example, provide a contract with a flat wage for the low ability agents and an incentive contract for the high – ability agents. Production Game puts specific functional forms into the game to illustrate equilibrium.

Production Game VI a : Adverse Selection, with Particular Parameters

Players
The principal and the agent.

The order of play

0. Nature chooses the agent's ability a, unobserved by the principal, according to distribution F(a), which puts probability 0.9 on low ability, a = 0, and probability 0.1 on high ability, a = 10.
1. The principal offers the agent one or more wage contracts $W_1 = \{W_1(q=0), W_1(q=10)\}, W_2 = \{W_2(q=0), W_2(q=10)\}$..
2. The agent accepts one contract or rejects them all.
3. Nature chooses a value for the state of the world, θ, according to distribution $G(\theta)$, which puts equal weight on 0 and 10. Output is then q = Min (a + θ, 10). (Thus, output is 0 or 10 for the low – ability agent, and always 10 for the high – ability.)

Payoffs
If the agent rejects all contracts, then depending on his type his reservation payoff is either $\pi_L = 3$ or $\pi_H = 4$ and the principal's payoff is π principal = 0.
Otherwise, $\pi_{\text{agent}} = W$ and $\pi_{\text{principal}} = q - W$

An equilibrium is
Principal : Offer $W_1 = \{W_1(q=0) = 3, W_1(q=10) = 3\}$

$$W_2 = \{W_2(q=0) = 0, W_2(q=10) = 4\}$$

Low agent : Accept W_1.
High agent : Accept W_2.

As usual, this is a weak equilibrium. Both low and high agents are indifferent about whether they accept or reject a contract. But the equilibrium indifference of the agents arises from the open – set problem; if the principal were to specify a wage of 2.01 for W_1, for example, the low – ability agent would no longer be indifferent about accepting it.

This equilibrium can be obtained by what is a standard method for hidden – knowledge models. In hidden – action models, the principal tries to construct a contract which will induce the agent to take the single appropriate action. In hidden – knowledge models, the principal tries to make different actions attractive under different states of the world, so the agent's choice depends on the hidden information. The principal's problem, as in Production Game V, is to maximize his profits subject to

1. incentive compatibility (the agent picks the desired contract and actions); and
2. participation (the agent prefers the contract to his reservation utility.)

In a model with hidden knowledge, the incentive compatibility constraint is customarily called the self – selection constraint, because it induces the different types of agents to pick different contracts. The big difference is that there will be an entire set of self-selection constraints, one for each type of agent or each state of the world, since the appropriate contract depends on the hidden information.

First, what action does the principal desire from each type of agent? The agents do not choose effort, but they do choose whether or not to work for the principal, and which contract to accept. The low ability agent's expected output is 0.5(0) + 0.5 (10) = 5, compared to a reservation payoff of 3, so the principal will want to hire the low ability agent if he can do it at an expected wage of 5 or less. The high ability agent's expected wage of 5 or less. The high ability agent's expected output, is 0.5 (10) + 0.5 (10) = 10, compared to a reservation payoff of 4, so the principal will want to hire the high ability agent, if he can do it at an expected wage of 10 or less. The principal will want to induce the low ability agent to choose a cheaper contract and not to choose the necessarily more expensive contract needed to attract the high ability agent.

The participation constraints are

$$U_L(W_1) \geq \pi^{-} L;\ 0.5\, W_1(0) + 0.5\, W_1(10) \geq 3$$

$$U_H(W_2) \geq \pi^{-} H;\ 0.5\, W_2(10) + 0.5\, W_2(10) \geq 4$$

Clearly the contracts $W_1 = \{3, 3\}$ and $W_2 = \{0, 4\}$ satisfy the participation constraints. The constraints show that both the low output wage and the high output wage matter to the low ability agent, but only the high output wage matters to the high ability agent, so it makes sense to make W_2 as risky as possible.

The self selection constraints are

$$U_L(W_1) \geq U_L(W_2); 0.5\, W_1(0) + 0.5\, W_1(10) \geq 0.5\, W_2(0) + 0.5\, W_2(10)$$

$$U_H(W_2) \geq U_H(W_1); 0.5 W_2(10) + 0.5 W_2(10) \geq 0.5 W_1(10) + 0.5 W_1(10)$$

The risky wage contract W_2 has to have a low enough expected return for the low ability agent to deter him from accepting it; but the safe wage contract W_1 must be less attractive than W_2 to the high ability agent. The contracts $W_1 = \{3, 3\}$ and $W_2 = \{0, 4\}$ do this, as can be seen by substituting their values into the constraints.

$$U_L(W_1) \geq U_L(W_2); 0.5(3)+0.5(3) \geq 0.5(0)+0.5(4)$$
$$U_H(W_2) \geq U_H(W_1); 0.5(4)+0.5(4) \geq 0.5(3)+0.5(3)$$

Since the self selection and participation constraints are satisfied, the agents will not deviate from their equilibrium actions. All that remains to check is whether the principal could increase his payoff. He cannot, because he makes a profit from either contract, and having driven the agents down to their reservation utilities, he cannot further reduce their pay.

As with hidden actions, if principals compete in offering contracts under hidden information, a competition constraint is added : the equilibrium contract must be as attractive as possible to the agent, since otherwise another principal could profitably lure him away. An equilibrium may also need to satisfy a part of the competition constraint not found in hidden actions models : either a non pooling constraint or a non separating constraint. If one of several competing principals wishes to construct a pair of separating contracts C_1 and C_2, he must construct it so that not only do agents choose C_1 and C_2 depending on the state of the world (to satisfy incentive compatibility), but also they prefer (C_1, C_2) to a pooling contract C_3 (to satisfy non pooling). We only have one principal in Production Game VI, though, so competition constraints are irrelevant.

It is always true that the self selection and participation constraints must be satisfied for agent who accept the contracts, but it is not always the case that they accept different contracts.

If all types of agents choose the same strategy in all states, the equilibrium is pooling. Otherwise, it is separating.

The distinction between pooling and separating is different from the distinction between equilibrium concepts. A model might have multiple Nash equilibria, some pooling and some separating. Moreover, a single equilibrium even a pooling one – can include several contracts, but if it is pooling the agent always uses the same strategy, regardless of type. If the agent’s equilibrium strategy is mixed, the equilibrium is pooling if the agent always picks the same mixed strategy, even though the messages and efforts would differ across realizations of the game.

A separating contract need not be fully separating. If agents who observe $\theta \leq 4$ accept contract C_1 but other agents accept C_2, then the equilibrium is separating but it does not separate out every type. We say that the equilibrium is fully revealing if the agent's choice of contract always conveys his private information to the principal. Between pooling and fully revealing equilibria are the imperfectly separating equilibria synonymously called semi – separating, partially separating, partially revealing, or partially pooling equilibria.

12.3 ADVERSE SELECTION UNDER CERTAINTY : LEMONS I AND II :

Akerlof stimulated an entire field of research with his 1970 model of the market for Shoddy used cars ("Lemons"), in which adverse selection arises because car quality is better known to the seller than to the buyer. In agency terms, the principal contracts to buy from the agent a car whose quality, which might be high or low, is non contractible despite the lack of uncertainty. Such a model may sound like moral hazard with hidden knowledge, but the difference is that in the used car market the seller has private information about his own type before making any kind of agreement. If instead, the seller agreed to resell his car when he first bought it, the model would be moral hazard with hidden knowledge, because there would be no asymmetric information at the time of contracting, just an expectation of future asymmetry.

We will spend considerable time adding twists to a model of the market in used cars. If the model had symmetric information there would be no consumer surplus. It will often be convenient to discuss the game as if it had many sellers, interpreting a seller whom Nature randomly assigns a type as a population of sellers of different types, one of whom is drawn by Nature to participate in the game.

The Basic Lemons Model

Players
A buyer and a seller

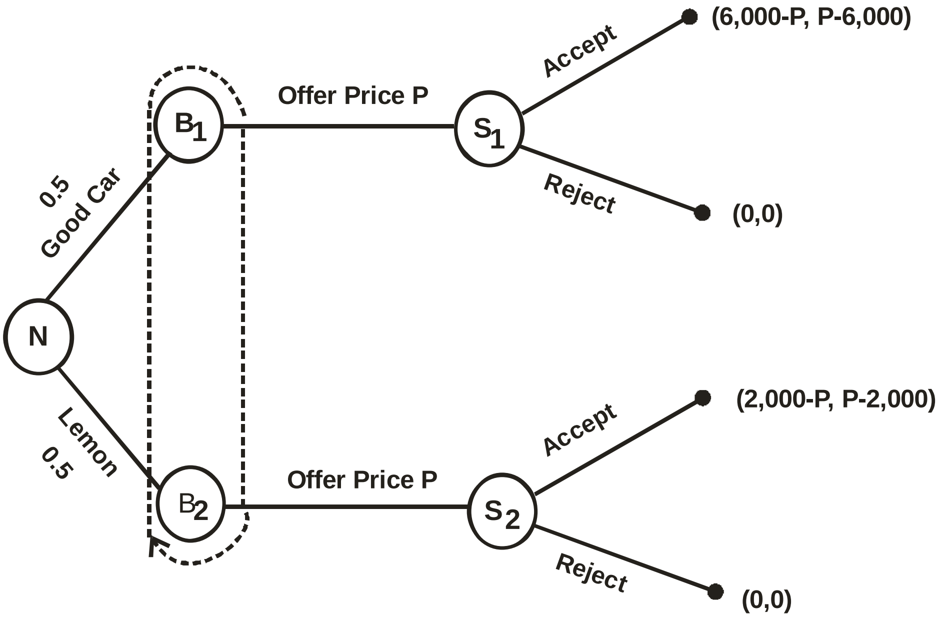
The order of play

0. Nature chooses quality type θ for the seller according to the distribution $F(\theta)$. The seller knows θ, but while the buyer knows F, he does not know the θ of the particular seller he faces.

1. The buyer offers a price P.
2. The seller accepts or rejects.

Payoffs

If the buyer rejects the offer, both players receive payoffs of zero. Otherwise, π buyer = V(θ) – P and π seller = P – U (θ), where V and U will be defined later.



Payoffs to : (Buyer, Seller)

Figure 12.1: An extensive form for Lemon I

The payoffs of both players are normalized to zero if no transaction takes place. A normalization is part of the notation of the model rather than a substantive assumption. Here, the model assigns the players' utility a base value of zero when no transaction take place, and the payoff functions show changes from that base. The seller, for instance, gains P if the sale takes place but loses U(θ) from giving up the car.

There are various ways to specify F(θ), U(θ) and V(θ). We start with identical tastes and two types (Lemons I), and generalize to a continuum of types (Lemons II). Next section specifies first that the sellers are identical and value cars more than buyers (Lemon III) next that the sellers have heterogeneous tastes (Lemon IV). We will look less formally at other modification involving risk aversion and the relative numbers of buyers and sellers.

12.3.1 Lemons I : Identical Tastes, Two Types of Seller

Let good cars have quality 6,000 and bad cars (Lemons) quality 2,000, So $\theta \in \{2,000, 6,000\}$, and suppose that half -------

the cars in the world are of the first type and the other half of the second type. A payoff combination of (0, 0) will represent the status quo, in which the buyer has $50,000 and the seller has the car. Assume that both players are risk neutral and they value quality at one dollar per unit, so after a trade the payoffs are π buyer = θ - P and π buyer = P - θ. The extensive form is shown in figure 12.1.

If he could observe quality at the time of his purchase, the buyer would be willing to accept a contract to pay $6,000 for a good car and $2,000 for a lemon. He cannot observe quality, and we assume that he cannot enforce a contract based on his discoveries once the purchase is made. Given these restrictions, if the seller offers $4,000 a price equal to the average quality, the buyer will deduce that the seller does not have a good car. The very fact that the car is for sale demonstrate it's low quality. Knowing that for $4,000 he would be sold only lemons, the buyer would refuse to pay more than $2,000. Let us assume that an indifferent seller sells his car, in which case half of the cars are traded in equilibrium, all of them lemons.

A friendly advisor might suggest to the owner of a good car that he wait until all the lemons have been sold and then sell his own car, since everyone knows that only good cars have remained unsold. But allowing for such behavior changes the model by adding a new action. If it were anticipated the owners of lemons would also hold back and wait for the price to rise. Such a game could be formally analyzed as a war of attrition.

The outcome that half the cars are held off the market is interesting, though not startling, since half the cars do have genuinely higher quality. It is a formalization of Groucho Mar'x wisecrack that he would refuse to join any Club that would accept him as a member. Lemons II will have a more dramatic outcome.

12.3.2 Lemons II: Identical Tastes, a continuum of Types of Seller

One might wonder whether the outcome of Lemons I was an artifact of the assumption of just two types. Lemons II generalizes the game by allowing the seller to be any of a continuum of types. We will assume that the quality types are uniformly distributed between 2,000 and 6,000. The average quality is $\bar{\theta} = 4{,}000$, which is therefore the price the buyer would be willing to pay for a car of unknown quality if all cars were on the market. The probability density is zero except on the support (2,000 – 6,000), where it is $f(\theta) = 1 / (6{,}000 - 2{,}000)$, and the cumulative density is

$$F(\theta) = \int_{2{,}000}^{\theta} f(x)\,dx .$$

After substituting the uniform density for $f(\theta)$ and integrating, we obtain $F(\theta) = \frac{\theta}{4{,}000} - 0.5$

The payoff functions are the same as in Lemons I.

The equilibrium price must be less than \$4,000 in Lemons II because, as in Lemons I, not all cars are put on the market at that price Owners are willing to sell only if the quality of their cars is less than 4,000, so while the average quality of all used car is 4,000, the average quality offered for sale is 3,000. The price cannot be \$4,000 when the average quality is 3,000, so the price must drop at least to \$3,000. If that happens, the owners of car with values from 3,000 to 4,000 pull their cars of the market and the average of those remaining is 2,500. The acceptable price fall to \$2,500, and the unraveling continues until the price reaches it's equilibrium level of \$2,000. But at P = 2,000 the number of cars on the market is infinitesimal. The market has completely collapsed!

Figure 12.2 put the price of used cars on one axis and the average quality of cars offered for sale on the other. Each price leads to a different average quality, $\bar{\theta}(P)$, and the slope of $\bar{\theta}(P)$ is greater than one because average quality does not rise proportionately with price. If the price rises, the quality of the marginal car offered for sale equals the new price but the quality of the average car offered for sale is much lower. In equilibrium, the average quality must equal the price, so the equilibrium lies on the 45 degree line through the origin. That line is a demand schedule of sorts, just as $\bar{\theta}(P)$ is a supply schedule. The only intersection is the point (\$2,000, 2,000).

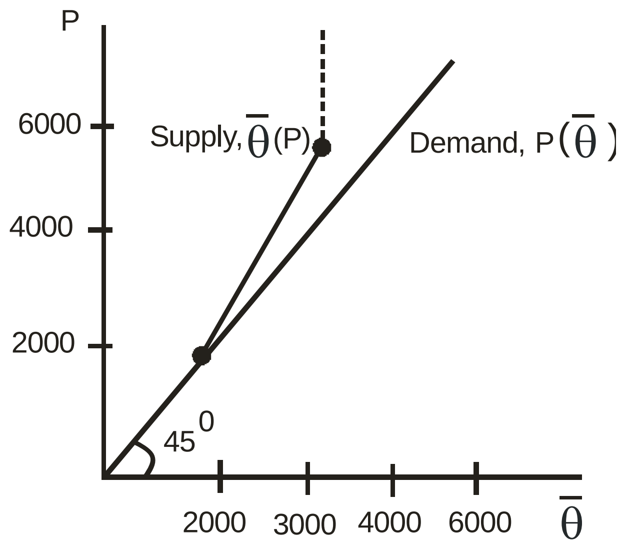


Figure 12.2: Lemons II: Identical tastes

12.4 HETEROGENEOUS TASTES : LEMONS II AND IV:

The outcome that no cars are traded is extreme, but there is no efficiency loss in either Lemons I or Lemons II. Since all the players have identical tastes, it does not matter who ends up owning the cars. But the players of this section, whose tastes differ, have real need of a market.

12.4.1 Lemons III : Buyers Value Cars More than Sellers

Assume that sellers value their cars at exactly their qualities θ, but that buyers have valuations 20 percent greater, and, moreover, outnumber the seller. The payoff if a trade occurs are π buyer $= 1.2\theta - p$ and π seller $= P - \theta$. In equilibrium, the sellers will capture the gains from trade.

In figure 12.3 the curve $\bar{\theta}(P)$ is much the same as in Lemons II, but the equilibrium condition is no longer that price and average quality lie on the 45 degree line, but that they lie on the demand schedule $P(\bar{\theta})$, which has a slope of 1.2 instead of 1.0. The demand and supply schedules interest only at (P = \$3,000, $\bar{\theta}$(P)=2,500). Because buyers are willing to pay a premium, we only see partial adverse selection; the equilibrium is partially pooling. The outcome is inefficient, because in a world of perfect information all the cars would be owned by the "buyers", who value them more, buy under adverse selection they only end up owning the low quality cars.

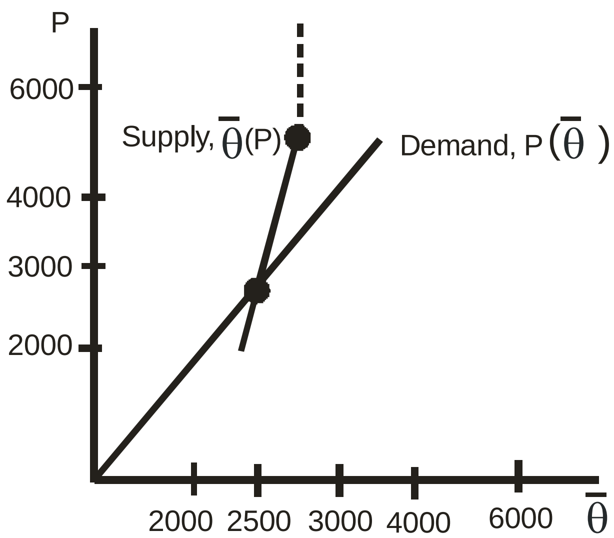


Figure 12.3 Adverse selection when buyers value cars more than sellers: Lemons III

12.4.2 Lemons IV : Sellers' Valuation Differ

In Lemons IV, we dig a little deeper to explain why trade occurs, and we model sellers as consumers whose valuation of quality have changed since they bought their cars. For a particular seller, the valuation of one unit of quality is 1 + E, where the random disturbance E can be either positive or negative and has on expected value of zero. The disturbance could arise because of the seller's mistake – he did not realize how much he would enjoy driving when he bought the car or because conditions changed – he switched to a job closer to home. Payoffs if a trade occurs are $\pi_{buyer} = \theta - P$ and $\pi_{seller} = P - (1+E)\theta$.

If E = -0.15 and $\theta = 2{,}000$, then $ 1,700 is the lowest price at which the player would resell his car. The average quality of cars offered for sale at price P is the expected quality of cars valued by their owners at less than P, i.e. $\bar{\theta}(P) = E(\theta 1 (1+E)\theta \leq P)$.

Suppose that a large number of new buyers, greater in number than the sellers, appear in the market, and let their valuation of one unit of quality be $ 1. The demand schedule shown in figure is the 45 degree line through the origin. Figure 12.4 show one possible shape for the supply schedule $\bar{\theta}(P)$, although to specify it precisely we would have to specify the distribution of the disturbances.

In contrast to Lemons I, II and III, here if $P \geq \$6{,}000$ some car owners would be reluctant to sell, because they received positive disturbances to their valuations.

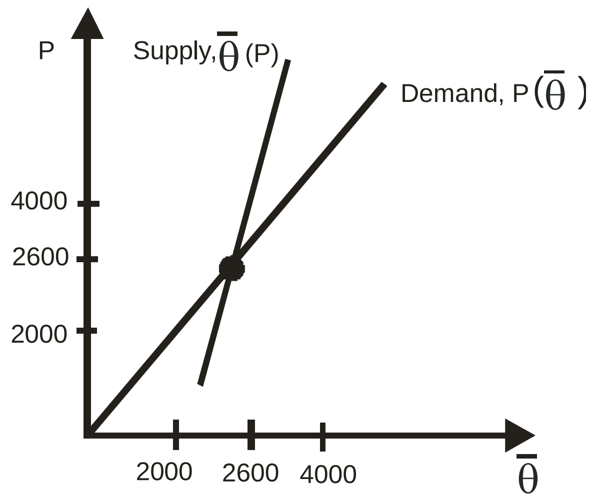


Figure 12.4: Lemons IV : Seller's Valuations differ

The average quality of cars on the market is less than 4,000 even at P = \$6,000. On the other hand, even if P = \$2,000 some sellers with low quality cars and negative realization of the disturbance still sell, so the average quality remains above 2,000. Under their cars so much they would pay to have them taken away.

The equilibrium drawn in figure is (P = \$2,600, $\bar{\theta}$ = 2,600). Some used cars are sold, but the number is inefficiently low. Some of the sellers have high quality cars but negative disturbances, and although they would like to sell their cars to someone who values them more, they will not sell at a price of \$2,600.

A theme running through all four Lemons models is that when quality is unknown to the buyer, less trade occurs. Lemons I and II show how trade diminishes, while Lemons III and IV show that the disappearance can be inefficient because some sellers value cars less than some buyers. Next we will use Lemons III, the simplest model with gains from trade, to look at various markets with more seller than buyers, excess supply, and risk – average buyers.

12.4.3 More Sellers than Buyers

In analyzing Lemons III, we assumed that buyers outnumbered sellers. As a result; the seller earned producer surplus. In the original equilibrium, all the sellers with quality less than 3,000 offered a price of \$3,000 and earned a surplus of up to \$1,000. There were more buyers than sellers, so every seller who wished to sell was able to do so, but the price equaled the buyers, expected utility, so no buyer who failed to purchase was dissatisfied. The market cleared.

If, instead, sellers outnumber buyer, what price should a seller offer? At \$3,000, not all would be seller can find buyers. A seller who proposed a lower price would find willing buyers despite the somewhat lower expected quality. The buyer's tradeoff between lower price and lower quality is shown in figure 12.3 in which the expected consumer surplus is the vertical distance between the price (the height of the supply schedule) and the demand schedule. When the price is \$3,000 and the average quality is 2,500, the buyer expects a consumer surplus of zero, which is \$3,000 - \$1.2. 2,500. The combination of price and quality that buyers like best (\$ 2,000, 2,000), because if there were enough sellers with quality θ = 2,000 to satisfy the demand, each buyer would pay P = \$2,000 for a car worth \$2,400 to him, acquiring a surplus of \$400. If there were fewer sellers, the equilibrium price would be higher and some sellers would receive producer surplus.

12.5 HETEROGENEOUS BUYERS : EXCESS SUPPLY

If buyers have different valuation for quality, the market might not clear, as Wilson (1980) points out. Assume that the number of buyers willing to pay $1.2 per unit of quality exceeds the number of sellers, but that buyer Smith is an eccentric whose demand for high quality is unusually strong. He would pay $ 100,000 for a car of quality 5,000 or greater, and $ 0 for a car of any lower quality.

In Lemons III without Smith, the outcome is a price of $ 3,000, an average market quality of 2,500, and a market quality range between 2,000 and 3,000. Smith would be unhappy with this, since he has zero probability of finding a car he likes. In fact, he would be willing to accept a price of $6,000, so that all the cars, from quality 2,000 to 6,000, would be offered for sale and the probability that he buys a satisfactory car would rise from 0 to 0.25. But Smith would not want to buy all the cars offered to him, so the equilibrium has two prices, $3,000 and $6,000, with excess supply at the higher price. Strangely enough, Smith's demand function is upward sloping. At a price of $3,000, he is unwilling to buy; at a price of $6,000, he willing, because expected quality rises with price. This does not contradict basic price theory, for the standard assumption of ceteris paribus is violated. As the price increases, the quantity demanded would fall if all else stayed the same, but all else does not quality rises.

12.6 RISK AVERSION

We have implicitly assumed, by the choice of payoff functions, that the buyers and sellers are both risk neutral. What happens if they are risk averse – that is, if the marginal utilities of wealth and car quality are diminishing? Again we will use Lemons III and the assumption of many buyers.

On the seller's side, risk aversion changes nothing. The seller runs no risk because he knows exactly the price he receives and the quality he surrenders. But the buyer does bear risk, because he buys a car of uncertain quality. Although he would pay $3,600 for a car he knows has quality 3,000, if he is risk averse he will not pay that much for a car with expected quality 3,000 but actual quality of possibly 2,500 or 3,500. He would obtain less utility from adding 500 quality units than from subtracting 500. The buyer would pay perhaps $2,900 for a car whose expected quality is 3,000 where the demand schedule is nonlinear, lying every where below the demand schedule of the risk – neutral buyer. As a result, the equilibrium has a lower price and average quality.

12.7 ADVERSE SELECTION UNDER UNCERTAINTY : INSURANCE GAME III

The term “adverse Selection”, like “moral hazard,” comes from insurance. Insurance pays more if there is an accident than otherwise, so it benefits accident – prone customers more than safe ones and a firm’s customers are “adversely selected” to be accident prone. The classic article on adverse selection in insurance markets is Rothschild & Stiglitz (1976), which begins, “Economic theorists traditionally banish discussions of information to footnotes” How things have changed! Within ten years, information problem came to dominate research in both micro economics and macro economics.

We will fallow Rothschild & Stiglitz in using state – space diagrams, and we will use a version of the Insurance Game of Section. Under moral hazard, Smith chose whether to be Careful or Careless. Under adverse Selection, Smith cannot affect the probability of a theft, which is chosen by Nature. Rather, Smith is either Safe or Unsafe, and while he cannot affect the probability that his car will be stolen, he does know what the probability is.

Insurance Game III

Players

Smith and two insurance companies

The Order of Play

0. Nature chooses Smith to be either Safe, with probability 0.6, or Unsafe, with probability 0.4 Smith knows his type, but the insurance companies do not.
1. Each insurance company offers it’s own contract (x, y) under which Smith pays premium x unconditionally and receives compensation y if there is a theft.
2. Smith picks a contract.
3. Nature chooses whether there is a theft, using probability 0.5 if Smith is safe and 0.75 if he is Unsafe.

Payoffs

Smith’s payoff depends on his type and the Contract (x, y) that he accepts. Let U’ > 0 and U’’ < 0

$$\pi\ \text{Smith}\,(\text{Safe}) = 0.5U\,(12 - X) + 0.5U\,(0 + Y - X)$$

$$\pi\ \text{Smith}\,(\text{Unsafe}) = 0.25U\,(12 - X) + 0.75\,(0 + Y - X)$$

The companies’ payoffs depend on what types of customers accept their contracts, as shown in the table 11.1.

Table 12.1 : Insurance Game III : Payoffs

Company Payoffs	Types of Customers
• 0	No Customers
• 0.5x + 0.5 (x – y)	Just safe
• 0.25x + 0.75 (x – y)	Just Unsafe
• 0.6 [0.5x + 0.5 (x – y)]+ 0.4 [0.25x + 0.75 (x – y)]	Unsafe and Safe

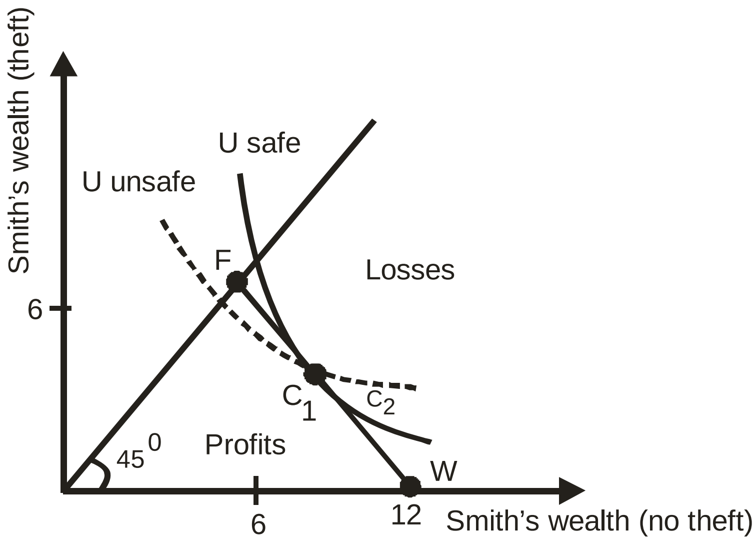


Figure 12.5: Insurance Game III

Smith is safe with probability 0.6 and Unsafe with probability 0.4. without insurance, Smith's dollar wealth is 12 if there is no theft and 0 if there is, depicted in figure 12.5 as his endowment in state space, W = (12, 0). If Smith is Safe, a theft occurs with probability 0.5, but if he is Unsafe the probability is 0.75. Smith is risk averse (because U' < 0) and the insurance companies are risk neutral.

If an insurance company knew that Smith was safe, it could offer him insurance at a premium of 6 with a payout of 12 after a theft, leaving Smith with an allocation of (6, 6). This is the most attractive contract that is not unprofitable, because it fully insures Smith. Whatever the state, his allocation is 6.

Figure 12.5 show the indifference curves of Smith and an insurance company. The insurance company is risk neutral, so its indifference curve is a straight line. If Smith will be a customer regardless of his type, the company's indifference curve based on its expected profits is wF (although if the company knew that Smith was Safe, the indifference curve would be steeper, and if it knew he was Unsafe, the curve would be steeper). The insurance company is indifferent between W and C_1, at both of which is expected profits are zero. Smith is risk averse, so his indifference curves are convex, and closest to the origin along the 45 degree if the probability of Theft is 0.5. He has two sets of indifference curves, Solid if he is Safe and dotted if he is Unsafe.

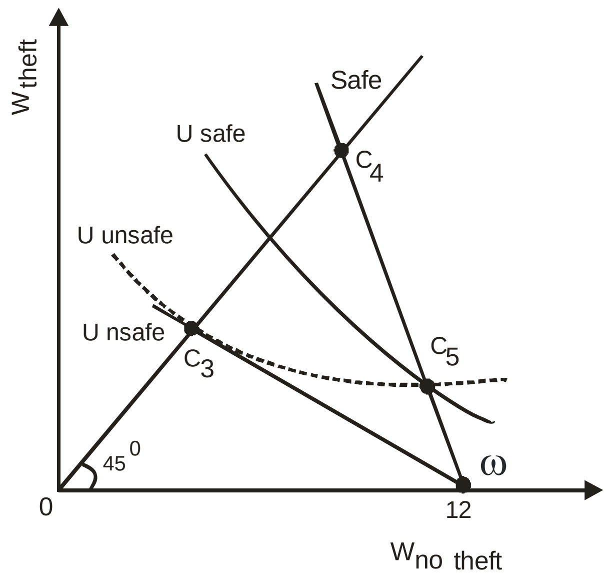


Figure 12.6: A separating equilibrium for Insurance Game III

Figure 12.5 shows why no Nash pooling equilibrium exists. To make zero profits, the equilibrium must lie on the line wF. It is easiest to think about these problems by imagining an entire population of Smiths, whom we will call customers". Pick a contract C_1 anywhere on wF and think about drawing the indifference curves for the Unsafe and Safe customers that pass through C_1, Safe customers are always willing to trade Theft wealth for No Theft wealth at a higher rate than Unsafe customers. At any point, therefore, the Slope of the Solid (Safe) indifference curve is steeper than that of the dashed (Unsafe) curve. Since the slopes of the dashed and solid indifference curves differ, we can insert another contract, C_2, between them and just barely to the right of wF. The safe customer prefer contract C_2 to C_1, but the Unsafe customers stay with C_1, so C_2 is profitable since C_2 only attracts Safes, it need not be to the left of wF to avoid losses. But then the original contract C_1 was not a Nash equilibrium, and since our argument holds for any pooling contract, no pooling equilibrium exists.

The attraction of the Safe customers away from pooling is referred to as cream skimming, although profits are still zero when there is competition for the cream. We next consider whether a separating equilibrium exist, using figure 12.6. The zero profit condition requires that the Safe customers take contracts on wC_4 and the Unsafes on wC_3.

The Unsafes will be completely insured in any equilibrium, albeit at a high price. On the zero profit line wC_3 the contract they like best is C_3, which the Safes are not tempted to take. The Safes would prefer contract C_4, but C_4 uniformly dominates C_3, so it would attract Unsafes too, and generate losses. To avoid attracting Unsafes, the Safe contract must be below the Unsafes in difference curve. Contract C_5 is the fullest insurance the Safes can get without attracting Unsafes : it satisfies the self selection and competition constraints.

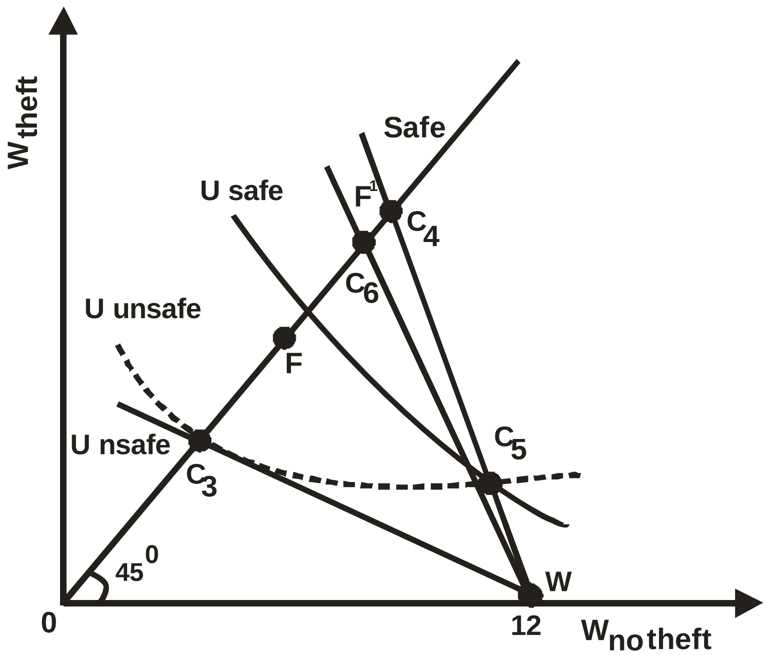


Figure 12.7: Curves for which there is no equilibrium in Insurance Game III

Contract C_5, however, might not be an equilibrium either. Figure 12.7 is the same as figure 12.6 with a few additional points marked. If one firm offered C_6, it would attract both types, Unsafe and Safe, away from C_3 and C_5, because it is to the right of the indifference curves passing through those points. Would C_6 be profitable? That depends on the proportion of the different types. The assumption on which the equilibrium of figure 12.6 is based is that the proportion of Safes is 0.6, so that the zero – profit line for pooling contracts is wF and C_6 would be unprofitable. In figure 12.7 it is assumed that the proportion of Safes is higher, so the zero profit line for pooling contracts would be wF' and C_6, lying to it's left, is profitable. But we already showed that no pooling contract is Nash, so C_6 cannot be an equilibrium. Since, neither a separating pair like (C_3, C_5) nor a pooling contract like C_6 is on equilibrium no equilibrium whatsoever exists.

The essence of nonexistence here is that if separating contracts are offered, some company is willing to offer a Superior pooling contract, but if a pooling contract is offered, some company is willing to offer a separating contract that makes it unprofitable. A monopoly would have a pure strategy equilibrium, but in a competitive market only a mixed strategy Nash equilibrium exists.

12.8 SUMMARY

Thus, in moral hazard with hidden knowledge the emphasis is on the agent's action rather than his choice of contract, and agents accept contracts before acquiring information. Under adverse selection, the agent has private information about his type or the state of the world before he agrees to a contract, which means that the emphasis is on which contract he will accept.

12.9 FURTHER READINGS

(1) Mas – Colell A.M.D. Whinston and J.R. Green : Microeconomic Theory Oxford University Press, 1995.

(2) Rasmusen E. : Games and Information, Blackwell, 1994.

12.10 QUESTIONS

(1) Explain the concept of adverse selection

(2) Explain how adverse selection is different from moral hazard.

(3) Explain adverse selection under uncertainty with identical tastes.

(4) Explain adverse selection under uncertainty with heterogeneous tastes.

❖❖❖❖

13

GENERAL EQUILIBRIUM

UNIT STRUCTURE

13.0 OBJECTIVES

After going through this unit you will be able to,

- Define general and partial equilibrium
- Explain the Walrasian equilibrium
- Describe efficiency in production and exchange
- Know three marginal conditions of Pareto Optimiy
- Discuss the stability of equilibrium
- Give the equations of general equilibrium analysis
- Do the critical evaluation of general equilibrium analysis

13.1 INTRODUCTION :-

The term ‘equilibrium’ has been derived from the Latin words and it means equal balance. It means ‘the state of rest’ or a state of motionlessness. Equilibrium has been defined by Machlup as “a constellation of interrelated variables so adjusted to one anther that no inherent tendency to change prevails in the model which they constitute". The concept 'equilibrium' has been used in economics to serve a variety of purposes. In welfare economics, the analytical focus of the present module, the term 'equilibrium' is used to denote the position which a person or an economic entity regarded as the best possible attainable position under the given circumstances and does not want to deviate from it (Scitovsky).

13.2 PARTIAL AND GENERAL EQUILIBRIUM :-

Partial equilibrium analysis is concerned with the study of individual economic units or quantities like individual prices, commodities, particular households, firms etc under ceteris paribus assumption. It is partial in the sense that it analyses only a part of the economic system. It analyses behaviour of decision making units (consumers and producers) and of markets in isolation of one another. In other words we are concentrating on the price of a single commodity in isolation from the price of other products. Marshallian method of economic analysis was based on the observation of the complexities of the laws of human action. It was imperative for Marshall to adopt a restrictive method of partial equilibrium in dealing with economic problems and theorizing. Alfred Marshall, the father of neoclassical welfare economics explained the technique in the following manner. 'The forces to be dealt with are so numerous, it is best to select a few at a time and to work out a number of partial solutions. This scientific device is older than the science. It is the method by which consciously or unconsciously sensible men have dealt from time immemorial with every difficult problem of ordinary life, says Marshall in his 'Principles of economics'. The most important element of the technique is 'ceteris paribus' assumption. The main objective of 'ceteris paribus' assumption is to simplify the conditions of analysis by assuming away the 'feedback effect'. Thus it becomes possible to analyze the behaviour of particular individual or households, a particular firm or industry and trace their equilibrium positions with a restricted range of data.

Partial equilibrium technique serves many useful purposes. It facilitates the isolation of a particular economic phenomenon from the complex economic world. Prof Stigler defines partial equilibrium as the "one which is based on a restricted range of data". In partial equilibrium analysis, changes in one variable are considered, keeping all other factors as fixed. Thus, the condition of 'other things being equal' i.e. ceteris paribus underlies at the root of partial equilibrium analysis.

Marshallian partial equilibrium approach made it possible to single out for attention one segment of the economy, at a time, neglecting its links with other parts. It rules out interdependence. The partial equilibrium approach lies at the centre of Mrshallian theory of value or price determination. According to Marshall, the price of a commodity in a competitive market is determined by the interaction of the forces of demand and supply. Changes in price demand and supply of one product or factor has nothing to do with demand, supply and price of other products or factors. In reality however markets are interdependent and interrelated. In an economy everything depends on everything else. This is a factor that is ignored by the partial equilibrium analysis. Partial equilibrium analysis presents a partial picture of the economy and it is too partial an analysis.

Marshall traces the demand behaviour through the law of demand. He states the law of demand as other things being equal, the demand for commodity rises when its price falls and vice versa. Hence Marshall has adapted the partial equilibrium approach.

13.3 GENERAL EQUILIBRIUM ANALYSIS :-

In contrast, general equilibrium aims at analyzing economic system in an integrated manner by recognizing the interrelationships and mutual interdependence between the input and the output markets. An analysis that takes into account the interdependence of price is called general equilibrium. For example the price of wool is not determined strictly by its own demand and supply curve. It is influenced by a vast multitude of interrelated factors like demand for yarn, clothes and so on.. The price of wool may also be influenced by the price of its substitutes such as linen, silk etc. The first scientific treatment of general equilibrium was made by Leon walrus (1883-1910) of the Lussane School. The ideas of Walras were later absorbed by Pareto (1883-1923) who further developed the general equilibrium analysis and cast it in a suitable mathematical mould. The basic purpose of general equilibrium analysis is to develop an analytical framework and prescribe the conditions for general equilibrium in the economy.

General equilibrium is a type of equilibrium in which a number of economic variables are studied to see the interrelations and interdependence among the variables for the proper understanding of the economy as a whole. Whereas in the partial equilibrium analysis, only two variables are taken into account, in general equilibrium analysis, all the relevant variables are brought to play their part. General equilibrium can be defined as a state of the economy in which all economic units and all the markets are in equilibrium. General equilibrium analysis is Leon Walras' claim to immortality, says Baumol in his book "Economic Theory and Operational Analysis". With regard to pricing under perfect completion there are two kinds of approaches, i.e. Marshallian 'partial equilibrium' discussed above and Walrasian general equilibrium. According to Marshalian Partial equilibrium analysis demand for a commodity is determined/defined by its price alone, under ceteris paribus assumption.

Symbolically state Dx is a function of price alone, other things remaining constant.

Dx f(P)

i.e Demand for commodity x is a function of price of x

Marshallian approach was critiqued as 'too partial an approach'. Walrasian General Equilibrium analysis as pointed out by Stonier and Hague is a 'study of' multi-market equilibrium'. In multi-market equilibrium analysis of price is not determined independently as in Marshallian partial equilibrium analysis. Thus, interdependency is taken into account in this model.

13.4 WALRASIAN EQUILIBRIUM: EXCHANGE AND PRODUCTION :-

The Edeworth box diagram can be used to show the conditions necessary for general equilibrium. For an economic system to achieve a general equilibrium, three conditions must be satisfied. These three conditions refer to general equilibrium in exchange, in production and between exchange and production. We assume a competitive economy in which consumers maximize utility and producers maximize profits. The analysis that follows is restricted to a two dimensional diagram where two consumers, 2 products, and 2 inputs will be used.

Assumptions

1. Constant Returns to scale
2. Full employment
3. Perfect Competition
4. Homogeneous units of productive service
5. Tastes and Income of the consumers are given
6. Constant Production techniques
7. Mobility of factors of production

13.4.1 GENERAL EQUILIBRIUM IN EXCHANGE

General equilibrium in exchange is diagrammatically represented in fig 5.1 with the aid of the Edge worth box diagram. We assume two individuals, A and B and two goods X and Y. Before exchange takes place A possess OAXi of X and OAYi of Y.

Individual B possesses OBXii of X and OBYii of Y. The initial position can be represented by Q in the
box diagram.

a) Efficiency in Exchange:

An efficient distribution of commodities between consumers (equilibrium of consumption) requires that:

$$MRS^{A}_{XY} = MRS^{B}_{XY} = \frac{P_Y}{P_X}$$

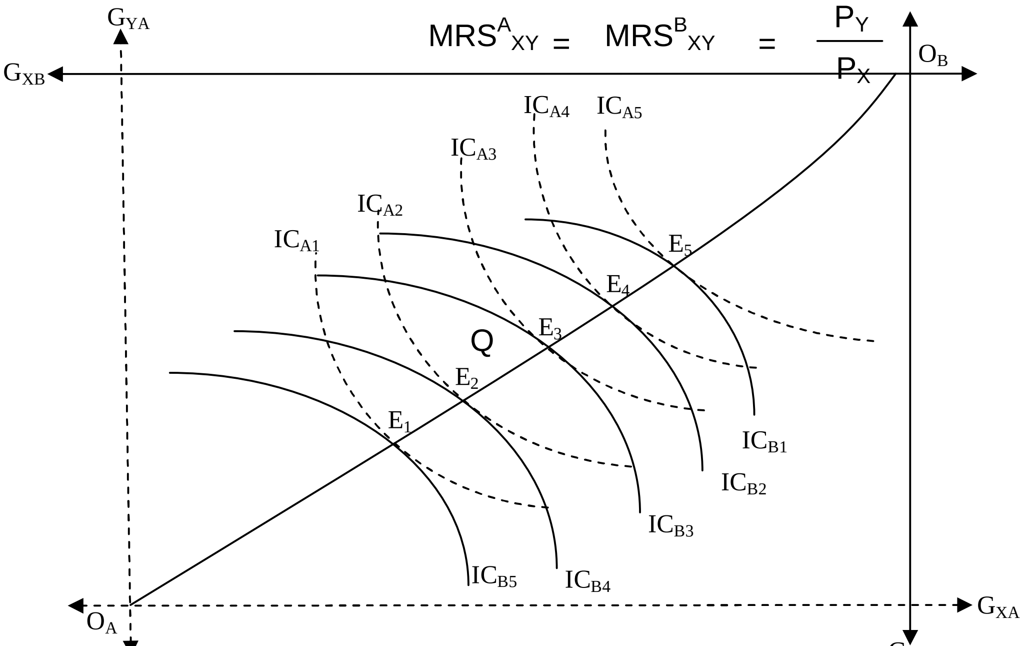


Figure: General Equilibrium in exchange

General equilibrium in exchange in exchange requires that consumers goods are so allocated between the consumers that each consumer is in equilibrium with MRS being equal to commodity price ratio and that the total product is exhausted. With A on his indifference curve, ICi, and B on his indifference curve, IC2. Both individuals can benefit from exchange, since each can move to a higher indifference curve by trading. For example, if A exchanges some of his Y for some of B's X so that they possess combinations of X and Y which will take them to point "S", then both would be on higher indifference curves. Thus , any movement from point Q to any within the shaded area bound by the two intersecting indifference curves (ICi and IBi) will lead to greater satisfaction for both individuals. General equilibrium of exchange will occur at some point such as S, on the contract curve OAOB, that connects all tangency points of A's and B's indifference curves and that denotes all potential equilibrium points. Once on the contract curve, no further Paeto improvement is possible (one person can be made better off-and another person worse off) and therefore no further exchange takes place.

Hence an equilibrium has been attained Since the points on the contract curve represent the tangency points of A's and B's indifference curves, and since the slope of an indifference curve is given by the MRS (Marginal rate of substitution) of the two goods, when equilibrium is achieved, the MRSxy is the same for both the individuals. Every point on the contract curve is efficient because one person cannot be made better off without making the other person worse off. What is true of two individuals is true for any number of individuals and general equilibrium of exchange prevails when the MRS for all individuals are equal. This result is achieved in a perfectly competitive economy

MRSA = MRSB

x,y x,y

This important result also falls when there are many goods and many consumers. An allocation of goods is efficient if the goods are distributed so that the MRS between any two pairs of goods is the same for all consumers and also equal to the price ratios. This competitive equilibrium is efficient where MRSA = Px MRSB.x
Y.X Py = Y

13.4.2 GENERAL EQUILIBRIUM IN PRODUCTION

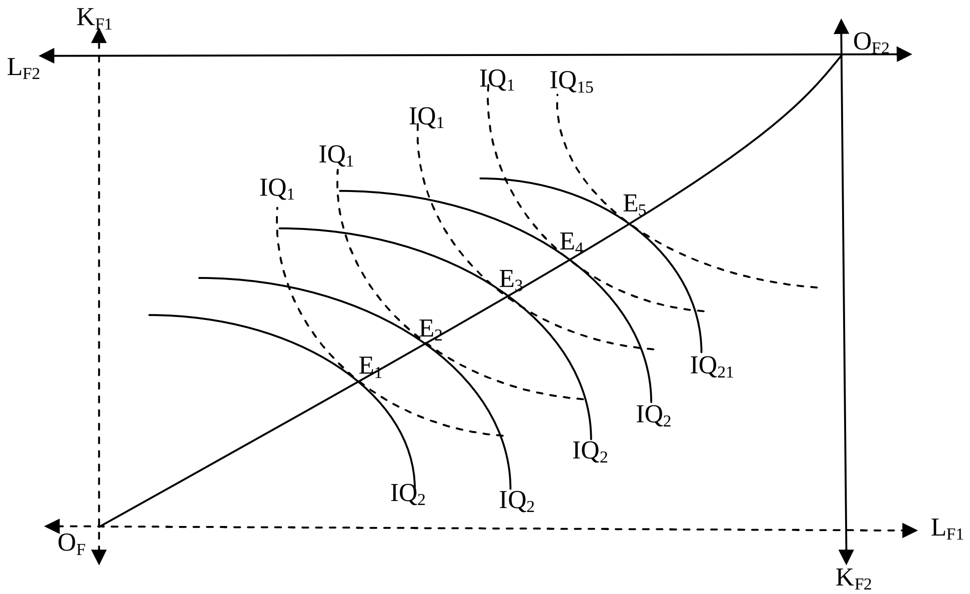
In a manner parallel to the analysis of general equilibrium in exchange general equilibrium in production is determined in the box diagram in figure 5.2. Figure 5.2 shows production isoquants for goods X and Y and the dimensions of the box represent

The amount of labour (L) and Capital (K) possessed by producers of X and Y. Initially the producer X has OxL of L and OxK of K and producer Y has OyK2 of K. This initial position is represented in the box diagram by F in figure 5.2. The relevant question is whether the two producers can trade L and K in such a way that more of both X and Y can be produced as a result or at least more of either X or Y with the same quantity of the other good.

B Efficiency in Production:

An efficient allocation of resources among firms (equilibrium of production) requires that:

$$MRTS^{F1}_{LK} = MRTS^{F2}_{LK} = \frac{i}{w}$$



In the box diagram a movement from F to any point in the shaded area bound by two intersecting isoquants x1 and y2 would lead to a greater output of both X and Y.For example at point 'V' each producer would be on a higher isoquant and each would be able to produce more of his respective goods. Hence the economy under consideration would produce more of both goods with the same inputs. As point 'V' is on the contract curve connecting all tangency points of the X and Y isoquants, general equilibrium in production has been established/reached. It implies that it is impossible to increase output without decreasing the output of other goods. General equilibrium in production is attained when the MRTS (Marginal Rate of Technical substitution) for L and K is the same for the two products. The competitive equilibrium must lie on the production contract curve and the competitive equilibrium is efficient in production.

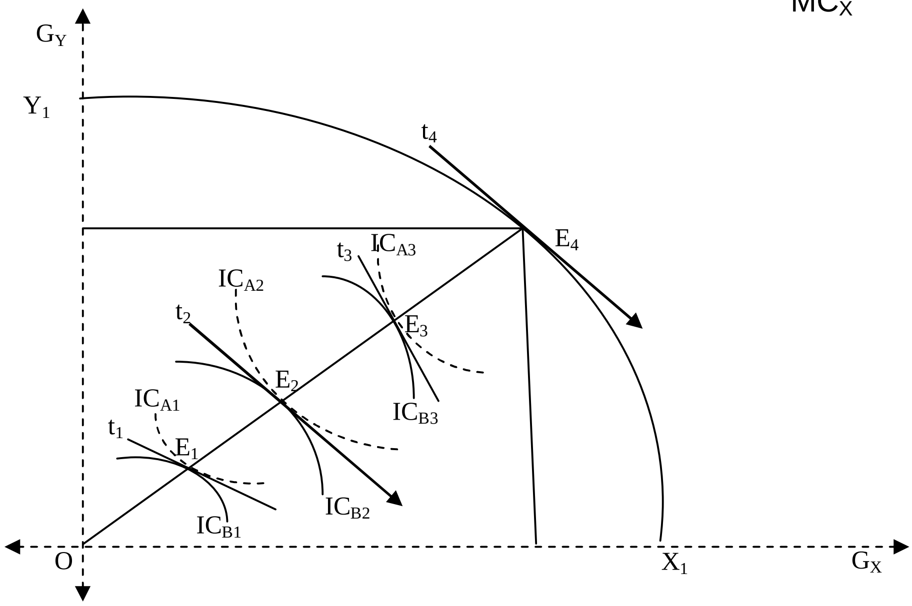
13.4.3 GENERAL EQUILIBRIUM between EXCHANGE and PRODUCTION

To have a general equilibrium in the economy general equilibrium in exchange must be consistent with general equilibrium in production. The condition necessary for general equilibrium between exchange and production is MRS=MRT (Marginal Rate of Substitution must be equal to Marginal Rate of Technical substitution).General Equilibrium in exchange occurs when both the individuals have the same MRS, which is a point on the contract curve. When MRS= MRT there is general equilibrium between exchange and production

C) Efficiency in Product-Mix:

An efficient combination of products (simultaneous equilibrium of production and consumption) requires that

$$MRS^{(A+B)}_{XY} = MRT^{F1+F2}_{XY} = \frac{MC_Y}{MC_X}$$



13.5 EFFICIENCY: THREE MARGINAL CONDITIONS OF PARETO EFFICIENCY

General equilibrium is achieved on the contract curve of the Edgeworth box diagram. Contract curve represents all the tangency points of the two individuals' indifference curves, thus at all tangency points, the slope of the indifference curves are the same and hence the two individuals have the same MRS. We know that each individual will maximize the satisfaction by equating his MRS for ant two goods to the ratio of their prices. Pareto optimiy for production is attained on the contract curve of the Edeworth box diagram for production (see FI x).

13.5.1 EXISTENCE OF EQUILIBRIUM

In connection with general equilibrium says Koutsoyiannis three problems arise. The three problems are:

1. Existence of equilibrium. Does a general equilibrium solution exist?
2. Uniqueness of equilibrium. If an equilibrium solution exists, is it unique?
3 Stability of equilibrium. If an equilibrium solution exists, is it stable?

EXISTENCE OF EQUILIBRIUM:

The price at which quantity demand (Qd) is equal to quantity supplied (Qs) is the equilibrium price. At such a price there is neither excess demand nor excess supply. The equilibrium price can be defined, thus as the price at which excess demand is zero.

13.5.2 STABILITY OF EQUILIBRIUM

Will equilibrium be stable? By this mean: if something temporarily disturbs the equilibrium, will the underlying forces tend to restore equilibrium. The ball resting inside a U shaped bowl is upside down and the ball is perched upon it, the ball's equilibrium is unstable. The question concerning stability can be swiftly answered: The control mechanism which brought about equilibrium in the first place will restore it, if it is disturbed. A stable equilibrium exists if the demand function cuts the supply function from above. In this case an excess demand drives the prices up, while an excess supply drives the prices down

Figure 22.3 Unique stable equilibrium (Koutsiyanis p489)

13.5.3 UNIQUENESS OF EQUILIBRIUM

The next important question raised is equilibrium exist that is stable. But will there be only one such equilibrium? Can there be multiple equilibrium? If they do we might be able to improve social welfare by shifting the economy from one equilibrium to another.(Read: P.R.G. Layard and A.A. Walters)

From the above discussion it is clear that the existence of equilibrium is related to the problem of whether the consumers or producers' behaviour ensures that the demand and supply curves intersect(at a positive price).The stability of equilibrium depends on the relationship between the slopes of the demand and supply curves. The uniqueness is related to the slope of the excess demand function.(Koutsoyiannis,19790).

13.5.4 EQUATIONS OF GENERAL EQUILIBRIUM ANALYSIS

Read: Baumol, Economic Theory and operational analysis

Under General equilibrium:
1. All demand forces equal supply forces
3. Price of each commodity is equal to its marginal utility
4. Price ratio between two goods is equal to marginal rate of substitution between them
5. Price and MC for each firm are equal
6. Cost of production for each firm is at minimum

Let us put the General Equilibrium Model in the form of equations
Suppose an economy has 2053 commodities and money is the 2054^{th} item.

Now, if hats are item no 12, then the demand for hat will be given by the following expression:

D12 = f (P1, P2, P3…….. P12 ……. P 2054, A, M)
where A= holding of physical assets
M= Stock of cash in existence

This implies:
- Demand for hats with reference to its price depends on price of other goods

- These goods are related goods e.g. substitutes, complements etc
Similarly, the supply function of hats can be stated as follows:

S 12 = f (P1, P2, P3…….. P2 ……. P2054, A, M)

The economy is in equilibrium when the supply of every commodity is equal to its demand.
For 2054 items, the following equations must hold good:

S1= f (P1, P2, P3... P2054 A, M) **= D1** = f (P1, P2, P3 …P2054, A, M)
,, “
,, “
,,
,,
,,
‘’
S(2054)= f (P1, P2, P3... P2054, A, M) **= D2054** = f (P1, P2, P3 …P2054, A, M)

Walrasian Identity says that every demand is matched by an equal supply

Sigma Pi Si+= Suma PiDi(aggie plez type it properly the walasian identuty

13.5.5 REDUNDANT EQUATIONS

Of the 2054 items price of money is a peculiar animal’ as stated by Baumol. The price of any item is the number of dollars it takes to purchase a unit of that good. The unit of money here is dollar so that the number of dollars it takes to purchase a unit of money is exactly one. P2054 rather than being a variable is ‘a number’1’ and in reality we have therefore only 2053 equations.

Given below are the four sets of equations for General Equilibrium covering all sectors

Ist Set

Price of any item is equal to the number of dollars it takes to purchase that good i.e. the number of dollars it takes to purchase one unit of that good.

G (a) = f (P'm, P'n, P'o ……. Pa, Pb, Pc)
G (b) = f(p'm.p'n,p'o…………pa,pb,pc
G(c) = f(p'm,p'n,p'o………….pa,pb,pc

Where a, b, c = goods
m, n, o= inputs
P'm , P'n and P'o = price of inputs
Pa, Pb, Pc = Price of goods
Hence, price of good A = price of inputs + price of other goods

IInd Set

This set deals with allocation of an economy's public resources.

Δm = amGa + bmGb + cmGc
Δn = anGa + bnGb + cnGc
Δo = aoGa + boGb + coGb
Hence, this determines how much of factor x is used in production of a, b, c…. etc.

IIIrd Set

This set gives us the cost of various consumer goods including the factor costs i.e. the total cost of producing a commodity

P'a = amP'm + anP'n + aoP'o
P'b = bmP'm + bnP'n + boP'o
P'c = cmP'm + cnP'n + coP'o

IVth Set

This set gives us the cost of production per unit of a commodity

P'a= Pa
P'b= Pb
P'c=Pc

Hence, per unit cost of commodity a, b, c is set by price of the three.

13.5.6 CRITICAL EVALUATION

It is based on highly unrealistic assumptions of perfect competition. It is static in nature with technology, consumers' tastes etc held constant. It doesn't explain satisfactorily how general equilibrium(GE) is brought about. It fails to achieve equity in the distribution of national income. General equilibrium analysis doesn't consider important factors such as government regulations, trade union etc which stand in the way of general equilibrium(GE) through market adjustment. According to Schumpeter the general equilibrium(GE) analysis is Walrus' claim to immortality. The fundamental idea of this model is in its interdependency. Its practical application can be seen in the input-output analysis of Leontieff. According to Eric Roll by inventing the general equilibrium(GE) model, Walrus became an economists' economist. The beauty of Marshallian solution is that it showed interdependence of demand and supply. However, Walrus went a step further by describing general equilibrium(GE) of the economy as a whole with emphasis on the importance of all prevailing prices.

Walrasian General Equilibrium fails to take note of certain features which impede the adjustment mechanism in taking the economy in the direction of General equilibrium. The government for example may be the biggest force preventing the establishment of general equilibrium (GE). Government policies concerning minimum wage may prevent the wage from finding their true value. Trade union can also impede the appropriate adjustment in wages when the cost of living registers a decline. Economists like J.R.Hicks attempted to remove the static approach. Hicks investigated what happens when changes take place in interlinked demands, supplies and prices. Others tried to reconstruct the model by bringing in certain forms of imperfect competition. Further Walrasian approach is a highly static approach as it assumes that taste of the consumers and the production coefficients of the consumers are fixed. The Walraisn model presupposes that everything happens in a predetermined manner. Walrasian G E model has been built upon the basis of perfect competition in all the markets of the economy. This is a highly unrealistic assumption as in vast majority of the markets what really exists is not perfect but imperfect competition.

Despite the above mentioned limitations, Walrsian system represents a capitalist system in equilibrium. Prodction in such a system is both efficient and responsive to the wants of the consumers. However though the GE model achieves efficiency it fails to achieve equity in the distribution of national income. It leads to unjust and inequitable distribution of national wealth.

However this fundamental theory led to many results of practical importance. The input output model is an important application of general equilibrium analysis. Leontieff in his famous book 'Structure of American Economy' explains the interdependence of the various sectors of the economy. It is a useful tool for forecasting. It is used in economic planning. Input output analysis gives a practical shape to general equilibrium analysis. Walrasian general equilibrium enriched economics, both at the theoretical and practical level. If Walras were to come out of his grave he has no reason to be disappointed as his work is furthered byTinbergen,Burgeson(Holland),Samelson (US),J.R.Hick s(UK),Gunnar Myrdal(Sweden).

13.6 SUMMARY :-

There are several reasons why the study of general equilibrium theory is important. It is the most complete existing model of economic behaviour. It makes the student aware of the tremendous complexity of the real world. General equilibrium theory can be helpful in the resolution of macroeconomic controversies.

13.7 QUESTIONS

1. Define and explain the concept of partial equilibrium.
2. Define and explain the concept of general equilibrium.
3. Distinguish between partial and general equilibrium.
4. Explain the Walrasian equilibrium in production and exchange.
5. Describe the three conditions of Pareto efficiency.
6. Explain the stability and uniqueness of the equilibrium.

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14

WELFARE ECONOMICS

UNIT STRUCTURE

14.0 OBJECTIVES

After going through this unit you will be able to,

- Explain fundamental theorems of welfare economics.
- Know paretian welfare economics.
- Define Pareto's social optimum.
- Know Kaldor – Hicks compensation Criteria.
- Discuss the theory of second best.
- Define and explain externalities.
- Define and explain public goods.
- Do the measurement of welfare.

14.1 INTRODUCTION :-

Welfare economics is that branch of economic science which aims at evaluating the social desirability of alternative social states. In other words, it attempts to lay down propositions by which one can rank, on the scale of better or worse, alternative economic situations open to society. The various prescriptions of welfare economics can be interpreted in terms of one single objective : the economic welfare of the community.

This last point leads us to define welfare economics somewhat differently. We can say Professor Oscar Lange that 'welfare economics is concerned with the conditions which determine the total welfare of a community.' Elsewhere the same scholar puts very aptly: 'welfare economics establishes norms of behavior which satisfy the requirements of social rationality of economic activity.'

14.2 FUNDAMENTAL THEOREMS OF WELFARE ECONOMICS

Welfare economics is concerned with the evaluation of alternative economic situation from the point of view of society's wellbeing. It lays down criteria on the basis of which policies can be adopted to maximize social welfare.

Pigou' Social optimum (A.C Pigou (1877-1959)

A.C. Pigou was a favorite pupil of Marshall and an outstanding economic theorist of England. His book 'Economics of welfare' was published in 1912 and he tried to establish economic welfare as a special branch of economics. He was a professor at Cambridge University and also the teacher of J.M Keynes.

Welfare Economics/ Social Optimum

According to Pigou, welfare is derived from utilitarianism. Bentham's utilitarian ethics is the source of inspiration for Pigou's ideas of welfare. Pigou defined economic welfare as *that part of social welfare which can be brought directly or indirectly into the relationship with the measuring rod of money*. Changes in economic welfare can be measured with the help of money. His approach towards welfare was following the cardinal approach as opposed to the ordinal approach followed by Paretian welfare economics.

Pigou made a distinction between economic welfare and social/general welfare. Economic welfare is a part of social welfare. Even if economic welfare remains constant, general welfare may change as it is affected by social, political and cultural institutions. General welfare is also affected by pattern of income distribution. For e.g. public bath increase welfare whereas public bars reduce welfare. General welfare changes in the same direction as economic welfare however; there is no relationship between the magnitudes of the two. Economic welfare consists of the utility derived from the exchange of goods and services whereas; general welfare is a wide complicated and impractical notion.

Pigou states two conditions of economic welfare under "Dual Criteria" which states:

i) Welfare increases when national income rises

ii) For maximization of welfare, distribution of national income is highly important.

The dual criteria for improvement in welfare consists of a rise in national income without reducing the share of the poor and any reorganization of the society which results in an increase in the share of the poor without reduction of national income.

Pigou derives welfare proposition from the following assumptions:

i) Rationality: Each individual tries to maximize his satisfaction out of his limited monetary resources.

ii) Interpersonal comparison of utility: Satisfaction derived from consumption of goods and services can be compared.

iii) Diminishing Marginal Utility (MU) of money: MU goes on decreasing as stock of money rises.

iv) Man's equal capacity: Different people derive the same satisfaction out of the same real income.

This concept of welfare is criticized on the following grounds:

- Welfare cannot be measured cardinally and this approach is considered inferior to Pareto's ordinal approach.
- National income is not an accurate measure of welfare.
- The assumption of man's equal capacity for satisfaction is ethical and makes the study normative.
- Pigou does not lay down the objective conditions of welfare.

Still, Pigouvian contribution to welfare economics has laid the foundation for modern welfare economics.

Diversion between Private Marginal Product (PMP) and Social Marginal Product (SMP)

PMP and SMP are annual flows resulting from marginal increase in the given quantity of resources.

PMP is that part of the total product which is enjoyed by the person investing in such resources. SMP is net product of physical things due to the marginal increment of resources in any given use/place accruing to whomsoever. Marginal net product is the difference made by withdrawing any unit at the margin.

When PMP and SMP are equal, welfare maximizes. However, there are certain factors that cause a divergence between the two. These are enumerated below:

- Self interest leads to equality between PMP of resources invested in various uses but this cannot be applied to SMP
- It also arises because of uncharged disservices or uncompensated services.
- Variation in cyclical fluctuation, war and rise of new industries cause a divergence between PMP &SMP costs and benefits.
- External effects / economies or diseconomies: External economies in the form of cheap inputs due to expansion of a firm lead to benefits enjoyed by the whole industry. Hence, SMP exceeds PMP benefit whereas, PMP costs exceed SMP costs.
- Interdependence of utilities enjoyed by different individuals: An increase in consumption of a good or service that has favorable effect on consumption of other consumers causes economies of consumption. For e.g. TV installation (leading to neighbors enjoying TV shows). This leads to PMP costs exceeding SMP costs but SMP benefits exceeding PMP benefits.
- Diseconomies of production: For e.g. a factory emulated smoke and causes health hazards and air pollution in its process of production. In this case, SMP costs exceed PMP costs and SMP benefits are lower than PMP benefits. External diseconomies of consumption also produce the same effect.

To bring equality between PMP and SMP Pigou suggested state intervention. When value of SMP is lower than PMP, social control is necessary. For e.g. the factory emulating smoke can be transferred outside the residential area. Sometimes, nationalization may become necessary in case of oligopoly or imperfect competition. Taxes can be imposed in case of external diseconomies and incase of external economies subsidies can be

Provided. Transfer of resources to a more fruitful use becomes necessary when PMP is less in one use as compared to other ones.

Critics have pointed out following deficiencies in Pigou's arguments. It is based on the unrealistic cardinal measurement of utility. National income cannot be accepted as a reliable index of welfare. There is a need to look beyond GNP/GDP. The hypothesis of man's equal capacity for satisfaction is more ethical than scientific.

14.3 PARETIAN WELFARE ECONOMICS

Vilfred Pareto an Italian economist of great repute, provided us with an alternative theory of welfare analysis.Pareto provided us with a positive criterion for increase or decrease in economic welfare. This was indeed a land mark in the history of welfare economics. Pareto refused to believe that utility was cardinally measurable units nor did he admit the possibility of making inter-personal comparisons of utility. Paretian welfare economics thus represented a complete break with the past. It is therefore called 'New welfare economics'. In recent years Pareto's analysis has been extended, modified, refined and polished by several leading economists like J.R.Hicks, Nicholas Kaldor,and Tibor Scitovosky.

14.3.1 PARETO'S SOCIAL OPTIMUM

Paretian social optimum refers to that situation where no one in society can move into a position that he prefers without causing proposed their compensation criterion in separate articles in 1939someone else to move into a position which that person prefers less. It is defined as that position from which it is not possible by any reallocation of resources to make any one better off without making some one worse off. The social optimum is a position "where it is impossible to make a small change of any sort such that ophelimitics of all the individuals, except that remain constant are either all increased or all diminished"

In order to attain Pareto optimality different marginal conditions have been suggested:The basic marginal conditions of Pareto optimality may be summarised as follows

1) Marinal condition for exchange optimality
   MRSAx,y=MRSBx,y

2) Production efficiency. Marginal condition for Production optimality is

MRTSx,y=MRTYXl,k=MRTYl.k.It means that marinal rate of technical substitution(MRTS) between any pair of factors must be equal for all commodities and all firms.

Marginal Condition for general optimality.

MRTSx,y=MRSAx,y=MRSBx,y

It implies that marinal rate of technical substitution(MRTS) between any pair of goods mut be equal to the marinal rte of substitution between them for any pair of consumers.

3) Efficiency in Product mix.
4) Optimum degree of specialization
5) Optimum factor-product relationship

14.4 KALDOR-HICKS COMPENSATION CRITERIA

On account of the several strictures passed against both the Pigouvian and Paretian criteria, attempts have been made by economists to reconstruct welfare theory on a scientific basis. Pareto provided us with a value free criterion.Pareto skillfully avoided value judgments and inter-personal utility comparisons.Kaldor and J.R.Hicks have tried to construct a new criterion of welfare on the foundation provided by Pareto,by introducing the principle of compensation

Kaldor and Hicks proposed their compensation criteria independently in 1939. Eventhogh their criteria are very much alike there is a minor difference between the two. Kaldor evaluates compensation from gainers' point of view , while Hicks does it from losers' point of view According Kaldor criteria if an economic change makes some people gain and some others lose, and gainers are able to compensate losers and yet for the change better off than they were originally, then the economic change results in increase in social welfare. In Hick's view if an economic change makes some people gain and some others lose, and losers are not able to compensate the gainers and to prevent them from voting for the change, then the change is socially desirable. The Kaldor-Hicks Compensation criteria may be stated as follows. If gainers of any economic change evaluate their gains at G and losers evaluate their losses at L, and if G>L, then gainers would be able to compensate the losers and yet retain a net gain.

14.5 SCITOVSKY'S COMPENSATION TEST

Tibor Scitovsky pointed out a contradiction in Kaldor-Hicks criterion. Scitovsky says, it is possible that state Y is better than state X in terms of Kaldor-Hicks criterion, as such the society moves into state Y. But once the society moves into Y, according to Scitovsky the same Kaldor_Hicks criterion may support the return of the society from state Y to state X on grouns of welfare. This contradiction is called scitovsky pradox.. According to Scitovsky a particular policy change will be desirable for society if the gainers are able to overcompensate the losers so that the latter do not prefer to o back to their earlier position. The gainers should not only able to bribe the losers into accepting the change, but the losers on their part should not be able to bribe the gainers into going back to the old state.

The main criticism against the compensation criteria is that

14.6 THE THEORY OF SECOND BEST

All the conditions of Paretian equilibrium are satisfied only in a perfectly competitive system with no externalities. In the real world, however no economic system or any sector thereof satisfies the marginal conditions necessary for the realization of the Paretian equilibrium. If perfect competition in the traditional sense is not possible some economists have propounded the theory of the second best. If perfect competition is ruled out the goal of economic policy should be' as close to perfect competition as possible'. Such a policy is being pursued in several capitalist countries.

The ideas underlying the theory of second best was recognized and discussed by different scholars in different contexts. It was generalized by Lipsey and Lancaster. According to the theory of second best the first best solution to welfare maximization is obtained when all the conditions of Pareto optimality are simultaneously satisfied. If any of the marginal conditions is not satisfied somewhere in the economy the first best solution (pareto optimality) cannot be obtained. Due to institutional constraints like monopolies and imperfect market conditions, externalities and indivisibilities, one or more of the first order conditions may not be satisfied. For a long time economists believed that the greater the number of marinal conditions satisfied, the close would be the solution to pareto optimum.. Lipsey and Lancaster later devastated this belief. They suggested that if one of the Paretian optimum conditions cannot be achieved, then an optimum situation can be achieved only by departing from all the other Paretian conditions. According to them if any of the Pareto optimaility conditions is not satisfied, the attempt to reach Pareto optimum- the first solution- must be abandoned and the second best solution should not be attempted. Henderson and Quandt says that “A best welfare position is unattainable….It is relevant to enquire whether a second best can be attained by satisfying the remaining Paretian conditions. In their theory of second best

,Lopsey and Lancaster have proved that if the first solution is not realized, then there is nothing to choose between the second best and so on. This is contrary to to the earlier belief that if the best is unattainable, then second best may be attained even if Pareto optimailit conditions are not satisfied.

14.7 EXTERNALITIES

The foregoing conclusion that perfect competition leads to Pareto optimality is based upon the assumption that there are no external effects in consumption and production. The assumption implies that 1) production of each producer is independent of others; and 2) utility function of each individual will affect the activities of others. Such effects are known as external effects or externalities. If externalities are present Pareto optimality may not be attained even under conditions of perfect competition.

The term externalities refer to external economies and diseconomies. External economies are the gains that arise from the activities of an economic unit and accrue to other members of the society by the activities of an economic unit for which market system not provide compensation to those who suffer. External economies and diseconomies arise in both production and consumption. It is important to analyze the effects of external economies and diseconomies in production and consumption on welfare maximization. To understand the effects of external economies in production consider the following examples. Construction of roads and railways reduces the cost of transportation and the advantages accrue to the industrial units which do not bear the cost of road and railway constriction. Afforestation schemes increase the rainfalls and oxygen gas in the air reduces air pollution and maintains ecological balance, which benefits the citizens in eneral and farmers in particular. But none of these bears the cost of production of afforesation activity.

The price system works efficiently because price conveys information to both producers and consumers. However, sometimes market prices may not reflect the activities of both producers and consumers. There is an externality when a production or consumption activity has an indirect effect on other consumption and production activities that is not reflected directly in market prices. It is called external as the effects on others are external to the market. For instance if a steel plant dumps effluent in a river there is an externality because the steel production does not bear the true cost of waste. If the cost of production reflected the effluent cost the price of steel will differ.

14.7.1 EXTERNALITIES IN CONSUMPTION

The assumption that utility level of one consumer is independent of the consumption pattern of the other is a far cry from reality. The 'bandwagon effects' show that changes in fashion crates strong imitation patterns. The demonstration effect developed by James Dusenburry shows that the tendency 'to keep up with the Joneses' creates conspicuous consumption patterns. Snob appeal/behaviour is a common tendency among consumers and the utility of a snob is greatly influenced by the purchases of other people. The change of cars frequently by some consumers decreases the utility of others who cant afford it (Koutsoyiannis, 1979).If externalities in consumption exist, thev equalization of marginal rate of substitution of commodities among consumers does not lead to Pareto optimality.

14.7.2 EXTERNALITIES IN PRODUCTION

Allocation of resources to production is not socially optimal when there exist a divergence between private and social cost. There are several types of external costs to the producing firms.(K 542 to bein)

14.8 PUBLIC GOODS :

Another source of market failure arises from the public good which is good that can be made available cheaply to many consumers. But once it is provided to some consumers, it is very difficult to prevent others from consuming it. Public goods are characterized by jointness in consumption and the 'exclusion principle does not apply. The defining characteristic of a public good is that it is non-rival and non-exclusive: consumption of it by one individual does not actually or potentially reduce the amount available to be consumed by another individual. Examples include radio and Television broadcast and national defense. Any individual can listen to or watch the output of broadcasting station, without preventing any other individual who possesses a radio or television receiver from consuming the same output. Any individual an increase his consumption of television broadcast up to the total number of hours broadcast without reducing any other individual's actual or potential consumption. Defense is a non-optional public good in that all inhabitants of the country consume the total quantity provided, and if one inhabitant is to be defended, all will be. On the contrary, private goods are the goods which have the characteristic that with a given output, an increase in one individual's consumption of a private good reduces the amount available for consumption by other individual. Many public goods are non-excludable. For example defense and police services. If it is impossible to exclude nonpayer from consuming a public good firms may not be able to collect revenue to cover the cost of producing the public good. Market may fail to provide an efficient amount of public goods even then they are excludable, arising from another characteristic of public goods: non-rivalry. Since the good is a public good an additional unit consumed by one individual does not reduce the amount available for consumption by any other individual. This means that no consumer is competing against any other consumer for a particular unit. The consumer, thus, the market is not competitive despite the large number of buyers and sellers. With public goods, the preference of free riders makes it really difficult or even impossible- for markets to provide goods efficiently.

14.9 EQUITY EFFICIENCY TRADE-OFF :-

The equity aspects of policies have always to be considered as well as their efficiency aspects. It is therefore important to focus on the that influence the distribution of income. The question what can we say about the desirable properties of a social welfare function such as S(Ua,Ub) is a relevant one. It is difficult to assume that n individual has a preference ordering over all possible. The utility function is purely ordinal and simply provides a numerical representation of the preference ordering. We could hardly expect to say which situation is actually preferable unless we could in some way compare the needs of two individuala.For example, if a cake were to be divided between A and B, A might prefer the division whereby A gets three- quarters of the cake to the division whereby A only gets one-half;, but B might have the opposite ordering. According to arrow's impossibility theorem to make ethical judgments we need more information. Our welfare function must contain independent variables that are comparable. These variables must enter into the function in a way that is symmetrical.

14.9.1 COMPENSATING VARIATION

Compensating variation and compensated demand curves are directly linked. Suppose price rise from pi0 to pii, other price remaining the same. How much compensation is needed to make the consumers as well as before. Obviously an amount equal to the change in the cost of securing u0. This is the natural measure of welfare change, except that since welfare has decreased we naturally measure welfare change by the negative of this cost difference. This measure is known as the compensating variation (Walters and Layrd,1987).

14.9.2 COMPENSATION CRITERIA

According to the Paretian criterion of social welfare, social welfare increases if any reallocation of resources makes at least one person better off without making any other individual worse off. In reality most economic changes make some people better –off at the cost of some others. Paretian criteria does not evaluate such economic changes. Some economists, viz Kaldor, Hicks, and Scitovsky have however devised compensation criteria that attempts to overcome the limitations of the Paretian criteria of maximum social welfare. This has come to be labeled as the new welfare Economics'.

14.9.3 CONCEPT OF COMPENSATING VARIATION.

What is compensation variation? The Compensating variation(CV) is the amount of money we can take away from an individual after an economic change, while leaving him as well as he was before it. For a welfare gain, it is the amount he would be willing to pay for the change. For a welfare loss, it is the amount he would need to receive as compensation for the change.

According to Paretian criterion social welfare increases if any reallocation of resources makes atleast one individual better off without making any individual worse off. It is however difficult to imagine an economic change that does not affect any individual adversely. I reality most economic changes make some people better off at the cost of some others. Pareto criterion does not evaluate such economic changes. Some economists like Kaldor, Hicks and Scitovsky have however devised compensation criteria that attempt to overcome the limitations of the Paretian criterion of maximum social welfare. This has come to be called the NEW Welfare economics.

14.9.4 THE KALDOR-HICKS COMPENSATION CRITERION

Though they the came up with their criteria separately in 1939 their criteria is jointly referred to as Kaldor Hicks criteria. The main point of difference between their criteria is that Kaldor evaluates compensation from gainers' point of view while Hicks does it from losers' point of view/angle. According to Kaldor, if an economic change makes some people gain and some others are able to compensate the losers and yet are better off than they were originally, then the change increases social welfare. According to Hicks.

The main problem with Kaldor-Hicks criteria is that it refers to only potential rather than actual compensation. Moreover it uses money value of gains and losses in evaluating economic efficiency of a change.

14.10 EXTERNALITIES AND THE DIVERGENCE BETWEEN PRIVATE AND SOCIAL COSTS:-

The external economies in production create a divergence between private and social gains. The divergence between the private and social costs results in non-optimisation of production. Under perfect competition a firm produces a commodity, say x is in equilibrium when its

MCx=Px

14.11 MEASUREMENT OF WELFARE :-

14.11.1 CONSUMER SURPLUS

Consumer Surplus is the difference between the price you are willing to pay and you actually pay. Samuelson calls it a 'conjunctural benefit'. This is especially important in the system of welfare economics and public policy. Consumer surplus is a utility surplus, which Marshall tries to measure with money. To prove the existence of consumer surplus, Marshall makes certain assumptions.

Assumptions

1. Ceteris Paribus (. i.e. fashions, tastes, styles, incomes, etc… all remain constant)
2. The marginal utility of money to individuals is the same.
3. Commodity in question has no substitute

Measurement of consumer surplus has been derived from the measure law of diminishing marginal utility. The consumer is in equilibrium when

MUx = Px equilibrium has been attained. In the table that follows consumer is in equilibrium when he purchases 6 units

Marshall constructed a table (below), with the number of units consumed, the marginal utility, i.e. the utility achieved from the consumption of each extra unit, the price of each extra unit and the consumer surplus. In the table, we can see that with each additional unit the marginal utility declines proportionately. It is in this regard, since the C.S. is nothing but the

MU–P/unit that we see this too declines proportionately.

Units	Marginal Utility	Price	Consumer Surplus
1	20	10	10
2	18	10	8
3	16	10	6
4	14	10	4
5	12	10	2
6	10	10	0

Therefore the 6^{th} unit of the commodity, the P = CS and the CS is zero.

CS $= \sum MU - \sum (Px.Nx)$

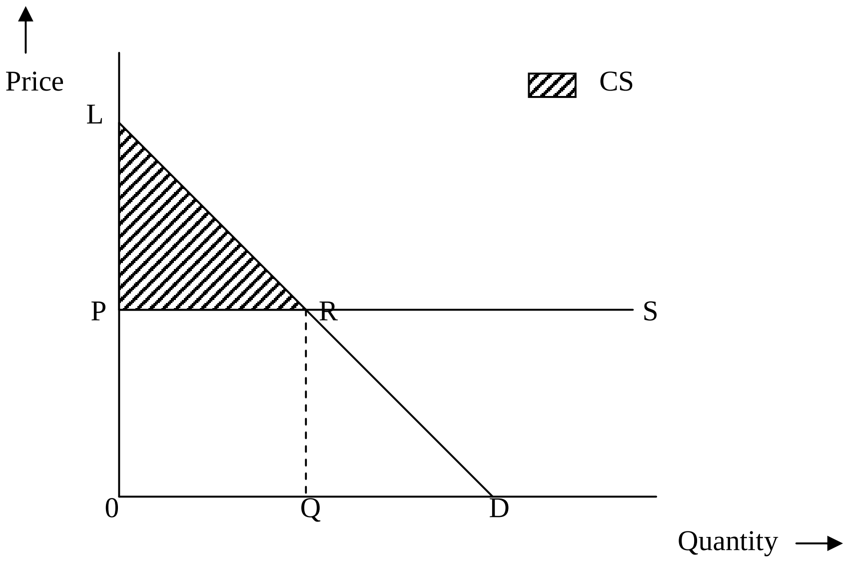
$= 90 – (10*6)$

$= 90 – 60$

$= 30$

Marshall also uses a;

Diagram to represent consumer surplus



PS indicates the real price, OQ is the number of units purchased and OQRL is the amount of money the consumer is prepared to spend to secure OQ units of the commodity. Lastly, OQRP is the actual amount of money spent by the consumer to buy the good.

Therefore, the shaded area – LRP is the consumer surplus

14.11.2 CRITICISM

The concept is not valid as the assumptions are arbitrary and does not hold good in practice. The critics condemn the assumption of constant marginal utility of money. Marshall further ignored the inter-dependence between goods. i.e the case of substitutes and complements. The magnitude of consumers' surplus depends on the availability of substitutes. Marshall was aware of this and suggested that substitute products like tea and coffee may be clubbed together as a single commodity. The concept is based on cardinal measurement of utility. Utility is a psychological experience and cannot be measured in cardinal terms. It cannot be easily measured as the area under the demand curve. According to Samulson consumers surplus is a 'conjunctural benefit'. All of us reap the benefit of an economic world we never made' (samuelson).

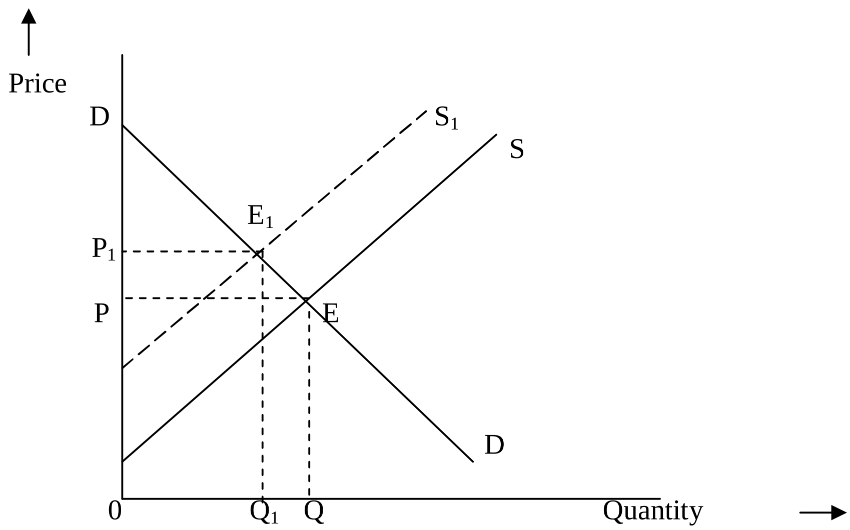
The concept of consumer's surplus is hypothetical and imaginary. In case of commodities like water and air, measurement of consumer's surplus is not possible. In the case of luxuries utility is derived from the prevalence of a high price.

At the same time, there is a relation between substitutes and complimentary goods, which cannot be ignored.

Nonetheless – it is an important tool in the measurement of welfare and utility and is used to measure how much is foregone in externalities as well.

Consumer Surplus also helps us look at the utility that is absorbed with a tax placed by the government. In recent times, these estimates of consumer surplus have been treated as important tools in cost-benefit analysis.

<u>Diagram showing the fall in consumer surplus with the addition of tax</u>



Under increasing costs, S → S_1 shifts by the extent of the tax. After the addition of the tax, the price increases from P to P_1, at the same time, the quantity consumed reduces from Q to Q_1. The consumer surplus originally was EPD and this reduces to E_1P_1D. In this case, gain in government revenue greater than loss in consumer surplus. Therefore increasing cost industry must be taxed. In the case of constant cost industry loss in consumers' surplus is greater than government revenue and therefore tax on constant cost industry is not justified and in the case of decreasing costs, loss in consumer surplus is greater than gain in government revenue and a tax would not be justified. Recently the gains in consumers' surplus have been treated as benefits in cost benefit analysis.

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14.12 SUMMARY

Welfare economics, thus, aims at evaluating the social desirability of alternative social states. In short, Welfare Economics attempts to lay down propositions by which one can rank, on the scale of better or worse, alternative economic situations open to society.

14.13 QUESTIONS

1. State and explain the Fundamental Theorems of Welfare Economics.
2. Describe the Paretian Welfare Economics.
3. Define and explain Kaldor – Hicks compensation criterion.
4. What is the Theory of Second Best?
5. Write short notes on:
    a. Public goods
    b. Externalities

❖❖❖❖

15

TOPICS IN APPLIED MICROECONOMICS

UNIT STRUCTURE

15.0 OBJECTIVES

After going through this unit, you will be able to Understand :

- The labour – leisure choice model
- The model of fertility analysis
- The modal analysing the education
- Estimation of production function in the context of Indian industries
- The problem of Commons

15.1 INTRODUCTION

Till the moment we have made theoretical analysis of various microeconomic model. In the present unit we will try to make the use of such models. In other words, we will study some topics in applied microeconomics. Some of the topics are Labour – Leisure Choice, Fertility Analysis etc.

15.2 THE LABOUR – LEISURE CHOICE :

The decision as to how many hours out of the day to devote to work is an important choice made by individuals. We model this choice by assuming that consumers desire leisure, L, as well as the consumption of goods individually. We simplify the model by asserting that, utility is a function of income, Y, and leisure :

$$U = U(Y, L).$$

Income is produced by working (24 – L) hours at wage W per hour. In addition, nonwage income Y^0 occurs individual. Nonwage income can be negative, as in the case of contractual debt obligations. The utility maximum problem is therefore,

Maximize	$U = U(Y, L)$
Subject to	$Y = W(24 - L) + Y^0$

This situation is pictured fig 6.1. The individual is endowed with 24 hours of leisure and nonwage income, assumed positive of Y^0. The budget line passes through the paint, $(24, Y^0)$, and slope – W. The consumer maximizes utility at some point, A, Where the indifference curves are tangent to the budget line. An increase in W is represented by rating the budget line clockwise through the endowment paint, resulting in a new maximum position. B, on a higher indifference curve.

The Lagrangian for this model is

$$=U(Y, L) + \lambda(Y^0 - Y + W(24 - L) \text{ ------------------------ } (1)$$

The first – order conditions are

$$U_Y - \lambda = 0 \text{ -- } (2)$$

$$U_L - \lambda_W = 0 \text{ --- } (3)$$

and the constraint

$$Y^0 - Y + W(24 - L) = 0 \text{ ---------------------------------- } (4)$$

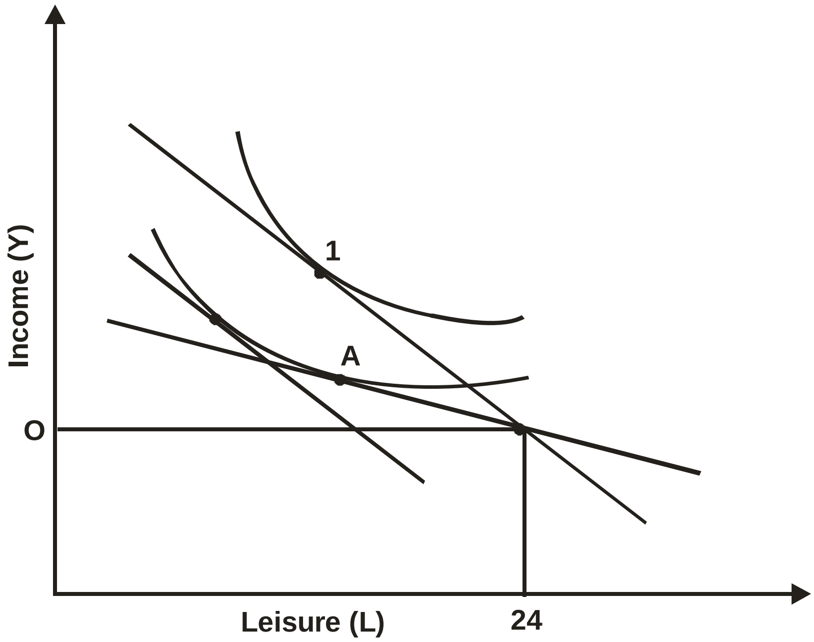

Fig. 15.1 : The Labour – Leisure Choice

A consumer is endowed with 24 hours of leisure and nonwage income Y^0. At some wage rate W, the utility maximum occurs at point A. An increase in W produces a pure substitution effect from A to C and an income effect from C to B. Assuming leisure is a normal good, the income effect acts in the opposite direction of the substitution effect; since the consumer sells leisure.

From 2 and 3, $U_L / U_Y = W$. This says that the marginal value of leisure, in terms of income forgone, is the wage rate. If a person can choose how many hours to work, then the decision not to work an additional hours entails giving up an hour's income, W.

Assuming the sufficient second – order conditions hold, the Marshallian demand functions.

$$L = L^M(W, Y^0) \text{ -- } (5)$$

$$Y = Y^M(W, Y^0) \text{ --- } (6)$$

& an expression for the Lagrange multiplier.

$$\lambda = \lambda^M(W, Y^0) \text{ -- } (7)$$

are implied. We can interpret λM as the marginal utility of nonwage income.

What is the effect on L and Y of an increase in the wage rate W? We already know that mathematically, no refutable implication is available. An increase in the wage rate raises the opportunity cost of leisure; We should expect on this amount the individual to substitute away from leisure, i.e. toward more work. However, this is just the pure substitution effect; As the wage rate increases, income also increases. If leisure is a normal good, we should expect the person to consume more leisure, i.e. to work less. Let us derive the associated Slutsky equation.

The Hicksian, or utility – held – constant demand, functions for this model are derived from the expenditure minimization problem,

Minimize $\quad Y^0 = Y - W(24 - L)$ ----------------------- (8)

Subject to $\quad U(Y, L) = U^0$ -------------------------------- (9)

In this model, Y^0 is no longer a parameter, it is the value of the objective function. The utility level is now a parameter. The Lagrangian for this model is

$$= Y - W(24 - L) + \lambda\left(U^0 - U(Y, L)\right)$$

Assuming the first and second – order conditions hold, the Hicksian demand functions.

$$Y = Y^U(W, U^0) \text{ --- (10)}$$

$$L = L^U(W, U^0) \text{ --- (11)}$$

are implied. The associated expenditure function is derived by substituting these solutions in to the objective function :

$$Y^*(W, U^0) = Y^U(W, U^0) - W\,[24 - L^U(W, U^0)] \text{ --- (12)}$$

The Hicksian & Marshallian demand functions for leisure are related to each other the fundamental identity.

$$L^U(W, U^0) \equiv L^M\left(W, Y^*(W, U^0)\right) \text{ ---------------------- (13)}$$

Differentiating both sides with respect to W,

$$\frac{\partial L^U}{\partial W} \equiv \frac{\partial L^M}{\partial W} + \left(\frac{\partial L^M}{\partial Y^0}\right)\left(\frac{\partial Y^*}{\partial W}\right) \text{ --------------------------- (14)}$$

Applying the envelope theorem to Eq.12

$\partial Y^* / \partial W = -(24 - L^U)$. Thus, rearranging (10 – 70), slightly.

$$\frac{\partial L^M}{\partial W} \equiv \frac{\partial L^U}{\partial W} + (24 - L^U)\left(\frac{\partial L^M}{\partial Y^0}\right) \text{ ---------------------- (15)}$$

An equation analogous to the traditional slutsky eq.

Notice in this case, however, the term multiplying the income effect is the amount of leisure "Sold", $24 - L^U$, ???? the amount of same good purchased. When the consumer comes to market with money income, which does not enter the utility function, the income effect for normal (noninferior) goods reinforces the substitution effect. In this case, since the consumer is selling leisure, not buying it, the income effect acts in the opposite direction of the substitution effect, for normal goods. There is ample evidence that leisure is a normal good. (How does winning one of the various state lotteries how in existence effect the winner's time spent wasting?) Since $\left(24 - L^U\right)$ is positive, the income effect is positive, while the pure substitution effect $\partial L^U / \partial W$ is necessarily negative. Because of this, the slope of the Marshallian (uncompensated) demand for leisure, $\partial L^U / \partial W$ is less predictable than the slope of the Marshallian demands for ordinary goods and services.

A recurring public policy question concerns the effects of tax rates on work effort. The 1986 U.S. tax changes lowered the marginal rates on federal income taxation to 28 to 33 %, from 50%. Some countries have tax rates in excess of 90%. It can be sun from the above analysis that lowering tax rates, which effectively raises the after tax wage rate, does not have an implied effect on hours wasted. Since the opportunity cost of leisure is now higher, the substitution effect produces less leisure. However, the individual is also wealthier, the income effect leads therefore to more leisure. The net effect is an empirical matter. (of course, at a tax rate of 100%, no effort will be forth – coming (legally); the income effect of lowering taxes at that margin will – certainly dominate, and induce greater effort).

The preceding model of labour – leisure choice is a special case of a model that appears in the literature on general equilibrium. Assume that, instead of the consumer bringing an amount of money income M to the market to purchase goods & services, the consumer comes to market with initial endowments at n + 1 goods $x_0^0, x_1^0, \ldots.., x_n^0$. The market sets prices of $P_0, P_1, \ldots., P_n$

for there goods and the consumer maximizes utility subject to the constraint that the value of the goods purchased equal the value of the initial endowment i.e. _______ .

$$\text{Maximize} \quad U(x_0, x_1, \ldots, x_n) \text{ -------------------------- } (16)$$

$$\text{Subject to } P_0\, x_0^0, \ldots.., P_n\, x_n^0 = P_0\, x_0 + \ldots.. + P_n\, x_n \text{ ----- } (17)$$

That is subject to

$$\sum_{i=0}^{n} P_i\, x_i^0 = \sum_{i=0}^{n} P_i\, x_i \text{ --- } (18)$$

The first – order conditions are obtained by setting the partials of the Lagrangian equal to 0 :

$$= U(x_0, \ldots.., x_n) + \lambda\left(\sum P_i x_i^0 - \sum P_i x_i\right) \text{ ---------------- } (19)$$

$$= U_0 - \lambda P_0 = 0 \text{ --- } (20)$$

$$= U_1 - \lambda P_1 = 0 \text{ -- } (21)$$

$$= U_n - \lambda P_n = 0 \text{ --- } (22)$$

$$\lambda = \sum P_i^0 x_i^0 - \sum P_i x_i = 0 \text{ --------------------------------- } (23)$$

15.3 FERTILITY ANALYSIS :

Gray S. Becker's most important foray into sociological country was to be a paper on the economics of fertility written for the National Bureau of Economic Research (1960b). Although political economy was once closely involved with demography (witness Malthus's famous essay), for much of this century the study of population was firmly in the hands of sociologists & untheoretical number – crunchers. A few tentative attempts had been made to relate birth rates to economic variables; but Becker's paper went way beyond this. Here the decisions to have childrens is firmly incorporated with in the familiar framework of neoclassical economics. More particularly, Becker adopts the startling & controversial position that children are in important respects analogous to consumer durables such as automobiles, TV sets & dishwashers, thus the economic theory which has proved fruitful in relation to there commodities, can be applied equally to human beings.

He argues that, at least under modern conditions, the raising of children to have children, despite the availability of effective contraception. Thus, if people choose to have children for the costs involved. These costs include such obvious things as food, clothing & schooling. Perhaps more importantly, however, they also include costs in firms of parental time, a scarce commodity which has alternative uses. Indeed, if one alternative is to use this time in the labour market, a value (its opportunity cost, in the jargon) can be put on it which will indicate that a very large proportion of the total costs of childrearing is amounted for by parental time.

The existence of these net costs indicates that children are some form of consumer goods, their spread over time indicates we are dealing with a consumer durable. They therefore have to compete with other consumer durables for a limited share of the compete with other consumer durables for a limited share of the household budget, more children means less – hi – fi equipment or a smaller car.

Once this rather bizarre comparison is admitted, it opens up the likelihood that decisions to have children will be affected by such variables as their '<u>price</u>' (in terms of alternatives foregone) and the size of the household budget. As we have indicated, Becker accepts Friedman's view that the usefulness of a hypothesis depends on it's ability to explain or predict. So how does Becker's approach fare in this respect?

Straightaway we are confronted with a problem. Broadly speaking, the demand for consumer durables tends to rise with income, on Becker's reasoning we might expect the demand for children to fallow a similar pattern. Yet there is much evidence to suggest that family size declines with income. How does Becker handle this apparent refutation of his approach? Are babies inferior goods ?!

One argument Becker offers in order to resolve this difficulty is interesting in the light of his later work. This is the argument that the cost at rearing children tends to rise – with family income, largely as a result of the higher opportunity cost of parental time. At any particular moment better – off families tend to be better educated & thus to have greater earning power, over time, all earnings tend to rise as income rises. The argument can be illustrated diagrammatically. In figure 6.2, an increase in income illustrated by a parallel outward shift of the budget constraint leads to increases in the 'consumption' of both competing consumer durables & babies, if the relative price of these commodities remains constant. At the point of tangency between a new (higher) in difference curve & the new budget constraint, more babies $\left(B_2\right)$ are chosen. If, however, the increased income results largely from higher wages paid to family members (a highly plausible assumption), this will raise the opportunity cost of time spent on rearing children, & thus increase their relative price. The budget constraint pivots, as in fig no. 15.2, & the new preferred combination of babies & other consumer durables may involve a smaller desired family size.

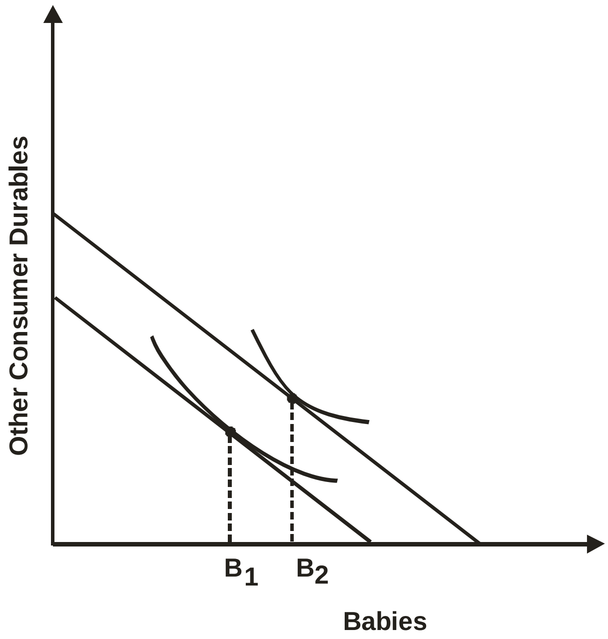

Fig. 15.2

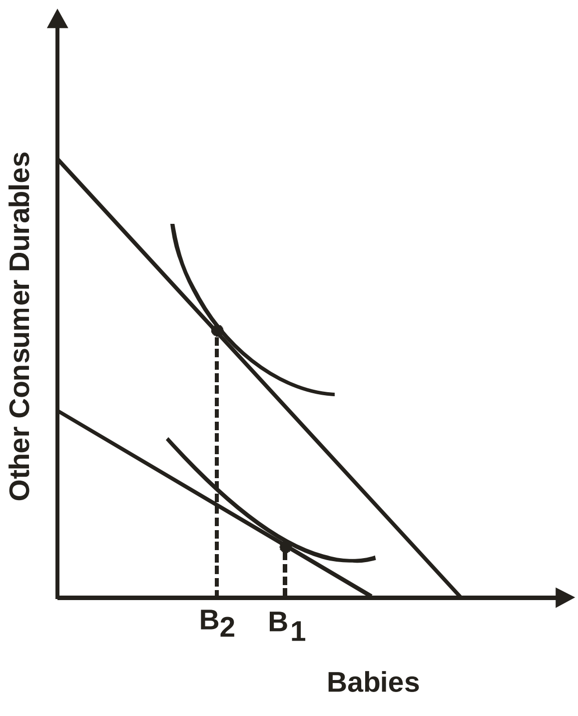

Fig. 15.3

It is ingenious, if not altogether convincing. There is a suspicion that evidence – Becker uses to support his arguments is highly selective, and moreover some of the generalizations he makes are amenable to alternative interpretations : for instance the observed inverse relation between education and family size could have nothing to do with the opportunity cost of parental time, but a lot to do with the different values & altitudes education might be expected to inculcate. However, Becker's approach is more plausible in relation to short – term variations in – fertility, Economic factors seems for more significant here than ad – hoc empirical generalizations are linked to a broader theoretical framework; this is why, like it or not, it has stimulated so much further work in this field.

15.4 ANALYSIS OF EDUCATION : HUMAN CAPITAL :

Although economists ever since Adam Smith have recognized that expenditure an education or training can be considered as an investment, this insight was not systematically used to explain labour – market behaviour until the early 1960s. Prior to this it was often assumed that individuals lacked the information or the foresight to make rational investment decisions in this area. A corollary of this seemed to be that governments should subsidise education.

A number of econometric studies in the 1950s suggested that education was an important element in the explanation of a country's growth performance, in addition the US Government became increasingly worried that its educational system might be lagging behind that of the USSR. As a consequence a good deal of attention was directed towards the analysis at the Economics of education. Already Becker had prepared a discussion paper (1960) which expressed scepticism of the view that there was a shortage of American college places. At the time, however, the theoretical framework for such a discussions was sketchy. It was not until Becker, Theodore Schultz, Jacob Mincer and others had contributed to a special issue of the Journal of political Economy on Investment in Human beings, in 1962, that the debate really took off.

Becker's contribution to this symposium was subsequently expanded into a book, Human Capital (1964, 1975), which was soon recognized as a classic & which made his reputation. In this book Becker's starting paint is the assumption that people spend on themselves or their children not just for present gratification, but also with the future in mind. Future gratification may be of a monetary or non-monetary kind, through Becker concentrates on the farmer. Further – oriented expenditures will normally only be – undertaken, he argues, if the present value of expected benefits (discounted by an interest rate reflecting the opportunity cost of capital) at least equals the present value of the cost of the expenditure. Such expenditure includes over casts such as fees & equipment, but a major element, he argues, is the value of earnings of foregone during the period of training.

This category of investment includes for more than the formal systems of education & training which the debate was centred on. In Becker's hands, the concept of human capital embraces such activities as the purchase of health care, time spent searching for the best pay offer rather than taking the first available job, migration, & the acceptance of low – paying jobs which have a large element of learning on the job. In Becker's model, in the long run, all such human capital formation is taken to the paint where the marginal returns to such activities are equal to the marginal cost of investment funds. In other words, in equilibrium (always Becker's concern) the rate of return on all investment activities – human & non – human – is equalized from this Becker dedness propositions which shed new light on a great many economic activities. Patterns of income distribution, the shape of age – earnings profiles, the duration of unemployment & the existence of male – female educational inequalities are all examples of issues which the approach illuminates.

To some writers, human capital theory also suggests a guide to policy, if the marginal rate of return on some form of training exceeds the cost of capita, for instance, this may be taken to provide a justification for the state to expand the provision of the training. Becker, however, has doubts about this, he paints out that such variations may simply reflect underlying variations in the non-monetary benefits and costs of particular activities.

Despite the acclaim which greeted Human Capital, Becker's work in this field is by no means universally accepted, Blang (1975), for instance, has attacked the weak empirical basis of much human capital theorizing. Studies indicate major variations in rate of return on different kinds of human capital, he claims & attacks Becker's attempts to explain this away by reference to auxiliary hypotheses about non – monetary factors as being the same kind of ad-hocery Becker deplores in others. We are again struck by the curious way in which Becker tends to use evidence to support or illustrated a hypothesis rather than genuinely to test it.

Another criticism relates to possible alternative explanations of some of Becker's empirical generalizations. For example critics have argued that the positive correlation between the married female participation rate and the level of education achieved can be attributed as much to attitude changes as to Bucker's explanation in terms of the high market value of educated women's time. Again, however, as in the case of fertility which we discussed in last section, it is doubtful whether such alternative explanations are as generally applicable.

Another whale set of criticisms which deserves more space than we can afford it here argues that Becker asks essentially the wrong questions. Educational systems are seen as devices for reinforcing social control. Thus market critics emphasize in particular the supposed structural necessity to maintain the divisions inherent in a complex society. Capitalism creates &maintains educational systems which fulfill its objectives. Becker's individualistic account of human capital investment cannot hope to deal with this. Criticisms on these lines are part of a wider critique of microeconomics.

15.5 ESTIMATING THE PRODUCTION FUNCTION IN THE INDIAN CONTEXT

The application of Econometric techniques to the study of Indian industries can be treated to the 1950's when Bhatia (1954), Dutt (1955) & Murthy & Sastry (1957) estimated production functions for Indian manufacturing. Since then there have been a large number of Econometric studies, there have been studies on various other economic aspects of Indian industries, including the cost, labour, demand & investment functions.

The object of this topic is not to provide a comprehensive survey of the vast econometric literature that has accumulated over the years. Rather the discussion is confined to certain selected topics. It discuss briefly certain methodologies that have been commonly adopted in the Econometric studies on India industries and by way of illustration, also present the results obtained by some of the studies.

15.5.1 Basic Data Sources :

In virtually all studies of production function estimation for Indian industries, the basic data have been drawn from the Census of Indian Manufactures (CMI) or the Annual Survey of Industries (ASI); CMI data are available from 1946 to 1958 & ASI data from 1959 onwards. Some studies have combined CMI & ASI data for estimation purposes. CMI & ASI have been the principal data source also for studies on the east & labour demand functions discussed later in this topic.

Both CMI & ASI relate to the organized sector of Indian manufacturing. The coverage of CMI is much narrower than that of ASI both in regard to the range of industries & the size of units covered. ASI covers units registered as factories. i.e. unite employing 10 or more workers without power are completely enumerated. They form the census sector of ASI. The remaining factories, constituting the sample sector of ASI, are covered on the basis of a probability sample.

The primary unit of enumeration in CMI & ASI is the factory & the published data are based on returns furnished by factories. Factories are classified into different industries according to there principal products (by value). This, sometimes, causes shifts of factories from one industrial class to another in successive surveys, there by affecting the compatiability of data over time. Inter – temporal comparability of ASI data is affected also by year-to-year variations in the response rate & therefore in coverage.

Changes in industrial classification have created difficulties in constructing comparable time – series at a disaggregated level. The classification used in ASI since 1973 is different from that used during 195 – 71, both of which are different from the classification used in CMI.

Since ASI & CMI provide aggregated data (at a different levels of aggregation). These are not useful for Econometric studies requiring data at the firm level. Therefore in such studies other sources of data have been used. Thus, studies on technical efficiency have utilized data for industrial enterprises gathered through surveys.

15.5.2 Measurement of Variables :

Various theoretical & empirical issue in the measurement of variables in the context of the production function & productivity studies for Indian industries have been discussed in Banerji (1975), Hashim & Dadi (1973), Dholkia (1977), Goldar (1986), & Ahluwalia (1991). This aspects needs therefore only to be dealt with briefly.

Studies on the production function & (productivity) have generally been undertaken in a two-input framework, taking value added as output & labour & capital as the two inputs. The inadequacies of the two input framework has been recognized in Econometric literature. It has been shown that the use of the value added form is justifiable only under very restrictive conditions (such as functional reparability), & if these are not satisfied then the use of the two input framework causes the estimates of parameters to be biased. It may be mentioned here that Williams & Laumnas (1981) have investigated this question for Indian industries & found that the functional reparability conditions necessary to justify due use of the value added form is not satisfied by the data.

Since the depreciation figures separated in CMI / ASI do not correctly represent the true capital consumption, gross value added has been generally preferred to the net value added as a measure of output. To correct for price changes, the yearly current values have been deflated (Single deflation) by price indices for manufactured products. Serious difficulties have often been faced in matching the industrial data with the price data & for this reason the deflators used are not wholly satisfactory. Also the price indices are based on product prices (inclusive or exclusive) & their use for deflating value added can be questioned.

Many studies have used the no. of employees as a measure of labour input. This obviously suffers from the limitation that variations in age, sex, education, skill & occupation composition of the labour force are not taken into account. Also, variations in hours of work are not reflected in such a measure of labour input, based on head count. Some studies (eg. Gupta 1989) have taken a weighted average of workers & non workers to construct a measure of labour input, using the wage rates of these two clauses as weights. This procedure can be criticized on the ground that due to imperfections in the Indian labour market differences in the rates of remuneration may not correctly reflect the differences in efficiency.

Though there are some paints of similarity, significant differences are found among studies in regard to the methodology adopted for the measurement of capital input. Wasking capital is generally excluded from the measure of capital input on the ground that the relationship between wasking capital & output is much less influenced by technological factors than the relationship between fixed capital & output. There are significant methodological differences among sudies regarding the way the reported figures on fixed capital are treated for obtaining the measure of capital input. It should be painted out these that – the published data on fixed capital (in CMI / ASI) are the book value of fixed assets at the end of the reference year, net of cumulative depreciation. The reported figures can capital grossly understate the market value at the capital stock because (1) the reported figures are at historical prices, & (2) the depreciation allowed by the income tax authorities are much higher than the true capital consumption. Some studies (eg. Narsimham & Febrycy – 1974) have used the published data directly for production function estimation without making any price correction. This is obviously inappropriate without making any price correction. This is obviously inappropriate for the study based on time series – data or pooled cross – section & time series data, some studies have directly deflated the fixed capital series obtained from CMI / ASI by a price index of capital assets. This type of blanket deflation is again not right because it ignores the fact that the capital stock figure reported for a particular year includes assets bought at different paints in time in the past. Most studies have used the prepetual inventory methods & thereby overcome this deficiency. In this method the annual investments are deflated rather than the stock existing at the end of the year the method is described below.

Let B_t denote the book – value of fixed assets at the end of year t. D_t the depreciation allowances made in that year (as given in CMI / ASI) & P_t the capital goods price index for that years, then the services on real (fixed) investment (I_t), may be derived as

$$I_t = (B_t - B_{t-1} + D_t)/P$$

Further, let K_0 be an estimate of the real capital stock for a benchmark year then the capital stock series, (K_t) may be derived using the following relationship.

$$K_t = K_{t-1} + I_t \qquad \text{or} \qquad K_T = K_0 + \sum I_t$$

Compared to the blanket deflation procedure which some studies have adopted the perpetual Inventory method of capital stock estimation is more sophisticated. But the application of this method in Indian studies has been deficient in some respects. Thus, in many of then studies, there are shortcomings in the way the benchmark estimate is obtained and or the deflator used for price correction. Also there is series for the discard of assets which very few studies have done.

It needs to be emphasized that proper measurement of output & inputs is very important for Econometric studies of producer behaviour yet, this aspect has generally not received, the attention if deserves adversely affecting the results. Among the studies that have exercised care in the measurement of output & inputs. It might be mention Ahluwalia (1991) & Goldar (1986).

15.5.3 Production function Studies :

The majority of studies an production estimation for Indian industries have used the Cobb-Douglas (CD) functional form some of the other functional forms used are the constant, Elasticity of Substitutions (CES), the Variable Elasticity of Substitutions (VES), & the Transcendental Logarithemic (Translog).

15.5.4 Cobb-Douglas Production Function :

The Cobb-Douglas (CD) production function has been widely used in empirical studies Allowing for technological change, the CD function for the two-input case, taking labour & capital as two inputs & value added as the input may be written as :

$$Y = A_0 e^{rt} L^{\alpha} K^{\beta}$$

Where Y is output (value added), L Labour, K Capital & t time. A is the efficiency parameter, α & β are the elasticities of a output with regard to labour & capital, respectively, & λ is the exponential rate of technological progress for the function to be well-behaved, α & β should lie between 0 and 1. The sum of α & β gives the returns to scale. If $\alpha + \beta$ is equal to one, there are constant returns to scale; $\alpha + \beta < 1$ implies decreasing returns to scale, while $\alpha + \beta > 1$ implies increasing returns to scale. The CD function implicitly assumes the elasticity of substitution between capital & labour to be unity.

The logarithmic transformation of the CD function yields an equation linear in parameters. Thus, taking logarithms & adding an error term u, the CD function may be written as :

$\text{In}\, Y = a + \alpha\, \text{In}\, L + \beta\, \text{In}\, K + \lambda t + u$ where $a = \text{In}\, A_0$. This may be estimated by the Ordinary Least Squares (OLS) technique.

In this context, it should be noted that the parameters of a production function can be estimated by fitting either the production function directly or the marginal productivity conditions. (derived under the assumption of Profit maximization). It has been shown that for an individual firm maximizing profits in a competitive market, estimation of the production function by the OLS technique would yield biased & inconsistent estimates because the disturbance term in the production function is not independent of the choice of inpuls. One way of overcoming this problem is to estimate the production function jointly with input demand functions, taking into account the joint distribution of the error terms.

Zellnea et al. (1966) have, however, established that under reasonable assumptions about the disturbance term & behavioural relations, the inpuls can be shown to be independent of the distribution erms of the production function, & hence the application of the OLS method for directly estimating the CD function will give consistent & unbiased estimates.

15.5.5 CD function : Two studies for Indian Industries :

The CD function has been estimated for Indian industries in a large number of earlier studies. I take up for discussion the estimates obtained. In two studies, namely, Ahluwalia (1991) & Goldar (1986). Goldar has estimated the CD formation using aggregate time series data & Ahluwalia both from aggregate time-series data & panel data. Her estimates based on panel data are taken up for discussion first.

Ahluwalia has used cross-section, time series panel data for 84 industries for the period 1959 – 60 to 1985 – 86. The ratio form of the CD function has been used for estimation purposes, which has the advantage that it provides a direct test of the hypothesis that the degree of homogeneity of the function (Returns to scale) is one.

In ratio form, the CD function may be written as :

$$\text{In}(Y/L) = \alpha + \beta\,\text{In}(K/L) + (\alpha + \beta - 1)\,\text{In}\,L + \lambda t + u$$

Since data for different industries are pooled to estimate the CD function, Ahluwalia introduces industry specific intercept dummies, allowing the intercept to differ among industries. Further, Ahlywalia defines a dummy variable D as taking value Zero up to 1982 – 83 & one thereafter, the product of this dummy – variable & the time variable is included in the regression equation to capture the change in rate of technological progress after 1982 – 83. One of the equations estimated by Ahluwalia is shown below (t – values in parentheses, intercept – dummies not shown).

$$\text{In}(Y/L) = -2.9 + \underset{(23.4)}{0.498}\ \text{In}(K/L) + \underset{(4.1)}{0.072}$$

$$\text{In}\,L - \underset{(-4.2)}{0.007}\,t + \underset{(8.6)}{0.007}\,D.t \quad (2.3.4)$$

n = 2240, R^2 = 0.87 period : 1959 – 60 to 1985 – 86. The model fits the data well as indicated by R^2. All the coefficients are statistically significant. The estimate of the capital coefficient is 0.498, while the estimate of the labour coefficient is 0.574. Both coefficients are between zero & one, as they should be. The estimated returns to scale parameter is 1.072. It is found to be significantly above one, indicating thereby the pressure of Economies of scale. The significant negative coefficient of time indicates a downward trend in total factor. Productivity, while a significant positive coefficient of the product of the dummy variable

& time indicates that there has been a reversal of this trend after 1982 – 83.

Ahluwalia's estimates of the CD function based on time-series data are shown in 6.1. Such estimates made by Goldar (1986) are also shown in the table. In both studies, the ratio form of the CD function has been used. It is seen from the table that the estimates of the returns to scale parameter are not significantly different from one, so that the hypothesis of Constant Return to scale is not rejected. Therefore, we may compare only the equations that have been estimated assuming the Returns to scale to constant. The estimates of β obtained in Ahluwalia's study are quite close to that in Goldar's In (range of 0.3 to 0.4). There, are however, marked differences in the estimates of λ, the rate of technological progress. In Goldar's study, it is found to be positive & statistically significant, while in Ahluwalia's it is very small & statistically insignificant.

Table 15.1

OLS estimates of the Cobb-Douglas production function for organised Indian Industry based on Time Series Data.

Author	**Period**	**Estimate of**		**λ**	**R^2**
		β	**α + β – 1**		
Ahluwalia (1991)	1960 – 82	0.321 (1.6)	- 0.315 (- 0.6)	0.013 (0.5)	0.90
	1960 – 82	0.387 (2.6)		- 0.001 (- 0.1)	0.91
	1960 – 79	0.281 (1.4)	- 0.332 (- 0.6)	0.016 (0.7)	0.90
	1960 – 79	0.357 (2.4)		0.002 (0.3)	0.91
Goldar (1986)	1659 – 79	0.281 (2.4)	0.313 (1.0)	0.007 (0.7)	0.95
	1959 – 79	0.314 (2.8)		0.017 (3.9)	0.95

Note-t-values in parentheses :

Same methodological comments on the CD function estimates presented in the two studies would be relevant. Ahluwalia's estimates of the CD function based on panel data involves a highly restrictive assumption that all the industries have an identical production function. She permits some flexibility in model specification by allowing the intercept to vary across industries, but the coefficients of labour & capital & the rate of technological progress are assumed to be the same for all industries. It needs to be noted. However as Ahluwalia paints out that this assumption is no different from what lies behind the estimation of the production function from aggregated time-series data. Another paints to be noted in this context is that the application of the OLS technique for estimating the CD function from panel data involves the assumption of homoscedasticity. This assumption may be questioned because the variance of the error term need not be the same for different industries as the member of firms vary from industry to industry. There may be other reasons for the error term to be heteroscedastic. This could have been tested & if the assumption of hemoscedastivity was not found to be instified, the Generalized Least Squares (GLS) method could have been used.

For the CD function estimate based on time-series data, it is important to check the residuals for any possible serial correlation. This has been done in both the studies end the tests carried out do not indicate any significant serial correlation. Another problem that may affect the CD function estimates (especially those based on time series data) its of multicollineasity. The results of the two studies seem to have been affected by this problem especially when the unrestricted CD functional from has been used. Consider Ahluwalias estimates of the unrestricted CD function shown in Table 5.1. It is seen that the overall explanatory power of the model is high but more of the coefficients of the explanatory variables is statistically significant. There is a similar problem with Goldar’s estimates. It seems muticollinearity has caused the estimated elasticity of output with regard to labour to be above unity, which violates the conditions of well – behaved production functions.

Next, some general comments may be made on the CD function estimates obtained in various studies for Indian industries & the way in which the estimated coefficients have been interpreted. It has been very common among the studies to interpret the sum of labour & capital co-efficients as a measure of Returns of Returns to scale. This interpretation can, however, be seriously questioned for studies based on aggregated data, especially time – series data. It may be argued that the concepts of returns to scale can be given an unambiguous meaning only at the micro level. The Relationship holds at a point of time & applies to a situation in which the character of inputs does not change. This is very different form the conditions that prevail when the analysis is carried out of the aggregate level using time-series data. Aggregate data tend to combine Economies of the size of the market. It is evident that for a proper measurement of Returns to scale, plant level data should be used for the estimation of the production function. The estimates of the production function based on industry level data or data for the aggregate manufacturing sector may not correctly show the extent of Economies of scale or diseconomies associated with plant size, and it is necessary to be very contains in drawing inferences about returns to scale based on such estimates of the production function.

It may be mentioned here that in many studies based on time-series data, estimates of the CD function have been found to be poor, being affected by the problem of multicollinearity, especially those studies that have used a time trend variable to capture technological progress. Due to muticollinearity, the coefficient of capital has proved to be low & statistically insignificant and in some cases even negative. The co-efficient of labour, on the other hand, had proved to be greater than one in some studies, again as a result of multicollinearity. Estimates of CD function parameters have been affected also by errors in the measurement of capital. This is a bigger problem for models in which a time variable is included, because this eliminates the trend components from various series & as a result the estimation bias caused by measurement errors of explanatory variables are eventuated. This has probably led to an underestimation of the capital coefficient and an overestimation of the coefficient of time.

15.5.6 CES Production Function :

The CES Production function with exponential Hicks – neutral technological change may be written as –

$$Y = A_0\, e^{u}\, [\delta L + (1 - \sigma)\, K - P] - U / P$$

In this equation, σ is the labour intensity (distribution) parameter, u is the degree of homogeneity (Returns to scale). λ is the rate of technological progress, & p is related to the elasticity of substitution σ (which is constant by assumption) in the following way $\sigma = 1/(1+P)$. The permissible range for p is – 1 to infinity, & thus σ should lie between 0 & infinity.

The CES function is more general than the CD function. The latter is a special case of the farmer, with the elasticity of substitution equal to one (p = 0 or $\sigma = 1$). The linear & the Leontief production functions are also special cases of the CES production function corresponding to σ equal to zero.

The CES production function cannot be transformed in a form that is linear in parameters. Thus to estimate the function directly a non-linear estimation procedure is required such as the non-linear least-squares or the maximum likelihood method. Kmenta (1967), has provided a method of linearizing the CES production function by obtaining a Tylor-series approximation to the function around p = 0. Kmenta's approximation to the CES function may be written as :

$$\text{In } Y = \text{In } A_0 + \lambda_1 + y\delta \text{ In } L + V(1-\partial) \text{ In } K - (1/2) PV\, \delta(1-\delta)(\text{In } L - \text{In } K)$$

The first four terms on the right hand side are those of the CD function. The last term accounts for non-unitary elasticity of substitution. The closer the elasticity of substitution to unity the better the approximation. It reduces to the CD function, if p = 0. The OLS technique to yield estimates of the parameters of the CES function.

The methods mentioned above provide direct estimates of the CES function. Indirect methods of estimation can also be used, based on conditions of producer's equilibrium. These methods involve a noaf stringent maintained by potheses such as the assumption of competitive product & factor markets. Under constant returns to scale these assumptions give rise to the following relationship between average productivity of labour & wage rate which is known as the SMAC function.

$$\text{In}\left(Y/L\right) = \text{Const.} + \sigma \;\; \text{in}\,(w) + \lambda(1-\sigma)t$$

where w denotes real wage rate, using data on Y, L & W, this equation may be estimated by the OLS technique to obtain estimates at σ & λ. Relaxing the assumption of constant Returns to scale, a generalized form of the SMAC function may be obtained as :

$$\text{In}\left(Y/L\right) = \text{const} + a \;\; \text{In}\,(w) + bt + c \;\text{In}\,(L)$$

Where

a = V / (V + P)
b = λ P / (V + P)
c = P(V – 1) / (V + P)

This equation may be estimated by the OLS method. The estimates of a, b & c can be used to derived the estimates of v, p & λ. It is possible to test whether σ is significantly different from one by using a sequential testing procedure on the estimated parameters of this equation. A major weakness of the above model is that the scale & technological progress parameters can not be identified unless σ differs from unity.

15.5.7 CES function : Studies for Indian Industries :

Direct estimation of the CES function for Indian Industries has been undertaken in only a few studies. Sankar (1970), Narasimham & Fabrycy (1974), & Barna & Leech (1987) have used non-linear estimation techniques to estimate the CES function. Narasimham & Fabrycy (1974), Bhasin & Seth (1977), & Barna & Leech (1987) have used the Kmenta approximation to the CES function. Much more common has been the use of the SMAC function. Besides being easier to estimate, this has the advantage that in its estimation data on capital input are not required.

Estimates of the SMAC function for Indian organised industry (factory sector) obtained in the study of Ahluwalia (1991) are shown below. These estimates have been made using time-series data at the aggregate level for the period 1959 – 85.

$$\text{In }(Y/L) = 2.252 + \underset{(7.1)}{0.879}\text{ In }(w) \underset{(-\,0.6)}{-\,0.133}$$

$$\text{In }(L) + \underset{(1.4)}{0.011}\text{ to}$$

Period = 1959 – 60 to 1985 – 86

$R^2 = 0.97$

$D^W = 1.74$

$$\text{In }(Y/L) = 0.465 + \underset{(10.8)}{0.936}\text{ In }(w) + \underset{(3.7)}{0.006}\text{ to}$$

Period = 1959 – 60 to 1985 – 86

$R^2 = 0.97$

$D^W = 1.82$

The estimates of the elasticity of substitution (1.01 & 0.94 in the two equations) are found to be statistically significant. These are not significantly different from unity, lending support to the use of the CD function to characterize production technology. Since the coefficient of the (L) is found to be statistically insignificant, the hypothesis of constant returns to scale is not related. When in (L) is dropped from the equation, the coefficient of L is found to be positive & statistically significant. This may be taken as indicative of significant technological progress.

Since different methods have been need to estimate the CES function for Indian Industries, it might be instructive to compare the estimates of elasticity of substitution obtained by the SMAC method with those obtained by Sankat's 1970 study estimating the SMAC function for 19 industries & for these industries also directly estimating the CES function using the Bayesian estimation technique. The period covered for the analysis is 1953 – 58. Compared to his estimates of elasticity of substitution based on the SMAC function, those based on the Bayesian estimation technique are generally higher. The estimates based on the SMAC function are significantly higher. The estimates based on the SMAC function are significantly below unity in a number of cases, but this is not so for the estimates obtained by the Bayesian method.

In several surveys of econometric literature relating to the CES function (for example Nerlove, 1967) It has been noted that in general, cross – section estimates of the elasticity of substitution are higher than the time – series estimates. (Cross – section estimates are about unity while the time series estimates are less than unity). This has been attributed to biases in the estimates of the elasticity arising from quality of labour differentials, dis-specification of the log structure, & simultaneity between production & input – use decisions. A more important explination offered for the pattern in the elasticity estimates tests on the 'putty – clay' nature of technology, as a result of which substitution possibilities get reduced once investment has occurred & capital is in place. According to this view, cross-section estimates may reveal ex-ante substitution possibilities, while time series estimate trend to reveal export substitution possibilities.

Turning to the CES function studies for Indian Industries the estimated elasticities of substitution do show a pattern similar to that described above. Thus, the elasticity estimates obtained in time – series studies of Diwan & Gujarati (1968) and Bhasin & Seth (1980) are in general much lower than those obtained in cross – section studies of Kari (1980) & Berna & Leerch (1987).

To conclude, most studies on the CES function for Indian industries have estimated the parameters indirectly by estimating the SMAC function. Though convenient, this approach has the limitation of involving certain highly restrictive assumptions such as competitive markets & absence of any lag in the adjustment of labour input to its desired level. The SMAC function studies also suffer from certain other inadequacies. It should be noted that errors in the measurement of labour input (say, due to quality differentials) get reflected in both labour productivity and the computed wage rate, this results in biases in the parameter estimates.

15.5.8 VES Production Function :

A major limitation of the CES production function is that it assumes the elasticity of substitution to be constant for all input combinations. Several functional forms have been suggested in the literature & used in empirical studies in which the elasticity of substitution is variable. These are called VES (Variable Elasticity of Substitution) Production functions. We take up for discussion one type of VES function, which has been suggested by Hildebrand & Liu (1965) & has been applied in a number of studies for Indian industries.

The functional form suggested by Hildebrand & Liu is as follows –

$$Y = \left[A(1-S)\,K^{n} + \delta K^{mn} L\,(t-m)^{n} \right]^{1/n}$$

where Y is output (value added), K capital. It reduces to the CES form for M = 0. It has the properties of a neoclassical production function, i.e. positive marginal product & downward slopping marginal product curve, over the relevant range of inputs. This function is homogeneous of agree one (i.e. constant returns to scale) & has variable elasticity of substitution.

An important implications of this functional form is that labour productivity becomes a long – linear function of Wage Rate & Capital Labour Ratio (under the condition of cost minimization & competitive markets)

$$\text{In}\,(Y/L) = a + b$$
$$\text{In}\,(w) + e\ \text{In}\,(K/L)$$

Using the parameters of the above equation, the elasticity of substitution may be obtained as :

$$\sigma = h/[1-(C/S_k)]$$

where S_k is the share of capital.

Kazi (1980) has stimated the VES function for g two digit and 16 there digit Indian Industries using cross-section (inter-state) data for 1974 & 1975. The results indicate the elasticity of substitution is varying & less than one. The same conclusion is reached in the study of Barma & Leech (1987) who have estimated the VES function for eleven industries using cross – sectional (inter state) data for 1969.

Two comments may be made on the VES function. First, while discussing the results of the studies on the SMAC function certain limitations of the estimates were noted, & these also apply to the VES function estimates. Secondly, wage rate & capital –

labour ratio are expected to be highly correlated & this may have affected the estimates of the VES function parameters. It should be noted, In particular, that if the true production function is a Cobb-Douglas one a linear relationship will arise between In (w) & in (K/L), & it would therefore not be possible to correctly estimates eq. above.

15.6 THE PROBLEM OF THE COMMONS : AN INTRODUCTION

Picture a posture open to all. It is to be expected that each herdsman will try to keep as many cattle as possible on the commons. As a rational being, each herdsman seeks to maximize lric gain. Explicitly or Implicitly, more or less consciously, he asks, what is the utility to me of adding one more animal to my herd? Adding together the component partial utilities, the rational herdsman concludes that the only sensible course for him to pursue is to add another animal to his herd. And another & another…… But this is that conclusion reached by each & every rational herdsman sharing a commons. There is the tragedy. Each man is locked into a system that compels him to increase his herd without limit – in a world that is limited. Ruin is the destination to ward which all men rush, each pursing his own best interest in a society that believes in the freedom of the commons. Freedom in the commons brings ruins to all.

It would be difficult to locate another passage of comparable length & fame containing as many errors as the one above. There is a germ of an idea, to be sure one which has been appreciated for long, and which I noted in previous topic : to wit, that a resource in finite supply is likely to be over used if available free of charge. But it would be wrong to suppose that each herdsman in professor Hardin's example will add cattle without limit. Animals are not costless, even to the herdsmen who own them. And such, private costs set limits on the number of animals each, herdsman finds most profitable to introduce into the common posture. But the paint remains that in the absence of a binding mutual agreement, each herdsm in will typically ignore the cost he imposes on the others when introducing another animal into the common are suppose then that the system will entertain too many animals in the posture, in the sense that it would be in the herdsmen's collective interest to curtail the number of animals. However, an excessive number of cattle in the common does not necessarily mean that it will be ruined for we cannot know without further information just how excessive the numbers will be. Whether or not the common will be ruined depends on a number of factors, an important one of which is the price of output (i.e. beef or milk) relative to the private cost of rearing cattle freedom in the commons does not necessarily bring ruins to all, in fact it may ruin more. Moreover, we shall note that if the resource ceases to remain a common property and if rent is charged by the usurper far access to the resource, each of the users could well be work off. The users may even become impoverished. The distributional consequences of an alteration in property rights – always bear close scrutiny. What is implied by professor Hardin's example is that each of the herdsmen could benefit if they jointly were to exercise some control over the common and if nobody were to enter the scene to collect resents for its use. The interesting question that arises then is why environmental resources are likely to be common property.

I noted that in usual parlance environmental resources are those resources which are regenerative but potentially exhaustible. I noted as well that the sustained flow of services provided by them can never exceed some finite rate. A striking further characteristic that many environmental resources possess can be described by saying that offer acute problems for defining & enforcing private rights to them. Thus, while property refers to rights, not all possible rights, especially private rights to many environmental resources, are possible pieces of property. This too was mentioned in earlier topic. The central implication of this feature is that in the absence of co-operation, actions not directly controlled by an agent – be the agent a firm, or an individual or a well – defined group acting in concert – affect the set of outcomes that the agent can attain by the use of actions that the agent does in fact control can attain by the use of actions that the agent does in fact control. Such phenomena are called 'externalities' in the Economics literature & they have been much discussed. The definition offered above may appear overly abstruse, however, the examples that follow will clarity.

Consider first underground water basins while it is easy enough to envisage different individuals in a community having titles to adjacent different matter. One usually does not know precisely how much water lie below a given surface area of land, even when these is a reasonably sharp estimate of the total stock in the entire basin. Add to this the fact that nothing is easier for a farmer than to extract water from under his neighbour's plot without anyone being the wiser, & one can see why private property rights on aquifers are difficult to define, let alone to enforce. One can of course define such rights by legislating that all water that lies under a given parcel of land belongs to the owner (or lessee) of the parcel. But this is not useful, since there is a tendency for water to migrate within the underground basin & thereby for water to migrate within the underground basin & thereby altering the extraction costs of neighbours, particularly so when pressure gradients are caused during the process of extraction. The source of the problem here lies in the uncertainty as to original location of a given quantity of water extracted at a given location.

In the face of this problem most communities have fallen back on the 'riparian doctrine' under which each owner of a parcel of land is allowed to extract as much water as the desires without regard to its effects on the owners of neighbouring parcels. The doctrine therefore provides no protection to a well – owner from the lowering of the water table under his land caused by his neighbour's actions. This suggests at once that in the absence of any intervention (eg. rationing at the well head through cooperation) the doctrine will encourage an excessive rate of overall water extraction, leading possibly to an eventual ruin of the basin. This possibly is particularly telling if in fact it is the community's long – term interest to keep the basin alive. But if the circumstances are such that this is a distinct possibility the question can be asked. Why the farmers do not see the impending destruction of the basin. The answer is that the farmers may know nothing about the natural rate of replenishment of the water basin & therefore may not know that the total annual rate of extraction exceeds this rate. In fact they may not know what the threshold level of ground water stock is. Moreover, & this is particularly important under the riparian doctrine no farmer on his own has much incentive to learn about the natural regeneration rate of ground – water basin. Under the riparian doctrine, each farmer is much like the traditional 'free – rider'. In upcoming chapter I shall analyse the kinds of policy measures that would be desirable in the face of such a farm of market failure.

Common – property marine fisheries suffer from this problem in an acute farm. The paint is that the oceans are not only a habitat for fish stocks, they are at the same time a sink into which pollutants are deposited. The problem is accentuated by the fact that much industrial efficient is discharged into coastal wetlands, such as the Wadden sea & the Indus delta, which provide a nursery to many marine species. Moreover, the fact that marine fisheries suffer from the thin problems of excessive pollution & overfishing.

It is as well to remark that while each of there twin problems has acquired special notoriety in the case of international waters, they occur in acute form within national boundaries as well. The apparent distribution of fisheries in the fresh water steams in Fuji, Japan, by discharges from paper mills is a case in paint. Over – fishing off the coast of Kerala, India, & in the Gulf of Thailand are others.

While marine pollution has drawn considerable attention in the literature for quite some time, the problem of overfishing from the commons would appear to have entered public consciousness only recently. If this is so it may be because unlike certain types of pollution which are tangible & are relatively easy to monitors, overfishing is not. As we noted above in a somewhat different context, no individual fisherman has incentives on his own to acquire sufficient information about the ecological implications of common – property fisheries. Today it is thought that something like 25 of the world's major fisheries are seriously depleted. In next chapter I shall present a formal entermporal model that will enable me to discuss what serious depletion aright mean. But it is as well to note here that as a stock gets depleted unit cost of catch typically rises, as fishermen have to travel further a field or obtain less catch at every attempt. It is in this manner that today's catch rates impose an Intertemporal Externality on the future, something which is not taken into account – or internalized, as Economists are prone to saying – if there is no internalized, as economists are prone to saying – if there is no intervention or an agreement in a common – property fishery. This often has serious distributional consequences at certain locations. If there is no intervention or an agreement in a common – property fishery. This often has serious distributional consequences at certain locations. If harvesting costs continue to rise at a particular location, what once formed a part of a poor man's food intake at that location may no longer remain so.

The magnitude of the problem of overfishing patently varies from case to case for marine fisheries with free entry the foregoing problem can arise via a seemingly convoluted process. In going problem can arise via a scearingly convoluted process. In free waters, where historical rights to the traditional fishermen are not respected, it can happen that large firms enter with modern fishing vessels. For the short run unit harvesting costs are thereby dramatically – reduces, this exacerbating the tendency towards overfishing. Meanwhile, the traditional fishermen, unable to compete with such equipment, are left impoverished for want of any catch. But in the long run, as a consequences of continual overfishing, harvest costs increase, despite – one should say, because of the use of modern harvesting techniques. Nor can one even necessarily argue that the introduction of modern harvesting techniques in the seas is at least partially blessed at the altar of intertemporal efficiency, for the market wage – rental ratio in many less developed countries is thought to be too high.

It is an observation of the utmost banality that the choice of production techniques is influenced by the institutional environment within which it is undertaken. In marine fisheries the slaughter of non-target animals in the process of each catch is a common place. This phenomenon, which is increasingly being taken very seriously by fisheries, experts, exacerbates the overfishing problem. Admittedly in the case of fisheries it will always prove difficult to monitor the extent to which non – target animals are killed by each unit of each catch. Nevertheless, it must also be granted that common property fisheries provide little incentives to individual fishing units to reduce, what are euphemistically called, 'incidental lakes'.

On occasion, where the fisheries are in international waters & the stock is depleted to low levels, the matter receives considerable publicity, as for example has been the case for some years with the blue whales. International fishing disputes in the North Atlantic, the Bay of Bengal & North East Pacific would seem to provide other cases in paint. At the widest international level the protracted Third United Nations Law of the Sea Conference has been prompted by a clear recognition of the property in mind that over 99% of marine fish production occurs in coastal waters. Rangh estimates suggest that something like 240 million metric tons of fish are produced annually in coastal waters & only about 1.6 million metric tons in the open oceans. In other words the open oceans, something like 90% of the Oceanic area, is essentially a '<u>biological desert</u>'. In view of this the recent more on the part of nations to extent their Exclusive Economic Zones to 200 nautical miles of territorial waters off the – coast line will clearly have a major redistributional effect. For example, if strictly enforced, Japan's traditional fishing grounds would be reduced by about 45%. Indeed it is estimated that a universal extension would appropriate about 90% of the world's current marine harvest to national control. About 15 nations stand to control over 40% of the enclosed oceanic space, with the first to controlling some 30%. But leaving aside the distributional consequences – as regards – such a reallocation of property rights on its own will not resolve the problem of excessive pollution & overfishing in waters with in the extended Exclusive Economic Zones. The management problem within these waters will be merely shifted from the international to the national scene. The problem may well be exacerbated by such a more on the part of nations. For, it past evidence is anything to go by, among the more successful attempts to protect fisheries that open seas by international agreement, the record of purely national regulations of limited resources does not make for particularly pleasant reading. An extension of the Exclusive Economic Zones is by no means a certain escape from this problem.

15.7 SUMMARY :

- The decision as to how many hours a day to devote for work an how many for leaisure can be analysied by the labour – leisure choice model.
- Under Fertility Analysis, Gray S. Becker, has advocated a firm decision to have children.
- Expenditure on education or training is an investment.

15.8 QUESTIONS

(1) Explain the labour – leisure choice.

(2) Write a note on fertility analysis.

(3) "Expenditure on education is investment." Explain.

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