QP Code : 21950

M.SC. (MATHS) PART-II
Algebra and Field Theory
(Rev) (JAN - 2019)

(3 Hours) [Total Marks:80

Instructions:

- Attempt **any two** questions from **Section I** and **any two** questions from **Section II**
- **All questions** carry **equal marks**
- **Answers** to **Section I and II** should be written in **same answer book**
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SECTION I (Attempt Any Two)

1. (a) Define a solvable group. Show that subgroup of a solvable group is solvable.

   (b) Let G be a group and let $G = G_1 \supset G_2 \supset \ldots \supset G_k = \{e\}$ be a normal tower such that each group G_i/G_{i+1} is simple and $G_i \neq G_{i+1}$ for $i = 1, \ldots, k-1$. Then show that any other normal tower of G having the same properties is equivalent to this one.

2. (a) Calculate the character tables of $\mathbb{Z}_2 \times \mathbb{Z}_2, \mathbb{Z}_2 \times \mathbb{Z}_3$ and $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$.

   (b) State Maschke's theorem. Is the converse of Maschke's theorem true? Justify your answer.

3. (a) Define submodule. Let R be a ring and let M be an R-module. Prove that a subset N of M is a submodule of M iff (i) $N \neq \phi$ and (ii) $x + ry \in N$ for all $r \in R$ and for all $x, y \in N$

   (b) Let $N_1, N_2, \ldots, N_k$ be submodules of the R-module M. Then show that following are equivalent:
   (i) The map $\pi : N_1 \times N_2 \times \ldots \times N_k \to N_1 + N_2 + \ldots + N_k$ defined by $\pi(a_1, a_2, \ldots, a_k) = a_1 + a_2 + \ldots + a_k$ is an isomorphism.
   (ii) $N_j \cap (N_1 + N_2 + \ldots + N_{j-1} + N_{j+1} + \ldots + N_k) = 0$ for all $j \in \{1, 2, \ldots, k\}$
   (iii) Every $x \in N_1 + \ldots + N_k$ can be uniquely written in the form $a_1 + \ldots + a_k$ with $a_i \in N_i$.

4. (a) Find the rational canonical form of
$$\begin{pmatrix} 0 & -1 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} c & 0 & -1 \\ 0 & c & 1 \\ -1 & 1 & c \end{pmatrix}$$

   (b) Find the Jordan canonical form of
$$A = \begin{pmatrix} 2 & -2 & 14 \\ 0 & 3 & -7 \\ 0 & 0 & 2 \end{pmatrix} \quad B = \begin{pmatrix} 0 & -4 & 85 \\ 1 & 4 & -30 \\ 0 & 0 & 3 \end{pmatrix} \quad C = \begin{pmatrix} 2 & 2 & 1 \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{pmatrix}$$

SECTION II (Attempt Any Two)

5. (a) Define algebraic extension. Prove that if F is a field of characteristic 0 and a, b are algebraic over F then there is an element c in $F(a, b)$ such that $F(a, b) = F(c)$.

   (b) Define Splitting Field. Show that if F is a field and $f(x)$ is a nonconstant elemet of $F[x]$ then there exists a splitting field E for $f(x)$ over F. Find splitting field for $f(x) = x^4 - x^2 - 2$ over $\mathbb{Q}$.

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6. (a) Define Normal Extension, Separable Extension of a field. Show that in every field of characteristic zero, each of its extension is separable."

   (b) Let F be a field of characteristic p. Then for any $a, b \in F$, show that $(a+b)^p = a^p + b^p$ and $(ab)^p = a^p b^p$.

7. (a) Define fixed field. Determine the fixed field of the automorphism $t \to t+1$ of rational function field $k(t)$.

   (b) Show that every element in a finite field can be written as sum of two squares.

8. (a) Define constructible number. If a and b are constructible numbers then prove that $a+b$ and $a-b$ are constructible.

   (b) Find the Galois group of $x^3 - 2$ over the field $\mathbb{Q}$.

M.SC. (MATHS) PART-II
Algebra - II. (Old)
(JAN - 2019)

(3 Hours) [Total Marks: 100

N.B.: (1) **Attempt any FIVE questions.**
(2) Figures to the right indicate marks for respective subquestions.

1. (a) Prove that a non-abelian group G of order p^3 has a center of order p (p a prime). (10)
   (b) Let G be a finite group of order $p^r m$, where p is a prime and $\gcd(p, m) = 1$. Show that G has a subgroup of order p^r. (10)

2. (a) Let N be normal subgroup of group G. Show that G has composition series if and only if N and G/N have the composition series. (10)
   (b) Let G be a nilpotent group and H a proper normal subgroup of G. Show that
   $$H \cap Z(G) \neq \{e\},$$
   where $Z(G)$ denote the center of G. (10)

3. (a) Let M, N be R-modules and let $\varphi : M \to N$ be an R-module homomorphism. Show that $\ker(\varphi)$ is a submodule of M and $M/\ker(\varphi) \cong \varphi(M)$. (10)
   (b) (i) Let M be an R-module and let $N_1, \ldots, N_k$ be submodules of M. The sum of $N_1, \ldots, N_k$ is denoted by $N_1 + \cdots + N_k = \{a_1 + \cdots + a_k : a_i \in N_i\}$ is a submodule of M (5)
   (ii) Let R be a ring with 1 and $F = R^n$ be a free module with basis $\{e_1, \ldots, e_n\}$. Let M be an R-module and let $m_1, \ldots, m_n \in M$. Show that there exist a unique R-module homomorphism $\varphi : F \to M$ such that $\varphi(e_i) = m_i$. (5)

4. (a) Let R be a PID and M be a finitely generated free module over R, of rank n. Show that every submodule of M is also free of rank $\leq n$. (10)
   (b) Let M be free R-module with basis $\{u_1, u_2, \ldots, u_n\}$. Show that $M \cong R^n$. (10)

5. (a) Let K/F be a finite separable extension. Show that $K = F(\alpha)$ for some $\alpha \in K$. (10)
   (b) Let K/F be a field extension and $\alpha \in K$ be algebraic over F. Show that there is a unique monic irreducible polynomial $m(x) \in F[x]$, which has α as a root. (10)

6. (a) (i) If $\sin\theta$ and $\cos\theta$ are constructible real numbers for an angle of magnitude θ, then show that θ is constructible. (5)
   (ii) Let α be a constructible real numbers. Show that $\sqrt{\alpha}$ is constructible. (5)
   (b) (i) Let F be a field of characteristic $p > 0$. If K is a finite extension of F such that $[K : F]$ is relatively prime to p, then show that K is separable over F. (5)
   (ii) Show that every irreducible polynomial over $\mathbb{Q}$ is separable. (5)

7. (a) Let F be a field of characteristic 0, and let $a \in F$. If K is the splitting field of $x^n - a$ over F, then show that $G(K/F)$ is a solvable group. (10)

   (b) Let K/F be a Galois extension and let $G = G(K/F)$. Show that there is a bijection between the set of subfields E of K containing F and set of subgroups H of G. (10)

8. (a) Show that the field $\mathbb{C}$ of complex numbers is algebraically closed. (10)

   (b) Show by an example that there exist polynomials of degree 5 in $\mathbb{Q}[x]$ that are not solvable by radicals over $\mathbb{Q}$. (10)

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M.SC. (MATHS) PART-II
Advanced Analysis and Fourier Analysis
(Rev) (JAN - 2019)

QP CODE : 21702

[Maximum Marks: 80

N.B. 1) Attempt any **Two** questions from each Section.
2) All questions carry equal marks.
3) Figures to the right indicate marks for respective sub-questions.
4 Answer to Section-I and Section-II should be written in the same answer book.

SECTION-I

1. (a) State and prove the Fubini's Theorem. (10)

(b) (i) Let A be closed rectangle in $I\!R^n$. Prove that the function $\chi_C : A \to I\!R$ is integrable if and only if the boundary of C has measure zero. (5)

(ii) Let $f : [0,1] \times [0,1] \to I\!R$ be defined by (5)

$$f(x,y) = \begin{cases} 0 & \text{if} \quad 0 \le x < \frac{1}{2}, \\ 1 & \text{if} \quad \frac{1}{2} \le x \le 1. \end{cases}$$

Show that f is integrable and $\int_{[0,1]\times[0,1]} f\,dx = \frac{1}{2}$.

2. (a) Let E be the subset of $I\!R^n$. Define the Lebesgue measure of a set E. If $E_1, E_2, \ldots,$ are disjoint measurable sets and $E = \cup_{j=1}^{\infty} E_j$, then show that $m(E) = \sum_{j=1}^{\infty} m(E_j)$. (10)

(b) (i) Prove that any closed subset of $I\!R^n$ is measurable. (5)

(ii) If $E \subset I\!R^n$ is measurable and $x \in I\!R^n$ then show that $m(x+E) = m(E)$. (5)

3. (a) Suppose f is non-negative measurable function defined on a subset of $I\!R^n$. Then show that there exists an increasing sequence of non-negative simple functions $\{\varphi_k\}_{k=1}^{\infty}$ that converges pointwise to f. (10)

(b) (i) If f and g measurable functions then prove that the integer power f^k, $f+g$ and fg are measurable. (5)

(ii) Suppose $A \subseteq E \subseteq B$, where A and B are mesurables sets of finite mesure. Prove that if $m(A) = m(B)$, then E is measurable. (5)

4. (a) State and prove Bounded Convergence Theorem. (10)

(b) (i) If f be bounded function supported on set of finite measure. If $\{\varphi_n\}_{n=1}^{\infty}$ is any sequence of simple functions bounded by M, supported on E, and with $\varphi_n(x) \to f(x)$ for almost every x then show that: (5)

(α) $\lim_{n\to\infty} \int \varphi_n$ exists.

(β) If $f = 0$ a.e., then $\lim_{n\to\infty} \int \varphi_n = 0$.

[TURN OVER

(ii) Let f be measurable function on set of finite measure E. Prove that f^+ and f^- are integrable over E if and only if $|f|$ is integrable over E. (5)

SECTION-II

5. (a) Suppose f is an integrable function on the circle with Fourier coefficients $\hat{f}(n) = 0$ for all $n \in \mathbb{Z}$. Then show that $f(\theta_0) = 0$ whenever f is continuous at the point θ_0. (10)

   (b) (i) State and prove the Dirichlet's theorem. (5)

   (ii) Let N-th Fejér kernel F_N be defined as $F_n(x) = \dfrac{\sin^2(Nx/2)}{N\sin^2(x/2)}$. Show that Fejér kernel is good kernel. (5)

6. (a) Show that the space $L^2(I\!R^d)$ is complete in its metric. (10)

   (b) (i) Prove that any Hilbert space has an orthonormal basis. (5)

   (ii) Let $\ell^2(\mathbb{Z})$ be the space of square summable sequence of two sided. If a, b denotes the sequences in $\ell^2(\mathbb{Z})$, we define the inner product and norm as follows. (5)

$$\langle a, b\rangle = \sum_{k=-\infty}^{\infty} a_k \overline{b_k}, \quad \text{and} \quad \|a\| = \left(\sum_{k=-\infty}^{\infty} |a|^2\right)^{1/2}.$$

   Then show that $\ell^2(\mathbb{Z})$ is a Hilbert space.

7. (a) If f is an L^2-periodic function, then prove that $\sum_{-\infty}^{\infty} |\hat{f}(n)|^2 \leq \|f\|^2$. (6)

   (b) Prove that $L^2[-\pi, \pi]$ is separable. (6)

   (c) Define the orthonormal basis for Hilbert space and hence determine whether the set $\left\{\dfrac{1}{\sqrt{2\pi}}, \dfrac{\cos nt}{\sqrt{2\pi}}, \dfrac{\sin nt}{\sqrt{2\pi}}\right\}$, where $t \in [-\pi, \pi]$ and $n \in \mathbb{N}$ is an orthonormal basis for Hilbert space $L^2[-\pi, \pi]$. (8)

8. (a) State and prove Weierstrass Approximation theorem. (8)

   (b) Define the Poisson kernel and prove that it satsfies the properties of good kernel. (6)

   (c) If $P_r(\theta)$ denotes the Poisson kernel, show that the function $u(r, \theta) = \dfrac{\partial P_r}{\partial \theta}$, defined for $0 \leq r < 1$ and $\theta \in I\!R$, satisfies: (6)

   (i) $\Delta u = 0$ in the disc.

   (ii) $\lim_{r\to 1} u(r, \theta) = 0$ for each θ.

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M.SC. (MATHS) PART-II
ANALYSIS - II.
(Old) (JAN - 2019)

Q. P. Code: 50663

[3 hours –Scheme A **Idol** students] Total Marks : 100

[2 hours –Scheme B] Total Marks : 40

N.B (1)Scheme A (IDOL) students will attempt any Five questions.

Scheme B students will attempt any Three questions

(2) **All** Questions Carry Equal Marks. Justify the answers with Mathematical justification.

<u>Write the scheme , under which yiu are appearing on your arnswerbook on first page</u>

Q.1.

(a) Show that for Lebesgue outer measure, if (A_n) is an increasing sequence of sets with respect to set inclusion and $A = \cup_{n\in\mathbb{N}} A_n$ then $\lim_{n\to\infty} m^*(A_n) = m^*(A)$.

(b) Show that lebesgue outer measure of an interval is the length of the inte3rval and the interval is a measurable set.

Q. 2

(a) Show that, $\underset{n\to\infty}{lim} \inf f_n$ is a measurable function if each (f_m) is a measurable function.

(b) Show that a nonnegative measurable functions is the pointwise increasing limit of as sequence of simple functions.

Q. 3

(a) State and Prove Fatou's lemma ?Show that the strict inequality may hold Justify.

(b) Show that a Riemann integrable function is Lebesgue integrable and the integrals have same value.

Q. 4

(a) Show that £^{2} $[a, b] \subset$ £^{1} $[a, b]$. Is £^{2} $[\mathbb{R}] \subset$ £^{1} $[\mathbb{R}]$? Justify..

(b) Show that a bounded measurable function is integrable over [a, b] . is it necessarily x sequence of simple functions

[Turn Over

Q. 5

(a) State and prove Fubini's theorem for lebesue integrable functions over $\mathbb{R}^{2}$.

b) Give an example to illustrate $\int_a^b(\int_a^b f(x,y)\mathrm{dx})\ \mathrm{dy} \neq \int_a^b(\int_a^b f(x,y)\mathrm{dy})\ \mathrm{dx}$

Q. 6

(a) i) Give an example of a lebesgue integrable function which is not Riemann integrable

ii) Give an example of a function so that the improper Riemann integral of f exists but the Lebesgue integral of f does not exist.

b) Show that a if a afunction g is almost everywhere equal to a measurable function, then g is measurable.

Q. 7

a) Show that $\pounds^1[\mathbb{R}]$ is a normed linear space and it is complete.

b) Define Fourier transform. State and prove Plancharel's theorem for $\pounds^2$.

Q. 8

(a) State and prove Bessel's inequality and Parsevel's identity for Fourier series.

(b) Give an example to show that the fourier series of a function f may not converge to f(x) at Some point x .

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M.SC. (MATHS) PART-II
Differential Geometry and Functional Analysis
(Rev) (JAN - 2019)

Q.P.Code:13237

[Marks: 80]

N.B.
1) All questions carry equal marks.
2) Solve any **Two** questions from section A.
3) Solve any **Two** questions from section B.

Section A

1. (a) i. Show that a matrix A represents a rotation of $I\!R^3$ if and only if $A \in SO_3$. (5)

   ii. Let $m : I\!R^n \longrightarrow I\!R^n$. Show that m is an isometry which fixes the origin if and only if $\langle m(x), m(y)\rangle = \langle x, y\rangle$ for all $x, y \in I\!R^n$. (5)

   (b) i. For any $x, y \in V$, where V is an inner product space, show that $\|x-y\|^2 = \|x\|^2 + \|y\|^2$ if and only if x is orthogonal to y. (5)

   ii. Let S be the rotation of the plane with an angle $\frac{\pi}{2}$ about the point $(1,1)^T$. Write S as a product of $t_a\rho_\theta$ where t_a is translation by a fix vector a and ρ_θ is a rotation by an angle θ . (5)

2. (a) i. Let γ be a regular curve in R^3 with nowhere vanishing curvature then show that the image of γ is contained in a plane if and only if torsion is zero at every point of the curve. (5)

   ii. Is the parametrized curve tractrix $\gamma : (0, \pi) \longrightarrow I\!R^2$ is given by (5)

$$\gamma(t) = (\ sint,\ cost + \ log\tan(\frac{t}{2}))$$

   regular? Justify. Note that t is the angle that the y axis makes with the tangent vector.

   (b) i. State and prove the Frenet-Serret formulae. (5)

   ii. Compute curvature k, torsion τ, tangent t, normal n and binormal b for parametrized curve $\gamma(t) = (\frac{4}{5}cost, 1 - sint, \frac{-3}{5}cost)$. (5)

3. (a) i. If $f : U \longrightarrow I\!R$ is a differentiable function in an open set U of $I\!R^2$ then show that the subset of $I\!R^3$ given by $(x, y, f(x, y))$ for $(x, y) \in U$ is a regular surface. (5)

   ii. Prove or disprove: The unit sphere is a regular surface. (5)

   (b) i. Let S be a regular surface. Show that there exist a vector subspace of dimension two which coincides with the set of tangent vectors $T_p(S)$ for $p \in S$. (5)

   ii. Let $f(x, y, z) = (x + y + z - 1)^2$. Locate the critical points and critical values of f. (5)

4. (a) i. State and prove Meusnier theorem. (5)

   ii. Let S_1 be the infinite strip in the xy plane given by $0 < x < 2\pi$ and S_2 be the circular surface $x^2 + y^2 = 1$ with the rulling given by $x = 1, y = 0$ removed. Prove or disprove the map $f : S_1 \to S_2$ is an isometry. (5)

[TURN OVER

(b) Consider the parametrized curve $\sigma(u, v) = (u + v, u - v, uv)$. Calculate

i. The coefficients of the first fundamental form (2)

ii. The coefficients of the Second fundamental form (2)

iii. The Gaussian curvature (2)

iv. The Principal curvatures (2)

v. The Mean curvature (2)

Section B

1. (a) If (A_n) is a sequence of nowhere dense sets in a complete metric space X, then prove that there exists a point in X which is not in any of the $A_n' s$. (10)

   (b) Let X be a compact metric space and $\mathcal{C}(X, \mathbb{R})$ denote the set of all continuous functions from X to $\mathbb{R}$. Prove that if a closed subspace of $\mathcal{C}(X, \mathbb{R})$ is bounded and equicontinuous then it is compact. (10)

2. (a) State and prove the lemma of Riesz. (10)

   (b) Prove that the closed unit ball of a normed linear space X is compact if and only if X is a finite dimensional normed linear space. (10)

3. (a) Prove that the dual space of l^1 is l^∞. (10)

   (b) Let X, Y are normed linear spaces over $\mathbb{R}$ where Y is a Banach space. If Z is a subspace of X and $T : Z \to Y$ is a bounded linear operator then prove that there exists a linear operator $S : X \to Y$ such that $S_{|Z} = T$ and $||S|| = ||T||$. (10)

4. (a) Let (T_n) be a sequence of bounded linear operators $T_n : X \to Y$ from a Banach space X into a normed linear space Y. Show that if sequence $(||T_n(x)||)$ is bounded for every $x \in X$ then sequence $(||T_n||)$ is bounded. (10)

   (b) Prove that a nonempty subset X of normed linear space N is a bounded subset of N if and only if $f(X)$ is a bounded set of numbers for every $f \in N^*$ (N^* is dual space of N.) (10)

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M.SC. (MATHS) PART-II
Differential Geometry
(Old) (JAN - 2019)

Q.P. Code : 37387

[Time: Three Hours] **[Marks:100]**

Please check whether you have got the right question paper.

N.B: 1. Attempt any five questions.
2. Each question carries twenty marks. Parts (a), (b) carrying **ten marks** each.

Q.1 a) i) Define an inner product on a n- dimensional real vector space $\mathbb{E}$;.
ii) State and prove the Schwarz inequality for an inner product <;> on $\mathbb{E}$;
iii) Prove that an n – dimensional (real) inner product space $(\mathbb{E}, <;>)$ has an orthonormal vector basis.

b) Prove that a n – dimensional inner product space $(\mathbb{E}, <;>)$is isomorphic with the Euclidean space $\mathbb{R}^n$ under an isomorphism which preserves the inner products.

Q.2 a) Let U denote an open subset if $\mathbb{R}^n$.
i) Define an alternating k –multilinear form ω on U and explain the C^∞ (U)- module structure of the set $\Omega^k(U)$ consisting of all the smooth k – multilinear, alternating forms ω on U.
ii) Define the exterior differentiation operator $d : \Omega^k(U) \to \Omega^{k-1}(U)$ and prove $d \circ d = 0$.

b) Define $\int_U \omega$ of an exterior n – form ω on its domain U and prove that the integral is independent of the coordinates on U.

Q.3 a) i) Define the curvature $k(s)$ and the torsion $\tau(s)$ of a curve $c(s)$, assumed to be arc – length parametrized.
ii) Derive the formulae:
1) $k(t) = \frac{\|\dot{c}(t)\times\ddot{c}(t)\|}{\|\dot{c}(t)\|}$
2) $\tau(t) = \frac{\det(\dot{c}(t),\ \ddot{c}(t),\ \dddot{c}(t))}{\|\dot{c}(t)\times\ddot{c}(t)\|^2}$ for a smooth, arbitrarily parametrized curve $t \mapsto c(t)$.

b) Calculate the curvature $k(t)$ and the torsion $\tau(t)$ for the curve $c(t) = (2t^2 + 1, 3t, t^3),\ t \in \mathbb{R}$.

Q.4 a) Let $c(t)$ be a smooth curve with the property that the tangent vectors $\dot{c}(t)$ make a constant angle with a fixed diredtion $\vec{e}$. Suppose the curvature $k(t)$ does not vanish anywhere prove that $\tau(t)/k(t)$ is constant.

b) Determine all the plane curves $c: (a, b) \to \mathbb{R}^2$ satisfying $k(t) =$ constant.

Q.P. Code : 37387

Q.5 State and prove the fundamental theorem for the curves (smooth) in $\mathbb{R}^3$.

Q.6 a) Define the following terms.

i) A smooth surface S in $\mathbb{R}^3$.

ii) The tangent plane $T_p(s)$ of the surface S at a point p of it.

iii) Orientability of a smooth surface S.

b) Give an example of a smooth surface S in $\mathbb{R}^3$ which is not orientable. Explain the smooth surface structure of your example and justify that the surface is not orientable.

Q.7 a) Let S be the surface in the form of the graph of a function (smooth). $f: U \to \mathbb{R}$, (U being an open subset of $\mathbb{R}^2$)

i) Derive an equation for the tangent plane $T_p(s)$ to the surface at a point $p = (a, b, f\ (a, b))\ (a, b) \in\ U$.

ii) Is S orientable? If it is orientable, describe an orientation of it.

iii) Describe the principal directions of S at a point p of S.

iv) Obtain a formula for the Gaussian curvature of S at a point $p = (a, b, f(a, b))$.

b) Obtain the Gaussian and the mean curvatures for a cylindrical surface of radius $r > 0$.

Q.8 Let S be smooth surface in $\mathbb{R}^3$ and let smooth surface in $\mathbb{R}^3$ and let p be a point of it.

a) Define the normal curvature of S at p in the direction $\vec{e} \in T_p(S)$.

State and prove the result regarding the relation between the normal curvature of S at p in the direction $\vec{e} \in T_p(S)$) and the principal cunatures of S at p.

b) State without proof, but explaining all the terms, the Theorema Egragium of Gauss.

M.SC. (MATHS) PART-II
Numerical Analysis
(Rev) (JAN - 2019)

Q. P. Code: 50442

[Total Marks:80

(1) Attempt any two questions from each section.

(2) All questions carry equal marks. Scientific calculator can be used.

(3) Answer to Section-I and Section-II should be written in the same answer book

Section-I

Que. 1 (a) Define: Absolute error, Relative error and Percentage error.
Use the series $\log_e\left(\dfrac{1+x}{1-x}\right) = 2\left(x + \dfrac{x^3}{3} + \dfrac{x^5}{5} + \cdots\right)$ to compute the value of $\log(1.2)$ correct to seven decimal places and find the number of terms retained.

(b) Convert the octal number $(5701.46)_8$ to the binary form and then convert to the hexadecimal form.

Que. 2 (a) Describe the Birge-Vieta method to determine a real number p such that $(x-p)$ is a factor of the polynomial equation $P_n(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + ... + a_{n-1}x + a_n = 0$, where $a_0 \neq 0$ and $a_0, a_1, a_2, ..., a_n$ are real numbers.

(b) Perform two iterations of the Newton-Raphson method to solve the following system of non-linear equations:
$4x^2 + 2xy + y^2 = 30$ and $2x^2 + 3xy + y^2 = 3$. Use initial approximation $x_0 = -3$ and $y_0 = 2$.

Que. 3 (a) Describe the Power method to determine the largest eigen value in magnitude of the square matrix $A = [a_{ij}]$ of order n.

(b) Use Crout's method to solve the following system of linear equations.

$$\begin{aligned} 10x + 3y + 4z &= 15 \\ 2x - 10y + 3z &= 37 \\ 3x + 2y - 10z &= -10. \end{aligned}$$

Que. 4 (a) Determine the step size h that can be used in the tabulation of a function $f(x)$, $a \leq x \leq b$, at equally spaced nodal points so that the truncation error of the quadratic interpolation is less than ϵ.

(b) Use Newton's divided difference formula to find the fourth degree curve passing through the points $(-4, 1245)$, $(-1, 33)$, $(0, 5)$, $(2, 9)$ and $(5, 1335)$.

Section-II

Que. 5 (a) Derive Newton-Cotes quadrature formula and use it to derive Trapezoidal rule for numerical integration.

(b) Derive two-point Gaussian quadrature formula to evaluate the integral $\int_{-1}^{1} f(x)dx$.
Use this formula to evaluate $\int_0^2 \dfrac{1}{x^3 + 2x + 5}\, dx$.

[Turn over

Que. 6 (a) Using the least-squares method, obtain the normal equations to find the values of a, b, c and d when the curve $y = d + cx^3 + bx^6 + ax^9$ is to be fitted for the data points $(x_i, y_i), i = 1, 2, 3......., n$.

(b) Obtain the least squares quadratic approximation to the function $y(x) = \sin x$ on $[0, \pi/2]$ with respect to the weight function $W(x) = 1$.

Que. 7 (a) Derive the Adams-Bashforth predictor formula to solve the differential equation $\dfrac{dy}{dx} = f(x, y)$ with $y(x_0) = y_0$.

(b) Given $\dfrac{dy}{dx} = x^2 + y^2 - 2$ with $y(-0.1) = 1.0900,\ y(0) = 1.0000,\ y(0.1) = 0.8900$ and $y(0.2) = 0.7605$. Compute $y(0.3)$ correct upto four decimal places using Milne's predictor-corrector method.

Que. 8 (a) Derive a numerical method (Crank-Nicolson's method) to obtain the numerical solution of one dimensional heat equation with initial and boundary conditions.

(b) The Laplace equation $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$ satisfies the conditions $u(x, 0) = 0$, $u(x, 4) = 8 + 2x$, $u(0, y) = \dfrac{1}{2}y^2$ and $u(4, y) = y^2$. Using Liebmann's method find the values of $u(i, j)$, $i = 1, 2, 3;\ j = 1, 2, 3$, correct to two places of decimals.

Q. P. Code: 50899

(3 Hours) [Total Marks:100

Internal (Scheme B) (2 Hours) [Total Marks:40

Note:

(1) **External (Scheme A) students answer any five questions.**

(2) **Internal (Scheme B) students answer any three questions.**

(3) **All questions carry equal marks. Scientific calculator can be used.**

(4) **Write on top of your answer book the scheme under which you are appearing.**

Que. 1 (a) Explain the terms: Inherent error, Round-off error and Truncation error. Find the truncation error for e^x at $x = \frac{1}{5}$ and $x = \frac{1}{9}$ if the first three terms are retained in the expansion.

(b) Convert the hexadecimal number $(1F5.B)_{16}$ to the binary form and then convert to the decimal form.

Que. 2 (a) Define the term rate of convergence of iterative method and also find the rate of convergence of the Chebyshev method.

(b) Perform two iterations of the Bairstow method to extract a quadratic factor $x^2 + px + q$ from the equation $x^4 + x^3 + 2x^2 + x + 1 = 0$. Use initial approximations $p_0 = 0.5$, $q_0 = 0.5$.

Que. 3 (a) Describe the Jacobi's method to obtain eigen values and eigen vectors of a real symmetric matrix $A = [a_{ij}]$ of order $n \times n$.

(b) Solve the following system of linear equations using Gauss-Seidel iteration method. $6x + 15y + 2z = 72$; $x + y + 54z = 110$; $27x + 6y - z = 85$. (Take 5 iterations.)

Que. 4 (a) Define interpolating polynomial and estimate the error in the interpolating polynomial.

(b) Find the cubic Lagrange's interpolating polynomial from the following data:

x:	0	1	2	5
f(x):	2	3	12	147

Que. 5 (a) Derive Newton-Cotes quadrature formula and use it to derive Trapezoidal rule for numerical integration.

(b) Evaluate $\int_0^{1.5}\int_0^{1} e^{x+y}\,dxdy$ using Simpson's three eight rule with $h = k = \frac{1}{2}$.

Que. 6 (a) Using the least-squares method, obtain the normal equations to find the values of a, b, c and d when the curve $y = d + cx^2 + bx^3 + ax^4$ is to be fitted for the data points $(x_i, y_i), i = 1, 2, 3......., n$.

(b) Using the Chebyshev polynomials, obtain the least squares approximation of second degree for $f(x) = 3x^4 - 5x^3 + 11x + 22$ on $[-1, 1]$ with respect to the weight function $w(x) = \frac{1}{\sqrt{1-x^2}}$.

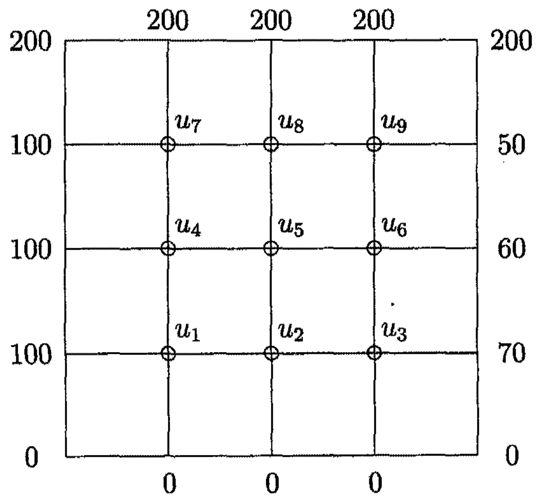
[Turn over

Que. 7 (a) Derive the Milne's predictor-corrector formula to solve the differential equation $\dfrac{dy}{dx} = f(x, y)$ with $y(x_0) = y_0$.

(b) Using Euler's modified formula, find an approximate value of $y(0.05)$ and $y(0.1)$, given that $\dfrac{dy}{dx} = x^2 + y$ with $y(0) = 1$. [Take $h = 0.05$].

Que. 8 (a) Derive a numerical method to find the numerical solution of one dimensional wave equation with initial and boundary conditions.

(b) Use Liebmann's method to solve the Laplace equation $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$ at the interior mesh points of the square region with boundary values given in the following figure.



[Take 2 iterations and obtain result correct upto three decimal places.]

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M.SC. (MATHS) PART-II
Graph Theory
(Rev) (JAN - 2019)

Q. P. Code : 40403

Marks: 80

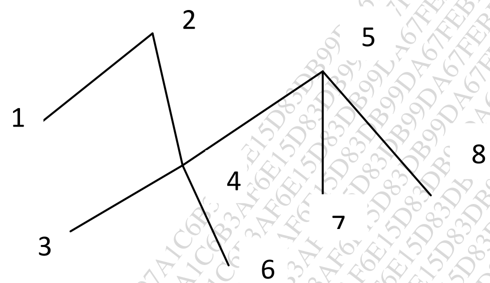
N.B. 1) Both the sections are compulsory.
2) Attempt **ANY TWO** questions from each section.

Section I

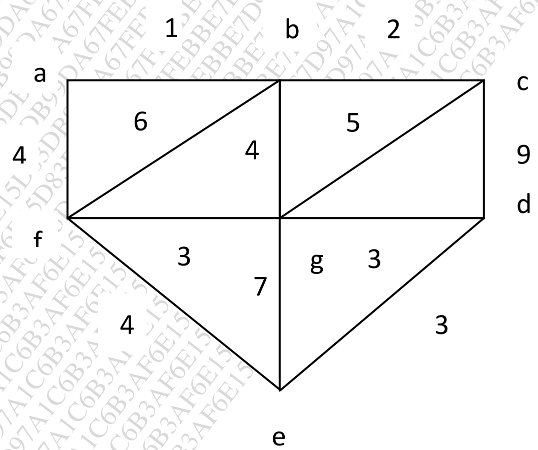
1. a) Let G be undirected graph. Then prove that G is bipartite if and only if it has no odd cycle. 10

b) Prove that the block graph of a connected graph is a tree. 10

2. a) Write an Algorithm of Prüfer coding and apply it on the given tree to find Prüfer sequence. 10



b) By applying Kruskal's algorithm find minimum cost spanning tree for the given weighted graph and prove that minimal cost spanning tree generated by Kruskal's algorithm is optimal. 10



a,b,c,d,e,f,g are vertices and 1,2,3,4,5,6,7,9 are the weights of edges.

TURN OVER

3. a) Prove that a graph is Eulerian if and only if it is connected and even. 10

   b) If a connected graph G has $n \geq 3$ vertices and degree of every vertex is at least $\frac{n}{2}$ then prove that it has Hamiltonian cycle. 10

4. a) For any graph G prove that $\alpha+\beta=p$. 10

   Where α: number of Independent set.,

   β: number of independent covering. and P: number of vertices.

   b) For Ramsay number R(p, q) prove that 10

   $R(p,q) \leq R(p-1,q)+R(p,q-1)$. If both summands on the right are even then prove that inequality is strict.

Section II

5. a) For any graph G, prove that $\chi(G) \leq \Delta(G)+1$. 10

   where $\chi(G)$: Chromatic number of graph G.

   $\Delta(G)$: Maximum degree of graph G.

   b) If two graphs are isomorphic then prove that their corresponding line graphs are also isomorphic. What can you say about the converse? Give justification. 10

6. a) State and prove Euler's formula for planar connected graph and show that complete graph on five vertices is non planar. 10

   b) Prove that every simple outer planar graph has a vertex of degree less than or equal to 2. 10

7. a) Prove that a diagraph D is unilaterally connected if and only if there is a directed walk not necessarily closed, containing all its vertices. 10

   b) Every Tournament D contains a directed Hamiltonian path. 10

8. a) Define Eigen value of a graph G and prove that if G be a connected graph with k distinct eigen values and let d be the diameter of G then k > d. 10

   b) Compute the spectrum of the complete bipartite graph with *m* vertices. 10

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M.SC. (MATHS) PART-II
Graph Theory
(Old) (JAN - 2019)

Q.P.Code:13372

(3 hours) Total Marks : 100

N.B. : 1) Answer any FIVE questions.

2) All questions carry EQUAL marks.

1. (a) Show that number of edges in complete bipartite graph $K_{m,n}$ is mn. Show further that number of edges in simple bipartite graph on p vertices is at most $p^2/4$..
   (b) Show that every (x, y) walk contains (x, y) path. Show further that any two longest paths in a a connected graph contains a vertex in common.
2. (a) Prove that number of spanning trees of K_n is n^{n-2}.
   (b) Let G be a $(p, p-1)$ graph. Then following conditions are equivalent ;
   (i)G connected,
   (ii)G is acyclic,
   (iii) G is tree.
3. (a) If G is Hamiltonian then show that for any proper non empty subset S of $V, \omega(G-S) \leq |S|$.
   (b) A graph G with $p \geq 3$ is 2-connected if and only if any two vertices of it are connected by at least two internally disjoint paths.
4. (a) Let M be a matching and Kbe a covering in a graph G such that $[M] = |K|$. Prove that M is maximum matching and K is minimum covering.
   (b) State Tutte's theorem on perfect matching. Show that every three regular graph without a cut edge has a perfect matching.
5. (a) Let $\pi_k(G)$ is number proper k colorings of G. Show that $\pi_k(G) = \pi_k(G-e) - \pi_k(G.e)$ for any edge e of G.
   (b) Define critical graph . Show that if G is k critical then it is connected and $\delta \geq k-1$.
6. (a) Define Eulerian graph. Show that a connected graph has an Euler trai if and only if it contains exactly two vertices of odd degree.
   (b) Define line graph of a graph. Show that line graph of connected graph is isomorphic to the graph if and only if it is cycle.
7. (a) State and prove Euler's theorem for planar graph. Hence show that every planar graph contains a vertex of degree at most five.
   (b) Define dual of a planar graph. Let G^*denote dual of a planar graph G. Show that if $G^* \cong G$ then $|E(G)| = 2|V(G)| - 2$. Construct such a graph on $n \geq 4$ vertices.
8. (a) For any two integers $\geq 2, l \geq 2$, show that $r(k,l) \leq r(k, l-1) + r(k-1, l)$. Show further that the inequality is strict if both the terms on the right side are even.
   (b) Show that $r(C_4, C_4) = 6$ and $r(K_{1,2}, C_4) = 5$.

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