(3 hours) **Total marks: 60**

N.B.

1. **Attempt any two questions from question numbers 1, 2, 3 and any two questions from question numbers 4, 5, 6.**
2. **Figures to the right indicate full marks**
3. **Simple non-programmable calculator is allowed.**

1 a. State the postulates of a Poisson process. If $\{N(t)\}$ is a Poisson process and $s < t$, then find $P(N(s) = k \mid N(t) = n)$. (05)

b. According to a survey, earthquake with magnitude at least seven occur on an average 18 times a year worldwide. What is the probability that two consecutive such earthquakes are at least 18 months apart. (05)

c. Customers arrive at a bank according to a Poisson process with rate 20 per hour. Given that 3 customers arrive in the first five minutes, what is the probability that two customers arrive in the first three minutes. (05)

2 a. Define Poisson cluster process by stating the assumptions. Obtain probability generating function of M(t) for the above process, where M(t) denotes total number of occurrences in an interval of length t. (08)

b. Customers arrive at a store in a group of 1 or 2 individuals with equal probability and arrival of groups is in accordance with a Poisson process with mean rate λ. Obtain probability generating function of M(t). Also obtain mean number of customers arriving in time t. (07)

3 a. Describe acceptance – rejection method of simulating continuous random variables. Give a method to generate random variable from beta distribution B (2, 4). (06)

b. Show that the use of antithetic variables will lead to a reduction in variance whenever $g(.)$ is a monotone function while computing $\theta = E(g(X))$. (09)

4 a. Suppose a lifetime random variable T has density function $f(t)$ and hazard function $h(t)$. Show that, first order derivative of $h(t)$ can be expressed as, (05)

$$h'(t) = h(t)[h(t) - \varphi(t)]$$

where $\varphi(t) = -\frac{f'(t)}{f(t)}$ and hence identify nature of hazard function of a lifetime random variable having survival function,

$$\bar{F}(t) = \begin{cases} e^{[1-e^{\theta t^{\alpha}}]}, & t > 0 \,;\, \alpha > 0 \,;\, \theta > 0 \\ 1, & \text{otherwise} \end{cases}$$

b. Let T be a lifetime random variable with survival function $\bar{F}$(t) and distribution function $F(t)$. If $F(t)$ belongs to IFR class of distributions then show that $\bar{F}(t)^{\frac{1}{t}}$ is decreasing function of time. (06)

c. Suppose a lifetime random variable T follows Weibull distribution with scale parameter α and shape parameterβ. Obtain hazard function of T and hence obtain nature of hazard function. (04)

5 a. Define, (06)

i. Quantile function
ii. Empirical quantile function.

Suppose a lifetime random variable T follows log-normal distribution, give the steps involved for obtaining Q-Q plot and explain how the parameters can be estimated based on this plot.

b. Define, (05)

i. Path set (PS).
ii. Cut set (CS).
iii. Minimal Cut Set (MCS).
iv. Minimal Path Set (MPS).
v. Structure function of a system.
vi. Monotone system.

c. i. Obtain structure function of a monotone system of n components when p MPS are given. (04)

ii. Obtain structure function of a monotone system of n components when c MCS are given.

6 a. Define hazard function and cumulative hazard function. Suppose the hazard function of the guidance system for on-board control of a space vehicle is thought to follow the following power function of time: (07)

$$h(t) = \alpha\mu t^{\alpha-1} + \beta\gamma t^{\beta-1}, t > 0 \,;\, \alpha > 0, \beta > 0, \mu > 0, \gamma > 0 \,.$$

Determine the reliability of the system. Does the system appear to have an underline structure of two separate components?

b. Define (08)

i) Series system
ii) Parallel system and
iii) k-out-of-n system.

Obtain reliability of the each of the above system of n components, when lifetime of each of the component is exponential with mean θ.

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