

**Institute of Distance and Open Learning**  
**MA/MSc(Mathematics) Part II**  
**Assignment 2019-20**

**Instructions:**

- All questions to be written and submitted in the assignment sheet provided by IDOL
- Answers to all five papers' assignments to be submitted separately
- The last date of submission is Monday 30th March 2020 before 5pm in Room No. 112, first floor, IDOL building, Kalina campus, Santacruz (E), Mumbai 400098

**Paper I: Algebra and Field Theory**

1. Define a solvable group. Show that subgroup of a solvable group is solvable.
2. Find the rational canonical form of the following matrices over  $\mathbb{Q}$  and determine whether they are similar:  $A = \begin{pmatrix} 2 & -2 & 14 \\ 0 & 3 & -7 \\ 0 & 0 & 2 \end{pmatrix}$   $B = \begin{pmatrix} 0 & -4 & 85 \\ 1 & 4 & -30 \\ 0 & 0 & 3 \end{pmatrix}$   $C = \begin{pmatrix} 2 & 2 & 1 \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{pmatrix}$
3. Show that if  $K$  is an algebraic extension of  $E$  and  $E$  is an algebraic extension of  $F$  then  $K$  is an algebraic extension of  $F$ .
4. Prove that  $\mathbb{Q}(\sqrt{2})$  and  $\mathbb{Q}(\sqrt{3})$  are not isomorphic.

**Paper II: Advanced Analysis and Fourier Analysis**

1. Let  $A$  be closed rectangle in  $\mathbb{R}^n$ . Prove that the function  $\chi_C : A \rightarrow \mathbb{R}$  is integrable if and only if the boundary of  $C$  has measure zero.
2. State and prove Bounded Convergence Theorem
3. State and prove the Dirichlet's theorem.
4. Prove that  $L^2[-\pi, \pi]$  is separable.

**Paper III: Differential Geometry and Functional Analysis**

1. For any  $x, y \in V$ , where  $V$  is an inner product space, show that  $\|x - y\|^2 = \|x\|^2 + \|y\|^2$  if and only if  $x$  is orthogonal to  $y$ .
2. Prove or disprove: The unit sphere is a regular surface.
3. State and prove the lemma of Riesz.
4. Prove that the dual space of  $l^1$  is  $l^\infty$ .

### **Paper IV: Numerical Analysis**

1. Perform two iterations of the Newton-Raphson method to solve the following system of non-linear equations:  $4x^2 + 2xy + y^2 = 30$  and  $2x^2 + 3xy + y^2 = 3$ . Use initial approximation  $x_0 = -3$  and  $y_0 = 2$ .
2. Use Newton's divided difference formula to find the fourth degree curve passing through the points  $(-4, 1245)$ ,  $(-1, 33)$ ,  $(0, 5)$ ,  $(2, 9)$  and  $(5, 1335)$ .
3. Obtain the least squares quadratic approximation to the function  $y(x) = \sin x$  on  $[0, \pi/2]$  with respect to the weight function  $W(x) = 1$ .
4. Derive a numerical method (Crank-Nicolson's method) to obtain the numerical solution of one dimensional heat equation with initial and boundary conditions.

### **Paper V: Graph Theory**

1. Prove that the block graph of a connected graph is a tree.
2. Prove that a graph is Eulerian if and only if it is connected and even.
3. Prove that every simple outer planar graph has a vertex of degree less than or equal to 2.
4. Compute the spectrum of the complete bipartite graph with  $m$  vertices.