(3 Hours)   [Total Marks: 100

Note: (*i*) All questions are compulsory.

(*ii*) Figures to the right indicate marks for respective parts.

Q.1 Choose correct alternative in each of the following: (20)

   i. The complete graph K_5 has ------- edges.
- (a) 9
- (b) 10
- (c) 11
- (d) 12

   ii. Fleury's Algorithm is used to construct ----
- (a) Euler path
- (b) Hamilton path
- (c) planar graph
- (d) shortest path

   iii. Pendant vertex in any graph has degree ----
- (a) 0
- (b) 1
- (c) 2
- (d) 3

   iv. Adjacency matrix of any simple graph is always ------- matrix
- (a) Row
- (b) Column
- (c) Square
- (d) Diagonal

   v. If a tree T has 100 vertices then the number of edges is
- (a) 100
- (b) 98
- (c) 101
- (d) 99

   vi. If a full binary tree has 10 internal vertices then the total no. of vertices is
- (a) 11
- (b) 20
- (c) 21
- (d) 10

   vii. In a tree, the number of path(s) between any two vertices is
- (a) two
- (b) Atmost two
- (c) one
- (d) Atleast one

   viii. A function $f(x)$ is said to be big $-O$ of $g(x)$ if there exist real constants c, k such that for all $x > k$ ....
- (a) $f(x) = c\, g(x)$
- (b) $g(x) \le c\, f(x)$
- (c) $|f(x)| \le c\, |g(x)|$
- (d) None of the above

   ix. Which of the following functions is of order less than n^2.
- (a) $n^2 - 7$
- (b) n^3
- (c) 1
- (d) None of the above

   x. To arrange the list of elements in ascending order we can use…
- (a) Linear search algorithm
- (b) BFS algorithm
- (c) Selection sort algorithm
- (d) None of the above

Q.2 *a)* Attempt any **ONE** question from the following: (08)

   i. Describe the Tower of Hanoi Problem. Discuss the solution for 3 discs and find the recursive algorithm for n discs.

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ii. Given an integer x and a list of n distinct integers in ascending order, write Binary Search algorithm for searching x in the list. Also take the trace of the algorithm for the following data: $x = 9, n = 5$, list: 3, 7, 10, 12, 15.
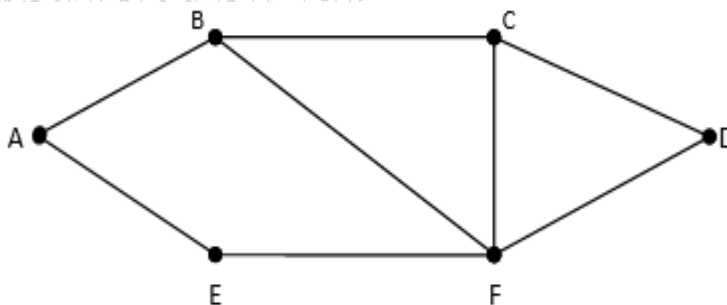
b) Attempt any **TWO** questions from the following: (12)

     i. Write the Euclid's algorithm to find the GCD of two positive integers a, b and trace your algorithm for $a = 15$ and $b = 12$.

     ii. Describe the characteristics of an Algorithm.

     iii. Design an algorithm to find first n terms of the Fibonacci sequence 0, 1, 1, 2, 3, 5, 8…. for a given $n \in \mathbb{N}$ (Give recursive or Iterative algorithm). Trace for $n = 4$.

     iv. Give an algorithm to find the minimum element of a finite set of n integers, and the position where it occurs first. Trace the algorithm for the set {7,2,1,9,1}.

Q.3 a) Attempt any **ONE** question from the following: (08)

   i. Find shortest path from b to e, for the following graph, using Dijkstra's algorithm.
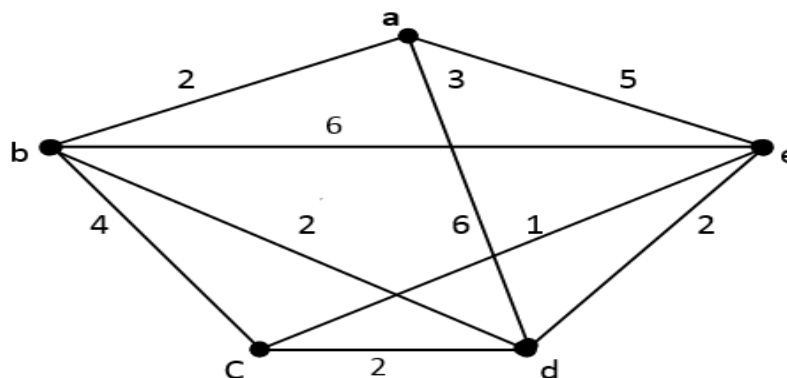
     ii. Write Fleury's algorithm, hence construct, Euler path for the following graph, using Fleury's algorithm.
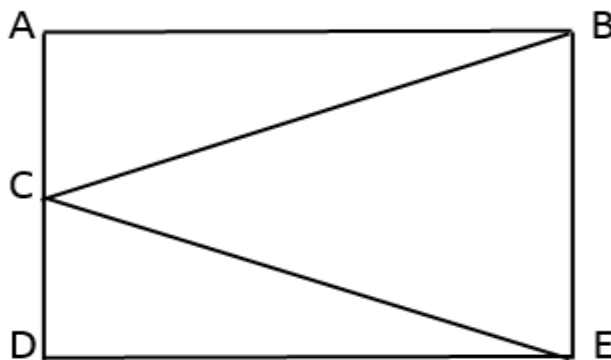

b) Attempt any **TWO** questions from the following: (12)

     i. Define Euler path and Euler circuit. Find least number of time it is necessary to lift a pencil from the paper when drawing the following graph. Justify your answer.

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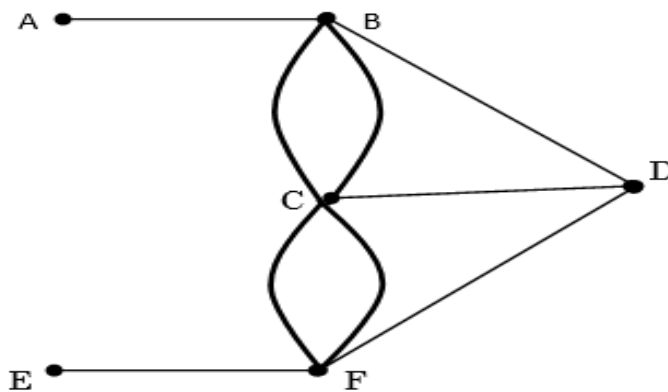
ii. State with an example: - Ore's theorem, Euler's formula for a planar graph and Kuratowskis theorem.

iii. For the following graph G

(p) Find four paths of length three each, from vertex C to vertex C.

(q) Find subgraphs $G - \{C\}$ and $G - \{B\}$.

(r) If a new vertex F and edges through F are added to the given graph, such that newly formed graph becomes a wheel, what will be the degree of vertex F?

iv. Represent graphs of $K_4, K_{1,3}$ and, $K_{2,2}$ by adjacency matrix and incidence matrix.

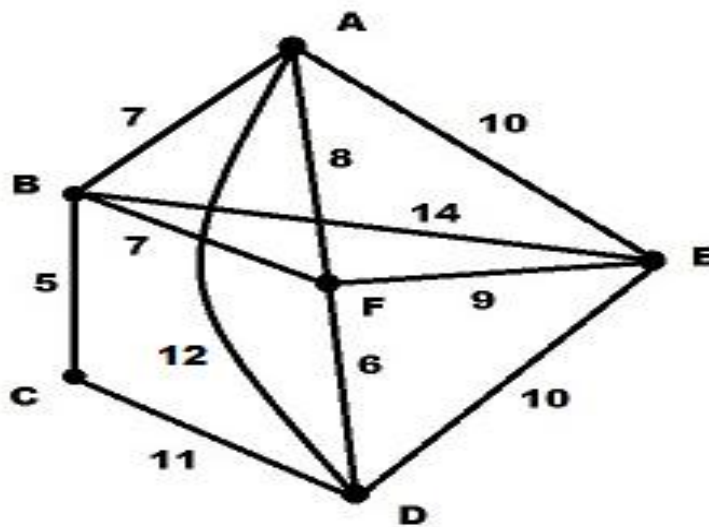Q.4 a) Attempt any **ONE** question from the following: (08)

i. Prove that a tree with n vertices has $n - 1$ edges.

ii. Use Huffman coding to encode the following symbols with the frequencies listed.
$A : 0.08, B : 0.10, C : 0.12, D : 0.15, E : 0.20, F : 0.35$
What is the average number of bits used to encode a character?

b) Attempt any **TWO** questions from the following: (12)

i. Define Tree, spanning tree and Full m-ary tree with an example for each.

ii. Prove that the number of vertices in a full Binary tree is always odd.

iii. Show that there is no tree with degree sequence (1,1,2,2,2,2,3,3,3,3,3,3).

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iv.   Use Kruskal's algorithm to find a spanning tree of minimum total weight in the graph below. Give the weight of your minimum spanning tree and show your steps.
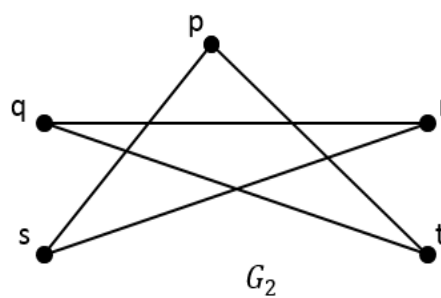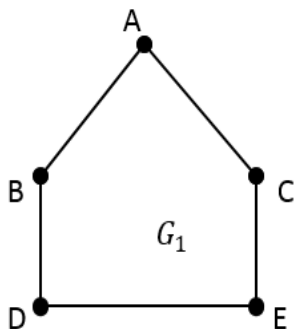

Q.5   Attempt any **FOUR** questions from the following: (20)

a)   Show that (i) $f(x) = x^2 + 2x + 1$ is big - O of x^2
      (ii) $f(x) = x^2$ is of order less than $g(x) = x^3$

b)   Design algorithms to exchange the values of a and b,
      (i) using temporary variable and (ii) without using temporary variable.

c)   Define Isomorphism of two graphs. Determine whether following graphs are Isomorphic, Justify your answer.

d)   Draw, if possible, the graph with 6 vertices with degree 1, 2, 2, 3, 3 and 4. If not possible, explain why such graph does not exist.

e)   Given the encoding scheme
      $a: 001, b: 0001, e: 1, r: 0000, s: 0100, t: 011, x: 01010$
      Find the word represented by
      (1)01110100011   (2)0001110000   (3)01001001011
      (4) 01100101010   (5) 0000001011

f)   Construct Binary search tree for following key sequence: -
      Jai, Guy, Joy, Anu, Jit, Evy, Amit, Tim, Ron, Kit, Toy, Roy, Kaj, Don.

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