

(Time: 3 Hours)

(Total Marks: 100)

N.B. 1. All questions are compulsory.

2. Attempt any two sub-questions from three sub-questions from Q.2, Q.3, Q.4, Q.5

3. Figures to the right indicate full marks.

4. Use of non-programmable calculator is allowed.

- Q.1 a. Attempt ANY FIVE sub-questions: (20)**
- i. A and A^c form partition of S. State true or false. Give reason.
 - ii. $P(A/B) \geq P(B)$. State true or false. Give reason.
 - iii. If a random variable x has a Uniform distribution with, range 1 to n, then its variance is ----
 - iv. The number of parameters involved in binomial p.d.f is/are.... State the parameter/s.
 - v. For a Poisson distribution Mean = 5 and Variance = 8. State true or false. Give reason.
 - vi. What is the median of a distribution in terms of Cumulative distribution function?
 - vii. In hypergeometric distribution units are selected with replacement.. State true or false. Give reason.
- b. Answer the following in one/ two sentences: (Any Five) (10)**
- i. If X is a discrete random variable then define Quartiles.
 - ii. Define a Bernoulli experiment.
 - iii. State mean of a hypergeometric distribution..
 - iv. If $P(A) = 1/12$, $P(B) = 1/3$ and $P(A \cap B) = 1/52$, find $P(A/B)$ and $P(B/A)$.
 - v. Give any 2 examples of a random variable which follows uniform distribution.
 - vi. When are two events mutually exclusive as well as independent?
 - vii. $\text{Cov}(X, Y) = 0$ implies that X and Y are two independent random variables. State true or false and give reason.
- Q.2 Attempt any TWO sub-questions: (20)**
- a. (05)**
 - i. Suppose A and B are independent events, defined on a sample space S, then prove that
 - A and B^c are independent
 - A^c and B are independent
 - ii. The odds that A speaks truth are 6:2 and odds that B speaks truth are 7:4. (05)
In what percentage of cases are A and B likely to contradict each other on an identical point?
 - b. (05)**
 - i. State and Prove Multiplication theorem of probability for two events A and B. (05)
 - ii. A bag contains 12 white and 18 black balls. Two balls are drawn in succession without replacement. What is the probability that first is white and the second is black? (05)

- c Widgets are manufactured in three factories A,B and C. The proportion of defective widgets from each factory are as follows. (10)
 Factory A : 0.01
 Factory B : 0.04
 Factory C : 0.02
 Factories A, B produce 30% of the widgets apiece, and the remaining 40% come from factory C. Imagine that an upset customer returns a defective widget to your company. As the manager finds out the probabilities that defective widget had come from 3 factories, which factory is liable for penalty?

Q.3

- Attempt any TWO sub-questions: (20)**
- a. i. Given Joint probability mass function of two discrete random variables, how does one examine whether they are independently distributed? (03)
 ii. A box contains 3 red marbles and 5 green marbles. Two marbles are taken at random without replacement. Let X be the number of green marbles obtained. Obtain the probability distribution of X and state the cumulative distribution function of X. (07)
- b. i. State and prove addition theorem of expectation for two random variables. (04)
 ii. Find the value of k if a random variable X has the following probability distribution: (06)

X	-2	-1	0	1	2	3
P(x)	0.1	K	0.2	2k	0.3	k

Hence obtain

- $P(X < 1)$
 - $P(X \geq 0)$
 - $P(-1 < X \leq 2)$
 - $P(-1 \leq X \leq 1)$
- c. A discrete random variable X has the following probability distribution (10)

X	1	2	3	4
P(x)	0.4	0.3	0.2	0.1

Obtain the first 3 raw moments. Hence find first three central moments. Also find mean, variance and coefficient of skewness.

Q.4

- Attempt any TWO sub-questions: (20)**
- a. i. Define a random variable X having discrete Uniform distribution. Derive its mean and variance. (07)
 ii. The mean and the variance of a binomial variate with parameters n and p are 2.4 and 1.44 respectively. Find $P(X=0)$ (03)
- b. If x is a Poisson variate with parameter “m”, write probability distribution of X. Derive mean and variance of X. (10)

- c. i. Derive recurrence relation for probabilities of binomial distribution with parameters n and p . (05)
- ii. A jar contains 25 pieces of candy of which 11 are yoghurt covered nuts and the rest are yoghurt covered raisins. Let X be the number of candies with nuts in a random sample of 7 pieces of candy selected without replacement. Compute $P(X > 5)$ (05)

Q.5 Attempt any TWO sub-questions: (20)

- a. i. Define Cumulative distribution function of a discrete random variable. Also state its properties. (05)
- ii. Five boys and 2 girls are to be seated in a row. Find the probability that
- Girls are together
 - Girls are not together
- (05)
- b. The joint p.m.f of random variables X and Y is as given below: (10)

$X \backslash Y$	1	2
1	0.1	0.2
2	0.2	0.1
3	0.3	0.1

Find correlation coefficient between X and Y

- c. i. Give three applications of Poisson distribution (03)
- ii. It is stated that 2% of optical lenses supplied by a manufacturer are defective. What is the probability that in a random sample of 200 lenses
- 3 or more are defective
 - At least two are defective
 - At most two are defective
- (Given $e^{-4} = 0.001832$, $e^{-0.4} = 0.6703$) (07)