M.SC (MATHS) PART-I (JUNE-2018)
ALGEBRA (REV-2016)
(PAPER-I) (JUNE- 2018)

Q. P. Code: 35867

Time: 3 Hours **Total Marks: 80**

Instructions:

- Attempt **any two** questions from **each section**
- **All questions** carry **equal marks**
- **Answer to section I and II** should be written on the **same answer book**.

SECTION I (Attempt any two Questions)

Q1. (a) Show that every linearly independent subset of a finitely generated vector space $V\ (F)$ is either a basis of V or can be extended to form a basis of V .

(b) Determine whether or not the following vectors form a basis of R^3.

(1,1,2) , (1,2,5) ,(5,3,4).

Q2. (a) Prove that two finite dimensional vector spaces over the same field are isomorphic if and only if they are of the same dimension.

(b) Let T be the linear operator on R^3 defined by

$T(x_1 , x_2 , x_3) = (3x_1 + x_3 , -2x_1 + x_2 , -x_1 + 2x_2 + 4x_3)$

Find the matrix of T in the ordered basis $(\alpha_1, \alpha_2, \alpha_3)$ where $\alpha_1 = (1,0,1)$,

$\alpha_2 = (-1,2,1)$, $\alpha_3 = (2,1,1)$.

Q3. (a) Show that similar matrices have the same minimal polynomial.

(b) Let T be the linear operator on R^3 which is represented in the standard basis by the

matrix $\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$. Prove that T is diagonalizable.

Q4. (a) Prove that if α and β are vectors in an unitary space,

$4(\alpha, \beta) = ||\alpha + \beta||^2 - ||\alpha - \beta||^2 + i||\alpha + i\beta||^2 - i||\alpha - i\beta||^2$

(b) Let W be any subspace of a finite dimensional inner product space V , then prove that

$V = W \oplus W^{\perp}$.

SECTION II (Attempt any two Questions)

Q5. (a) Show that the number of generators of a finite cyclic group of order n is $\phi(n), where\ \phi(n)$ is the Euler ϕ -function.

(b) If H and K are finite subgroup of a group G, then prove that

$$o(HK) = \frac{O(H)\ .\ O(K)}{O(H \cap K)}$$

Q6. (a) Show that every quotient group of a group is a homomorphic image of the group.

(b) If G is any finite group such that $p/o(G)$, where p is a prime number then show that G has an element of order p.

Q7. (a) Prove that a finite integral domain is a field.

(b) Let R be a commutative ring with unity. Prove that R is a field.

Q8. (a) Show that Z (set of all integers) is a P.I.D (Principal Ideal Domain).

(b) Show that an element in a U.F.D is prime if and only if it is irreducible.

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M.SC (MATHS) PART-I (JUNE-2018)
ALGEBRA (OLD)
(PAPER-I) (JUNE- 2018)

Q. P. Code: 11730

External (Scheme A) (3 Hours) Total Marks: 100
Internal (Scheme B) (2 Hours) Total Marks: 40

N.B.: Scheme A students should attempt any five questions.
Scheme B students should attempt any three questions.
Write the scheme under which you are appearing, on the top of the answer book.

Q1. (a) Show that every quotient group of a group is a homomorphic image of the group.

(b) Show that the number of generators of an infinite cyclic group is two.

Q2. (a) Using Sylow theorems prove that a group of order 28 is not simple.

(b) If G is any finite group such that p / o(G) , where p is a prime number then show that G has an element of order p.

Q3. (a) Prove that a group of order 99 is not simple.

(b) If Z is the Centre of a group G such that G/Z is cyclic, then show that G is abelian.

Q4. (a) Show that every Euclidian domain is a unique is a unique factorization domain.

(b) Show that if R is a unique factorization domain then the product of two primitive polynomial in R[x] is a primitive polynomial in R[x].

Q5. (a) If R is commutative ring with unity whose only ideals are {0} and R, then show that R is a field.

(b) Prove that J [$\sqrt{-3}$] is not a UFD. , J being the ring of integers.

Q6. (a) Show that similar matrices have the same minimal polynomial.

(b) Let T be a linear operator on a n dimensional vector space V (F) . Then show that T satisfies its characteristic equation.

Q7. (a) Let A is a linear transformation on a vector space V such that A^2 - A + I = 0.Then show that A is invertible.

(b) Find the Rank of the matrix. $\begin{bmatrix} 3 & -4 & -1 & 2 \\ 1 & 7 & 3 & 1 \\ 5 & -2 & 5 & 4 \\ 9 & -3 & 7 & 7 \end{bmatrix}$

Turn Over

Q8. (a) Find all (complex) characteristic values and characteristic vectors of the following matrix.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

(b) If α and β are vectors in an inner product space $V(F)$ and $a, b \in F$, then prove that

$$\text{Re}\,(\alpha, \beta) = \tfrac{1}{4} \| \alpha + \beta \|^2 - \tfrac{1}{4} \| \alpha - \beta \|^2$$

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M.SC (MATHS) PART-I (JUNE-2018)
ANALYSIS & TOPOLOGY (REV-2016)
(PAPER-II) (JUNE- 2018)

Q.P. Code : 35866

[Time: 3 Hours] **[Marks: 80]**

Please check whether you have got the right question paper.

N.B:
1. Attempt **any two** questions from **each section**.
2. All questions carry equal marks.
3. Answer to **section I and II** should be written in **same answer book**

Section – I (Attempt any two questions)

Q.1 a) Define metric space. If (X, d) is a metric space, then prove that in (X, d) :
- i) Union of open sets is open
- ii) Intersection of a finite number of open sets is open

b) Let (X, d) be a metric space. Show that the following statements are equivalent :
- i. A is dense in (X, d)
- ii. The only closed superset of A is X.
- iii. Every non-empty open set in (X, d) intersects A.
- iv. Every open ball in (X, d) intersects A.

Q.2 a) Let $f : (X_1, d_1) \to (Y_1, d_2)$ be a function. Prove that f is continuous at $x = p \in X$ iff whenever a sequence $(x_n) \in X$ converges to $p \in X$, the sequence $(f(x_n)) \in Y$ converges to $f(p) \in Y$.

b) Let f, g be continuous functions from (X, d) to $(\mathbb{R}, |\;|)$. Show that $A = \{x \in X : 2f(x) > 3g(x)\}$ is open in X.

Q.3 a) If $f : \mathbb{R}^n \to \mathbb{R}^m$ is differentiable at $a \in \mathbb{R}^n$ then show that its total derivative is unique.

b) Define partial derivative of a function. Find the partial derivatives of the function $f : \mathbb{R}^3 \to \mathbb{R}^2$ defined by $f(x, y, z) = (xy, z^2)$, if they exist.

Q.4 a) State and prove Mean value Theorem.

b) State Taylor's theorem for a function $f : \mathbb{R}^2 \to \mathbb{R}$. Find the Taylor expansions of the function $f(x, y) = x^3 + 2xy^2 - 3xy + 4x + 5$ about $(a, b) = (1, 2)$ up to the third order.

Section – II (Attempt any two questions)

Q.5 a) Define a topological space. Let $X = \mathbb{N}$, $I_n = \{1,2,3,\ldots,n\}$ and $J_n = \{n, n+1, n+2, \ldots\}$. Define $\tau = \{\phi, I_1, I_2, \ldots, \mathbb{N}\}$ and $\tau^1 = \{\phi, J_1 = \mathbb{N}, J_2, J_3, \ldots\}$. Prove that τ, τ^1 are both topologies on X.

b) There are 26 topologies on $X = \{a, b, c\}$. List at least 10 of them with justification.

TURN OVER

Q.6 a) Define a Hausdroff Topological space. Prove that every metric space is a Hausdroff space.

b) Show that continuous image of a connected space is connected.

Q.7 a) Define a compact space. Show that a finite union of compact sets is compact.

b) State Tube lemma. Let x, y be compact topological spaces. Prove that $X \times Y$ (with product topology) is also compact.

Q.8 a) Define local compactness. Is every locally compact set compact? Verify this for $\mathbb{R}$.

b) Define uniformly continuous function on a metric space. Show that $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \dfrac{1}{1+x^2}$ is uniformly continuous on $\mathbb{R}$.

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Q. P. Code: 11734

External (Scheme A) (3 Hours) Total Marks: 100
Internal (Scheme B) (2 Hours) Total Marks: 40

N.B.:Scheme A students should attempt any five questions.
Scheme B students should attempt any three questions.
Write the scheme under which you are appearing, on the top of the answer book.

Q1. (a) State and prove Blozano- Weierstrass theorem. [10]

(b) Show that every bounded monotonic sequence in R is convergent. [10]

Q2. (a) State and prove ratio test for convergence of a positive term series. [10]

(b) Discuss the convergence of $\sum_{n=0}^{\infty} \frac{Cos\ nx}{n^3+1}$. [05]

(C) State Leibnitz's test for alternating series and show that the series $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n}$ is conditionally convergent. [05]

Q3. (a) Find the total derivative of $f(x,y) = x^2\, y$ at (1,2) . [10]

(b) Find the partial derivative of $f(x,y) = \frac{xy}{x^2+y^2}$ at (0,0) given that f(0,0) = 0 . Discuss the continuity of f at (0, 0). [10]

Q4. (a) State and Prove Mean value theorem for a differentiable real value function of a two variable. [10]

(b) Expand using Taylor's theorem $f(x,y) = e^{x+y}$ at (0,0) upto and including second degree terms.

Q5. (a) If [a,b] is the closed interval in R and $f: [a,b] \to R$ is continuous , Prove that f is Riemann integerable on [a,b] . [10]

(b) When is a function $f: [0,1] \to R$ is said to be of bounded variation ? Show by means of an example that a continuous function need not to be of bounded variation . [10]

Q6. (a) If f is continuous on a [a,b] and if $F[X] = \int_a^x f(t)dt$ prove that $F'[X] = f(x)\ \forall\, x \in [a,b]$. [10]

Turn Over

(b) Find the extreme values of $f(x,y) = x^3\,y - y^2\,x^2$. [10]

Q7. (a) State and Prove Fubini's theorem for a double integral of a continuous real valued Function *f(x,y)* on a rectangle in *xy* plane . [10]

(b) Sketch the region of integration and evaluate using polar coordinates

$\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}}(x^2 + y^2)dydx.$ [10]

Q8. (a) State only Inverse Function theorem and use it to prove that if $f: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is one-one, continuously differentiable and has invertible Jacobian matrix at each point the *f* is an open mapping and $f^{-1}: f(\mathbb{R}^2) \rightarrow \mathbb{R}^2$ is differentiable. [10]

(b) Discuss the convergence of $\int_0^1 \frac{dx}{x\sqrt{1-x}}$ stating clearly the result used. [10]

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M.SC (MATHS) PART-I (JUNE-2018)

TOPOLOGY (OLD)

(PAPER -III) (JUNE- 2018)

QP Code : 21071

Scheme A (External)]	(3 Hours)	**[Total Marks:100**
Scheme B (Internal)]	(2 Hours)	**[Total Marks: 40**

Instructions:

- **Scheme A** students should attempt **any five** questions.
- **Scheme B** students should attempt **any three** questions.
- **All** questions carry **equal marks.**
- **Mention** clearly the **Scheme** under which you are appearing.

Q. 1. (a) (i) State without proof Zorn's lemma. (5)

(ii) If C is a relation on a set A, define a new relation D on A by $(b,a) \in D$ if $(a,b) \in C$. Show that C is symmetric if and only if $C = D$. (5)

(b) State and prove the Schröder-Bernstein theorem. **(10)**

Q. 2. (a) Let X, Y be topological spaces. Let $f : X \to Y$. Prove that the following are equivalent:

(i) f is continuous.

(ii) For every subset A of X, $f(\overline{A}) \subset \overline{f(A)}$.

(iii) For every closed subset B of Y the set $f^{-1}(B)$ is closed in X.

(10)

(b) (i) State the first countability axiom. Give an example of a space which satisfies the first countability axiom with correct reasoning. (5)

(ii) Define basis of a topological space. Prove that if X is any set, the collection of all one-point subsets of X is a basis for the discrete topology on X. (5)

Q. 3. (a) Let $(X, d), (Y, d')$ be metric spaces. Let $f : X \to Y$ be a continuous onto map. Prove that X is compact if and only if Y is compact. **(10)**

(b) (i) Prove that if in a metric space every sequence has a convergent subsequence then it is compact. (5)

(ii) Show that an open connected subset of $\mathbb{R}$ is path-connected. (5)

Q. 4. (a) Compute the first fundamental group of the sphere S^2. State clearly all results used. (10)

(b) (i) Define path-homotopy and prove that it is a symmetric relation. (5)

(ii) Prove that any covering map is an open map. (5)

[TURN OVER

Q. 5. (a) (i) Define Hausdorff topological space. Give an example of a space which is not Hausdorff with correct justification. (5)

(ii) Define T_1 topological space. Prove that a topological space X is T_1 if and only if for each $x \in X$, the singleton set $\{x\}$ is closed. (5)

(b) Define the terms open map, quotient map. Let X, Y be topological spaces and $p : X \to Y$ be a surjective, continuous, open map. Prove that p is a quotient map. (10)

Q. 6. (a) (i) Give an example of a covering map and a covering space with justification. (5)

(ii) State (without proof) homotopy lifting theorem. (5)

(b) Define homeomorphism. Determine whether $\mathbb{R}$ and $\mathbb{R}^2$ are homeomorphic topological spaces. (10)

Q. 7. (a) (i) Define T_2 topological space. Prove that if a space X is T_2, then the diagonal $\{(x, x) \in X \times X\}$ is closed in $X \times X$. (5)

(ii) Define connected topological space. Give an example of a topological space which is not connected with correct justification. (5)

(b) Define path-connectedness. Let X, Y be topological spaces with X path-connected. Let $f : X \to Y$ be continuous. Then prove that $f(X)$ is path-connected. (10)

Q. 8. (a) (i) Define the terms: second countable space, Lindelöff space. (5)

(ii) Define contractible space. Show that a contractible space is path-connected. (5)

(b) State tube lemma (without proof). Prove that if X, Y are compact topological spaces, then so is $X \times Y$. (10)

M.SC (MATHS) PART-I (JUNE-2018)
COMPLEX ANALYSIS (REV -2016)
(PAPER -IV) (JUNE- 2018)

Q. P. Code: 28234

Time: 3 Hours **Total Marks: 80**

Instructions:

- Attempt **any two** questions from **each section**
- **All questions** carry **equal marks**
- **Answer to section I and II** should be written on the **same answer book**.

SECTION I (Attempt any two Questions)

1) (a) Construct the Stereographic Projection Map.
 (b) Solve $z^5+32=0$.

2) (a) Prove that a Mobius Transformation is a composition of translation, rotation, inversion, and magnification.
 (b) Prove that the circle $|z-2|=3$ is mapped onto a circle $\left|w+\frac{2}{5}\right|=\frac{9}{25}$ under the transformation $w=\frac{1}{z}$.

3) (a) Let $0\notin G$ be an open connected set in $\mathbb{C}$ and suppose that $f:G\to\mathbb{C}$ is analytic. Then prove that f is a branch of logarithm if and only if $f'(z)=\frac{1}{z}$, $\forall z\in G$ and $e^{f(a)}=a$ for atleast one $a\in G$.
 (b) If $f(z)=u+iv$ is analytic and $u-v=e^x(\cos y-\sin y)$. Find $f(z)$ in terms of z.

4) (a) Let γ be such that $\gamma(t)=\gamma_1(t)+i\gamma_1(t)$ be a smooth curve and suppose that f is a continuous function on an open set containing $\{\gamma\}$. Then prove that

 (i) $\int_{-\gamma} f(z)dz=-\int_{\gamma} f(z)dz$

 (ii) $\left|\int_{\gamma} f(z)dz\right|\le\int_{\gamma}|f(z)||dz|$

 (iii) If $M=\underset{t\in[a,b]}{Max}|f(\gamma(t))|$ and $L=L(\gamma)$ (length of γ) then $\left|\int_{\gamma} f(z)dz\right|\le ML$

 (b) Evaluate $\int_0^{1+i} z^2\,dz$ along

 (i) The line $y=x$

 (ii) Along the parabola $y=x^2$. Is the integral independent of path?

SECTION II (Attempt any two Questions)

5) (a) State and prove Cauchy's Integral Formula.

(b) Evaluate $\int_C \frac{\sin^6 z}{(z-\pi/2)^3}\,dz$, using Cauchy's Integral Formula where C is the circle $|z| = 2$.

6) (a) Let f be meromorphic in a domain G and have only finitely many zeroes and poles. If γ is any simple closed curve in G such that no zeroes or poles of f lie on γ, prove that $\frac{1}{2\pi i}\int_\gamma \frac{f'(z)}{f(z)}\,dz = Z_f - P_f$, where Z_f *and* P_f denote respectively the number of zeroes and poles of f inside γ each counted according to their order.

(b) State and prove the Minimum Modulus Principle.

7) (a) Define (i) Essential Singularity (ii) Pole. Find the poles of *cosech 2z* within $|z| = 4$.

(b) Find all the possible Laurent Series expansions of $f(z) = \frac{1}{z(z+1)(z-2)}$.

8) (a) State and prove Rouche's theorem.

(b) Use Argument Principle to evaluate $\int_{|z|=\pi} \cot(\pi z)\,dz$.

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Q. P. Code: 28236

External (Scheme A) (3 Hours) Total Marks: 100
Internal (Scheme B) (2 Hours) Total Marks: 40

N.B.:
1. Scheme A students should attempt any five questions.
2. Scheme B students should attempt any three questions.
3. Write the scheme under which you are appearing, on the top of the answer book.

1) (a) Construct the Stereographic Projection Map.
   (b) Solve $z^5 + 32 = 0$.

2) (a) Prove that a Mobius Transformation is a composition of translation, rotation, inversion, and magnification.
   (b) Prove that the circle $|z - 2| = 3$ is mapped onto a circle $\left|w + \frac{2}{5}\right| = \frac{9}{25}$ under the transformation $w = \frac{1}{z}$.

3) (a) Let $0 \notin G$ be an open connected set in $\mathbb{C}$ and suppose that $f : G \to \mathbb{C}$ is analytic. Then prove that f is a branch of logarithm if and only if $f'(z) = \frac{1}{z}$, $\forall z \in G$ and $e^{f(a)} = a$ for atleast one $a \in G$.
   (b) If $f(z) = u + iv$ is analytic and $u - v = e^x(\cos y - \sin y)$. Find $f(z)$ in terms of z.

4) (a) Let γ be such that $\gamma(t) = \gamma_1(t) + i\gamma_1(t)$ be a smooth curve and suppose that f is a continuous function on an open set containing $\{\gamma\}$. Then prove that

   (i) $\int_{-\gamma} f(z)dz = -\int_{\gamma} f(z)dz$

   (ii) $\left|\int_{\gamma} f(z)dz\right| \le \int_{\gamma} |f(z)||dz|$

   (iii) If $M = \underset{t\in[a,b]}{Max}\left|f(\gamma(t))\right|$ and $L = L(\gamma)$ (length of γ) then $\left|\int_{\gamma} f(z)dz\right| \le ML$

   (b) Evaluate $\int_0^{1+i} z^2\, dz$ along

   (i) The line $y = x$

   (ii) Along the parabola $y = x^2$. Is the integral independent of path?

5) (a) State and prove Cauchy's Integral Formula.

(b) Evaluate $\int_C \frac{\sin^6 z}{(z-\pi/2)^3} dz$, using Cauchy's Integral Formula where C is the circle $|z| = 2$.

6) (a) Let *f* be meromorphic in a domain G and have only finitely many zeroes and poles. If γ is any simple closed curve in G such that no zeroes or poles of f lie on γ, prove that $\frac{1}{2\pi i}\int_\gamma \frac{f'(z)}{f(z)} dz = Z_f - P_f$, where Z_f *and* P_f denote respectively the number of zeroes and poles of *f* inside γ each counted according to their order.

(b) State and prove the Minimum Modulus Principle.

7) (a) Define (i) Essential Singularity (ii) Pole. Find the poles of *cosech 2z* within $|z| = 4$.

(b) Find all the possible Laurent Series expansions of $f(z) = \frac{1}{z(z+1)(z-2)}$.

8) (a) State and prove Rouche's theorem.

(b) Use Argument Principle to evaluate $\int_{|z|=\pi} \cot(\pi z) dz$.

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Q. P. Code : 40992

Duration: 3 Hours] [Marks: 80

N.B. 1) Attempt any two questions from section - I and any two questions from section - II.
2) All questions carry equal marks.
2) Answers of section I and section-II should be written in **different** answer books.

SECTION-I (Attempt any two questions)

1. (a) Determine which of the following is true for all sets A and B (Justify your answer): (10)
   i. $A - (A - B) = B$.
   ii. $(A \cap B) \cup (A - B) = A$

   (b) Determine whether following argument and their conclusion is valid logical inference or not? Justify your answer. (5)

   If pair of angles A and B are right angles then they are equal. The angles A and B are equal. Hence, the angles A and B must be right angles.

   (c) Show that "If an integer a is such that $(a-2)$ is divisible by 3, then (a^2-1) is also divisible by 3" using direct proof. (5)

2. (a) Define finite set. Show that if a set A is finite then there is no bijection of A with a proper subset of itself. (5)

   (b) Show that if $B \subset A$ and if there is an injection $f : A \to B$ then A and B have the same cardinality. (5)

   (c) Let A be a set then show that there is no injective map $f : P(A) \to A$ and there is no surjective map $g : A \to P(A)$ is a power set of A. (10)

3. (a) If $S \subseteq \mathbb{Z}_+$ such that (i) $1 \in S$; (ii) $n \in S$ implies $n+1 \in S$. Show that $S = \mathbb{Z}_+$. ($\mathbb{Z}_+$ is the set of all positive integers) (10)

   (b) If R is a partial ordering relation on A and S is partial ordering relation on B where A and B are disjoint sets. Is $R \cup S$ is partial ordering relation on the set $A \cup B$? Justify. (10)

4. (a) Define even permutation and odd permutation. Show that every permutation is either even or odd but not both. (10)

   (b) Show that order of a permutation of a finite set written in disjoint cycles is the least common multiple of the lengths of the cycles. (10)

[TURN OVER

SECTION-II (Attempt any two questions)

5. (a) Define probability space. If P and Q are probability measures on a measurable space, examine if $H = \alpha P + (1-\alpha)Q$, $0 < \alpha < 1$ is a probability measure. (05)

   (b) Show that Borel sigma field contains all intervals of the type $[x, \infty)$. (05)

   (c) $\{A_n\}$ is a sequence of events where (05)

   $$A_n = \begin{cases} A, & n = 1, 3, 5 \ldots \\ B, & n = 2, 4, 6 \ldots \end{cases}$$

   Find Limit superior and limit inferior. Show that $\lim A_n$ does not exist

   (d) The coefficients a, b, c of a quadratic equation $ax^2 + bx + c = 0$ are obtained by throwing three fair dice. What is the probability that equation has real roots? (05)

6. (a) State and prove any two properties of probability function. (05)

   (b) Distinguish between a)Field and sigma field b) Limit superior and limit inferior. (05)

   (c) Define $P(A)$ as $P(A) = \frac{1}{3}\delta_1(A) + \frac{2}{3}P_2(A)$. Then obtain $P(0, 0.7]$ if P_2 has density $f(x) = 2x,\ 0 < x < 1$. (05)

   (d) Ram selects any of the two routes A or B for going to the college with equal probabilities. He has to face heavy traffic on routes 30% of times if he chooses route A ,and 40% of times if he chooses route B. What is the probability that on a randomly selected day (a)he faces heavy traffic (b)he has selected route B given that he faced heavy traffic. (05)

7. (a) X has Poisson distribution with parameter λ ,find its mean and variance. (05)

   (b) For any r.v.'s X, Y show that $E[X+Y]2 \leq [\sqrt{(E(X^2)} + \sqrt{(E(Y^2)}]^2$. (05)

   (c) State properties of Characteristic function. (05)

   (d) Verify independence if the $P(0,0) = \dfrac{1}{9}, P(1,1) = \dfrac{1}{9}, P(0,1) = \dfrac{5}{9}$ and $P(1,0) = \dfrac{2}{9}$. Obtain conditional pmf of Y given $X = 1, V(Y|X = 1)$ (05)

8. (a) Show that $EyE[X/Y = y] = E[X]$ (05)

   (b) Show that $P[|X - E(X)| > C] \leq \sigma^2/C^2$ where σ^2 is variance of X and $C > 0$ (05)

   (c) A sample of size 36 is taken from this exponential population with mean 4 what will be the distribution of sample mean $\overline{x}$? Hence find $P[\overline{x} < 4.5]$. Given $P[Z < 0.125] = 0.548, P[Z < 0.75] = 0.758$, where Z has $N(0, 1)$. (05)

   (d) Examine whether the Weak law of large numbers holds for sequence of independent r.v.'s $\{X_k\}$: $X_k = \begin{cases} \pm k & \text{with probability } \dfrac{1}{2k^2} \\ 0 & \text{with probability } 1 - \dfrac{1}{k^2} \end{cases}$. (05)

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M.SC (MATHS) PART-I (JUNE-2018)

COMBINATORICS

(PAPER -V) (JUNE- 2018)

QP Code : 21067

Scheme A(External)	(3 Hours)	Total marks: 100
Scheme B(Internal/External)	(2 Hours)	Total marks: 40

N.B: 1) Scheme A students answer any five questions.
2) Scheme B students answer any three questions.
3) All questions carry equal marks.
4) Write on the top of your answer book the scheme under which you are appearing.

1. (a) State and prove Pascal's identity.
(b) 9 gentleman and 5 ladies be seated around table. In how many ways can the gentleman and ladies be made to sit in alternate seats.

2. (a) Find the number of integer solutions to the equation $x_1 + x_2 + x_3 = 11$, where $x_1 \leq 3$, $x_2 \leq 4$ and $x_3 \leq 6$.
(b) Show that (i) $S(n,2) = 2^{n-1} - 1$, (ii) $S(n,2) = {}^nC_2$, for $n \geq 2$, where $S(n,k)$ denotes Stirling numbers of second kind.

3. (a) How many positive integers between 1 and 160 are not divisible by either 2 or 3 or 5?
(b) Write any two applications of Pigeon hole principle, with justification.

4. (a) Compute the Mobius function of the linearly ordered set $(X_n, \leq)$ where $X_n = \{1,2,\ldots\ldots,n\}$.
(b) Solve the recurrence relation $a_n - 7a_{n-1} + 15a_{n-2} - 9a_{n-3} = 0$ given that $a_0 = 1$, $a_1 = 2$and $a_2 = 3$.

5. (a) How many eight digit numbers can be formed having digits only from $\{1,3,5,7,9\}$ in which 3 appears twice and 5 appears at least 5 times?
(b) Define Derangement (D_n) of finite objects. Show that $D_n - n\,D_{n-1} = (-1)^n, \forall\, n \geq 1$

6. (a) State and prove Baye's theorem.
(b) For independent Random variables X, Y show that $E(XY) = E(X)E(Y)$, where $E(XY)$, $E(X)$ and $E(Y)$ are expectations of XY, X and Y respectively.

7. (a) A student has to appear for an examination consisting of 3 questions selected randomly from a list of 100 questions. To pass he needs to answer all three questions. What is the probability that student will pass the examination if he knows the answers of 90 questions on the list. Make use of hypergeometric distribution.
(b) Let X and Y be independent random variables with binomial distributions $B(m,p)$ and $B(h,p)$ respectively. What is the distribution of $X+Y$?

8. (a) Define expectation of discrete random variable. Prove that :
(i) $E(X+b) = E(X)+b$, (ii) $E(aX) = aE(X)$, (iii) $E(X-a)^2$ is minimum for $a = E(X)$
(iv) $E(b) = b$, (v) $E(X+Y) = E(X)+E(Y)$
(b) Define variance of a discrete random variable. Compute the variance of random variable with normal distribution.

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Con. : 84-17.

M.SC (MATHS) PART-I (JUNE-2018)

DISCRETE MATHEMATICS & DIFFERENTIAL EQUATIONS (REV-2016)

(JUNE- 2018)

Q.P. Code 37367

(3 Hours) [Total marks: 80]

Instructions:

1) Attempt any two questions from each section.
2) All questions carry equal marks.
3) Answer to Section I and section II should be written in the same answer book.

SECTION-I (Attempt any two questions)

Q.1]

A) Prove that '$a \equiv b \pmod{n}$ if and only if a and b leave the same reminder when divided by n'. [10]

B) The sum of two positive integers is 100. If one is divided by 7, the reminder is 1, and if the other is divided by 9 the remainder is 7. Find the numbers. [10]

Q.2]

A) i) Prove that $S(n, 2) = 2^{(n-1)} - 1$. [05]

ii) Find the coefficient of $y^2z^2x^5$ and y^2z^2w. Find number of terms and sum of all coefficients in the expansion of $(w + x + y + z)^5$. [05]

B) State and prove principle of inclusion and exclusion. [10]

Q.3]

A) A train required 15 hours to complete the journey of 972 kms. from Pune to Indore. It is known that the speed of the train was 50 kms /hr. in first 3 hours and 40 kms /hr. in last 3 hours. Show that the train must have travelled at least 234 kms. Within a certain period of three consecutive hours. [10]

B) Show that every sequence of n^2+1 distinct real numbers contains a subsequence of length n+1 that is either strictly increasing or strictly decreasing. [10]

Q.4]

A) Show that a graph G is bipartite if and only if every circuit in G has even length. [10]

B) Find the disjunctive normal to the function $F(x, y, z) = (x \vee y) \wedge \bar{z}$. [10]

SECTION-II (Attempt any two questions)

Q.5]

A) Let f be continuous function on $\mathbb{R}$ satisfying Lispchitz condition , if $\emptyset$ and φ are two solution of initial value problem $y' = f(x,y)$; $y(x_0) = y_0$ on an interval I containg x_0 then show that $\emptyset(x) = \varphi(x)$. [10]

B) Compute the first four successive approximation, if $y' = x^2 + y^2$; $y(0) = 0$. [10]

Q.6]

A) Show that solution matrix $\emptyset$ of $y' = A(x)y$ is fundamental matrix if and only if $\det(\emptyset) \neq 0$. [10]

B) Solve: $\frac{dx}{dt} = 2x + 4y;\quad x(0) = 4,\qquad \frac{dy}{dt} = x - y;\quad y(0) = 5.$ [10]

Q.7]

A) Solve $(1 - x^2)y'' - 2xy' + p(p+1)y = 0$ using power series. [10]

B) Let $u(x)$ be any non-trivial solution of $u'' + q(x)u = 0$ where $q(x) > 0, \forall x > 0$ if $\int_1^\infty q(x)\,dx = \infty$ then show that $u(x)$ has infinitely many zeros on the positive x-axis. [10]

Q.8]

A) Solve $u_x.u_y = u;\ \ u(x,0) = x^2$. [10]

B) Find the following integral surface generated by the partial differential equation:
$x(y^2 + z)z_x - y(x^2 + z)z_y = (x^2 - y^2)z$, which contains the straight line $x + y = 0,\ z = 1$. [10]