Solution

1T00426 - T.E.(Biotechnology Engineering)(SEM-VI)(Choice Base) / 88845 - Process Control & Instrumentation.

Q2 a. Given $y(t) = te^{-t}$

Take Laplace on both the sides

$$Y(s) = \frac{1}{(s+1)^2} X(s)$$

Hence we have find the transfer function now they have stated X(s)is given stepinput

$$Y(s) = \frac{1}{(s+1)^2} \, {}^1\!/_s$$

$$Y(s) = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{(s+1)^2}$$

$A = 1, B = -1, C = -1$

Taking laplace inverse we have

$$Y(t) = \left(1 - e^{-t} - te^{-t}\right)$$

Q2b.

The step response equation is $Y(t) = A\left(1 - e^{-\frac{t}{\tau}}\right)$

Y(t) = y-y_s τ = 6 sec, A= (100-80) =20

i) y – 80 = 20$\left(1 - e^{-\frac{6}{6}}\right)$

y = 80+12.64=92.64 ^{o}C

ii) y – 80 =20 $\left(1 - e^{-\frac{t}{6}}\right)$

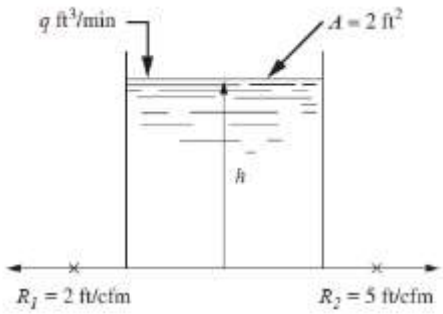
90-80 =20$\left(1 - e^{-\frac{t}{6}}\right)$

t = 4.15 sec

iii) 90% of ultimate response is 0.9 x100 =90 ^oC

Hence answer is same as above i.e. t = 4.15 sec

Q2c



$$q - q_1 - q_2 = A\frac{dh}{dt}$$

$$q - \frac{h}{R_1} - \frac{h}{R_2} = A\frac{dh}{dt} \qquad \text{------(1)}$$

At steady state:

$$q_s - \frac{h_s}{R_1} - \frac{h_s}{R_2} = A\frac{dh_s}{dt} \qquad \text{----------(2)}$$

Deviation variable form by subtracting equation(2) from equation (1)

$$(q - q_s) - \frac{(h - h_s)}{R_1} - \frac{(h - h_s)}{R_2} = A\frac{d(h - h_s)}{dt} \qquad \text{-----------(3)}$$

$Q = (q\text{-}q_s)$, $H = (h\text{-}h_s)$

Now eqn (3) becomes:

$$Q - \frac{H}{R_1} - \frac{H}{R_2} = A\frac{dH}{dt} \qquad \text{-------(4)}$$

Taking laplace transform of equation(4):

$$Q(s) - \frac{H(s)}{R_1} - \frac{H(s)}{R_2} = A\,sH(s) - H(0)$$

$H(0) = 0$

$$Q(s) - \frac{H(s)}{R_1} - \frac{H(s)}{R_2} = A\,sH(s)$$

$$Q(s) - H(s)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = A\,sH(s)$$

$$Q(s) = A\,sH(s) + H(s)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$Q(s) = H(s)\,\frac{\left(AR_1R_2\,s + R_1 + R_2\right)}{R_1R_2}$$

$$\frac{H(s)}{Q(s)} = \frac{R_1R_2}{\left(AR_1R_2\,s + R_1 + R_2\right)}$$

$$\frac{H(s)}{Q(s)} = \frac{2\times 5}{\left(10A\,s + 7\right)}$$

$$\frac{H(s)}{Q(s)} = \frac{10/7}{\left(\left(\frac{10}{7}\right)2\,s + 1\right)}$$

$$\frac{H(s)}{Q(s)} = \frac{1.43}{\left(2.85\,s + 1\right)}$$

Q3a **i.** The characteristic equation is:

$$s^4 + 4s^3 + 6s^2 + 4s + (1 + K) = 0$$

Arranging in Routh Array:

$$\begin{array}{lll}
s^4 & 1 & 6(1+k) \\
s^3 & 4 & 4 \\
s^2 & 5 & 1+k \\
s & 4-\frac{4}{5}(1+k) & \\
1 & 1+k &
\end{array}$$

For the system to be unstable:

$$4\left(1-\left(\frac{1+k}{5}\right)\right) < 0$$

$$1 < \frac{1+k}{5}$$

$$k > 4$$

$$1 + k < 0$$

$$k < -1$$

$$k > -1$$

The system is stable at:

$$-1 < k < 4$$

(ii) For two imaginary roots:

$$4 = \frac{4}{5}(1 + k); k = 4$$

Values of complex roots are:

$$5s^2 + 5 = 0$$

$$s = \pm i$$

(iii)

To find other two roots:

$$(s - s_1)(s - s_2)(s - s_3)(s - s_4) = s^4 + 4s^3 + 6s^2 + 4s + (1 + k)$$

Where $s_1 = +i$ and $s_2 = -i$

Solving and comparing the coefficients of both the sides for s_3 and s_4 :

$$s_3, s_4 = -2 \pm i$$

Q4a. Root locus for the open loop transfer function:

$$G(s)H(s) = \frac{K}{s(s^2 + 4s + 8)}$$

Soluion: (*i*) No. of open-loop poles $n = 3$

No. of open-loop zeros $m = 0$

(*ii*) No. of separate root loci $n - m = 3$

(*iii*) Open-loop plots are at

$$s = 0, s = -2 + 2j, s = -2 - 2j$$

(*iv*) Asymptotic angles are

$$\theta_A = \frac{(2q+1)\,180^\circ}{n-m},\ q = 0, 1, 2$$

$$q = 0,\ \theta_1 = \frac{180}{3} = 60^\circ$$

$$q = 1,\ \theta_2 = \frac{180 \times 3}{3} = 180^\circ$$

$$q = 2,\ \theta_3 = \frac{180 \times 5}{3} = 300^\circ$$

(*v*) Centroid $\quad \sigma_c = \dfrac{\sum P - \sum Z}{n - m} = \dfrac{-2 + 2j - 2 - 2j}{3} = -1.33$

(*vi*) Breakaway point:

$$1 + G(s)\,H(s) = 0; \quad 1 + \frac{K}{s(s^2 + 4s + 8)} = 0$$

$$K = -(s^2 + 4s^2 + 8s)$$

$$\frac{dK}{ds} = -(3s^2 + 8s + 8) = 0$$

$$s = \frac{-8 \pm \sqrt{64 - 96}}{2}$$

$$= \frac{-8 \pm 5.66j}{2} = -4 \pm 2.83\,j$$

(*vii*) Point of intersection with imaginary axis.

$$1 + \frac{K}{s(s^2 + 4s + 8)} = 0 \Rightarrow s^3 + 4s^2 + 8s + K = 0$$

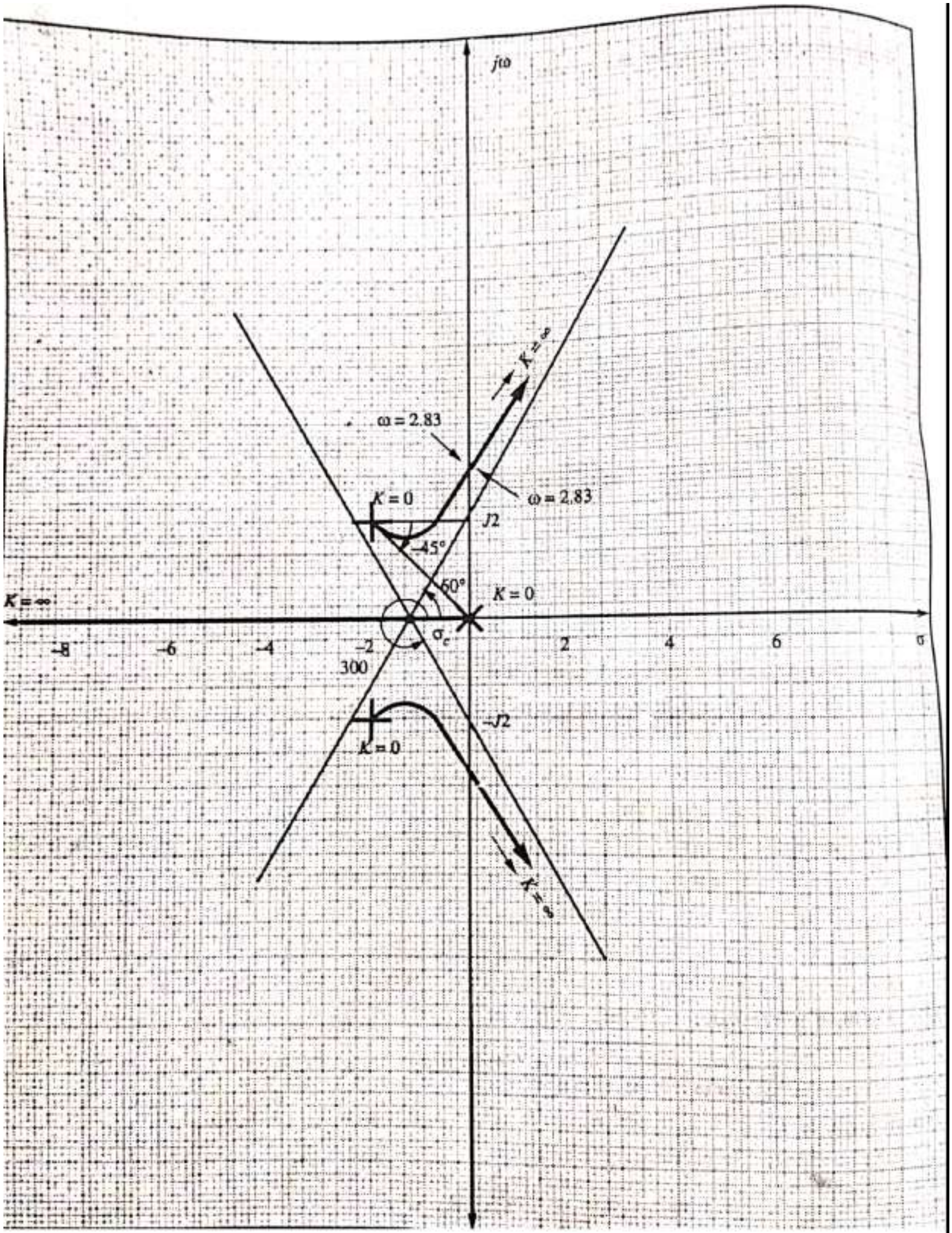
Routh array is

$$\begin{array}{c|cc} s^3 & 1 & 8 \\ s^2 & 4 & K \\ s^1 & \dfrac{32-K}{4} & 0 \\ s^0 & K & \end{array}$$

For sustained oscillation, $\dfrac{32 - K}{4} = 0 \Rightarrow K = 32$

Auxiliary eqn. is $4s^2 + K = 0$; $4s^2 = 32$; $s^2 = -8$; $s = \pm j\,2.83$

(*viii*) Angle of departure at $s = -2 + j\,2$ is $\theta_d = 180^\circ - (90^\circ + 135^\circ) = -45^\circ$



jω
K = ∞
ω = 2.83
ω = 2.83
K = 0
j2
−45°
60°
K = 0
K = ∞
σ_c
300
−8
−6
−4
−2
2
4
6
σ
−j2
K = 0
K = ∞

Q5a The Bode plot for $G(s)H(s)=\dfrac{100}{s(s+0.5)(s+10)}$

Step1: $G(s)H(s)=\dfrac{100}{s0.5(2s+1)10(0.1s+1)}=\dfrac{20}{s(2s+1)(0.1s+1)}$

Step2: K= 20

1 pole at origin $\dfrac{1}{s}$

Simple pole $\dfrac{1}{(2s+1)}$, τ_1 =2, wc_1 = 1/ τ_1 = 0.5

Simple pole $\dfrac{1}{(0.1s+1)}$, τ_1 =0.1, wc_1 = 1/ τ_1 = 10

Step3:Magnitude plot analysis

i) For K=20, 20 log K= 26.02dB
ii) For 1 pole at origin, straight line with slope -20 dB/decade will pass through w=1 and 20log K=26.02 dB line
iii) Overall magnitude plot will continue with slope -20 dB/decade till wc_1 =0.5. Now from wc_1 =0.5 onwards as simple pole is occurring, slope is going to change by -20 dB/decade hence resultant will have slope -40 dB/decade representing addition of K, 1/s and 1/(1+2s). This will continue wc_2 = 10
iv) At wc2 = 10 again simple pole is ocuuring and hence slope will further change by -20 dB/decade. So resultant will have slope of -60 dB/decade and continue as no further terms exist.

Step 4

Phase Angle plot :

$$G(j\omega)\ H(j\omega) = \frac{20}{j\omega(1+2j\omega)\left(1+j\frac{\omega}{10}\right)}$$

$$\angle G(j\omega)\ H(j\omega) = \frac{\angle 20+j0}{\angle j\omega\ \angle 1+2j\omega\ \ \angle 1+j\frac{\omega}{10}}$$

$$\angle 20 + j0 = 0°$$

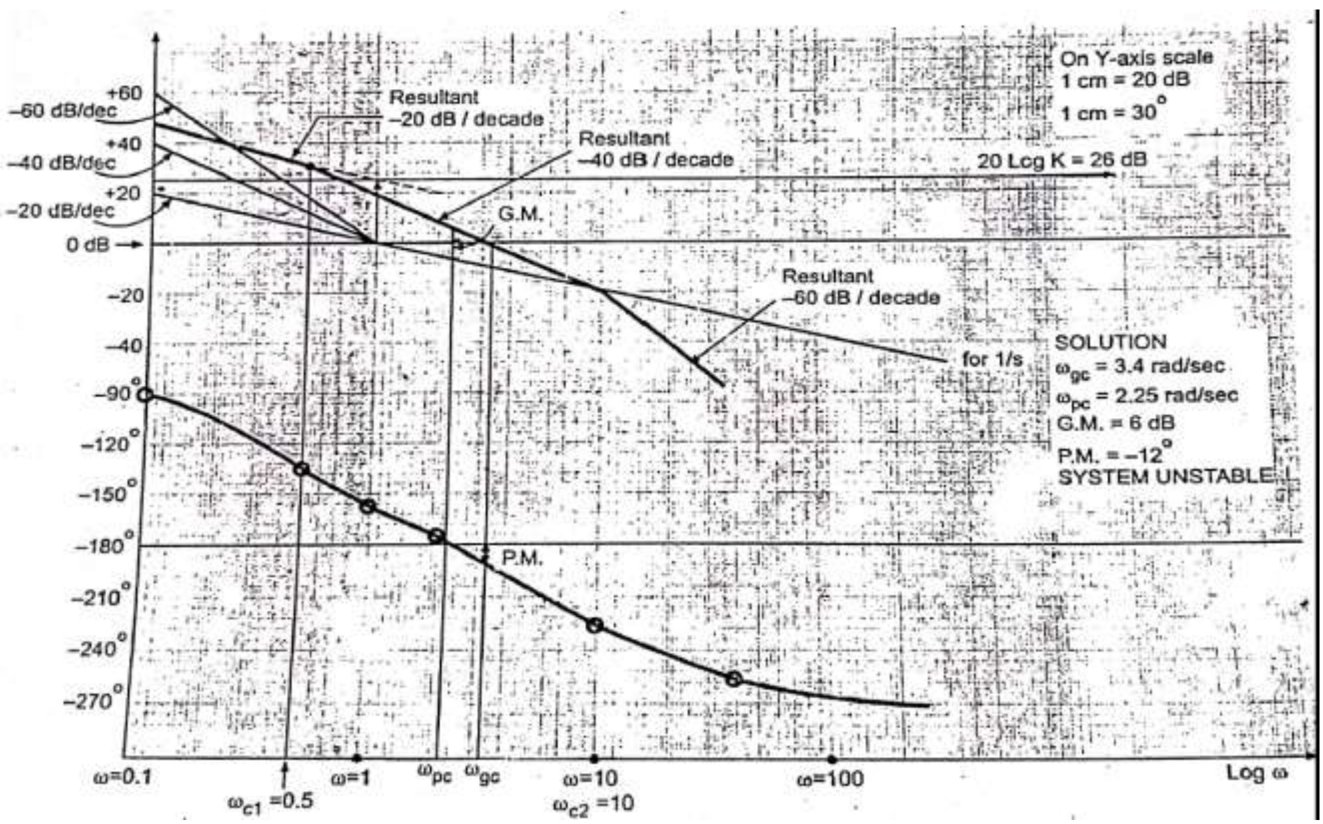
$$\angle\frac{1}{j\omega} = -90°, \quad \text{1 pole at origin}$$

$$\angle\frac{1}{1+2j\omega} = -\tan^{-1} 2\omega \qquad \angle\frac{1}{1+j\frac{\omega}{10}} = -\tan^{-1}\frac{\omega}{10}$$

∴ Phase Angle Table

ω	$\frac{1}{j\omega}$	$-\tan^{-1} 2\omega$	$-\tan^{-1} \frac{\omega}{10}$	ϕ_R
0.1	− 90°	− 11.3°	− 0.57°	− 101.87°
0.5	− 90°	− 45°	− 2.86°	− 137.86°
1	− 90°	− 63.43°	− 5.71°	− 159.14°
2	− 90°	− 75.96°	− 11.3°	− 177.26°
10	− 90°	− 87.13°	− 45°	− 222.13°
50	− 90°	− 89.42°	− 78.69°	− 258.11°
∞	− 90°	− 90°	− 90°	− 270°

Bode Plot:



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