Erlang C Formula

In an Erlang C telephone system, N channels are available. New calls are assigned a channel until all channels are full. When all the channels are occupied, a new call is queued until a channel can be served. This is in contrast to an Erlang B system in which new calls are blocked.

The assumptions made here are the same as in an Erlang B system, except the fourth one, where the calls that encounter congestion enter a queue and are stored there until a server becomes free. Here, the second assumption, that is, statistical equilibrium, implies that A ≤ N. If A ≥ N, calls are entering the system at a greater rate than they leave. As a result, the length of the queue will continually increase towards infinity. This is not statistical equilibrium.

The probability of delay formula or Erlang's delay formula can be given as follows:

\[ P_D = \frac{A^N}{A^N + N! \left( 1 - \frac{A}{N} \right) \sum_{k=0}^{N-1} \frac{A^k}{k!}} \]  

(2.18)

This is also called the Erlang C formula.

Erlang B Formula

We can find from the Poisson distribution that

\[ P(x) = \frac{A^x}{x!} e^{-A} \]  

(2.16)

where \( P(0) = e^{-A} \)

In Eq. (2.16), \( P(x) \) is the probability of \( x \) calls in progress. Here, \( x \) can have any value between zero and infinity and the sum of their probabilities must be unity. Thus, if call arrivals have a Poisson distribution, so does the number of calls in progress. This requires an infinite number of trunks to carry the calls. If the number of trunks available is finite, then some calls can be lost or delayed and the distribution is no longer Poissonian. The distribution that then occurs is derived using Erlang's formula.

Erlang determined the GOS (i.e., the loss probability) of a lost call system having \( N \) trunks when the offered traffic is \( A \). The solution was obtained on the basis of the following assumptions:

- Pure chance traffic
- Statistical equilibrium
- Full availability
- Loss of calls encountering congestion

The probability of congestion, or of a lost call, or the GOS \( B \) for the full availability group of \( N \) trunks and the offered traffic \( A \) erlangs is given by

\[ B = \frac{A^N}{N!} \frac{1}{\sum_{k=0}^{N-1} \frac{A^k}{k!}} \]  

(2.17)

This is also called the Erlang B formula.
Solution 2a:

The total bandwidth given is 36 MHz.

Channel bandwidth = 25KHz x 2 simplex channels = 50 kHz per duplex channel

Total Available Channels within 36 MHz range = 36 MHz / 50 kHz = 720 channels

A) If N = 7, the total number of channels available per cell is 720/7 = 102.85 (rounded to 102 channels)

B) If N = 12, the total number of channels available per cell is 720/12 = 60.