

Institute of Distance and Open Learning
MA/MSc(Mathematics) Part I
Assignment 2019-20

Instructions:

- All questions to be written and submitted in the assignment sheet provided by IDOL
- Answers to all five papers' assignments to be submitted separately
- The last date of submission is Monday 30th March 2020 before 5pm in Room No. 112, first floor, IDOL building, Kalina campus, Santacruz (E), Mumbai 400098

Paper I: Algebra

1. Prove that the $\mathbb{Q} - \{1\}$ with operation $*$ defined as $a * b = a + b - ab$, is an abelian group.
2. Let V be a finite dimensional vector space over a field F . Let B and B' be bases of V . Show that for every linear functional T on V the matrix of T with respect to B' is similar to the matrix of T with respect to B .
3. Prove that a finite integral domain is a field.
4. Show that similar matrices have the same minimal polynomial.

Paper III: Complex Analysis

1. Prove that if G is an open connected set and $f : G \rightarrow \mathbb{C}$ is differentiable with $f'(z) = 0 \forall z \in G$, then f is constant.
2. Prove that a Möbius Transformation is a composition of translation, rotation, inversion and magnification.
3. State and prove Cauchy's Integral Formula.
4. Find all the possible Laurent Series expansions of $f(z) = \frac{1}{z(z+1)(z-2)}$.

Paper IV: Discrete Mathematics and Differential Equations

1. Prove that $a \equiv b \pmod{n}$ if and only if a and b leave the same remainder when divided by n .
2. State and prove Euler's criterion for quadratic residue of p .
3. Show that the Legendre polynomial $P_n(x)$ of degree n is given by $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$.
4. Solve $(1 - x^2)y'' - 2xy' + p(p + 1)y = 0$ using power series.

Paper V: Set theory, Logic and Elementary Probability Theory

1. Define finite set. Show that if a set A is finite then there is no bijection of A with a proper subset of itself.
2. By using Zorn's lemma, prove that a nonzero unit ring contains a maximal proper ideal.
3. State and prove continuity property of probability.
4. Show that $E_y E[X/Y = y] = E[X]$