Q. P. Code: 28231

(3 Hours) [Total marks: 80]

Instructions:

1) Attempt any two questions from each section.
2) All questions carry equal marks.
3) Answer to Section I and section II should be written in the same answer book.

SECTION-I (Attempt **any two** questions)

1. A) Solve the linear Diophantine equations $247x + 91y = 39$.. [10]
   B) State and prove Euler's criterion for quadratic residue of p. [10]

2. A) Let $r, n \in N$ and $l = \min\{r, n\}$. then show that $n^r = \sum_{k=1}^{l} C(n,k)k!\,S(r,k)$. [10]

   B) For $n \in N$, How many square free integers do not exceed n? [10]

3. A) Give any sequence of $mn + 1$ distinct real numbers then prove that there exist either an increasing sequence of length $m + 1$ or decreasing sequence of length $n + 1$ or both. [10]
   B) During a month with 30 days a baseball team plays at least one game a day, but on more than 45 games. Show that there must be a period of some number of consecutive days during which the team must play exactly 14 games. [10]

4. A) Let G be a graph and e be an edge. Then show that e is a cut edge if and only if e is not on a cycle. [10]
   B) Let $d_1 \leq \cdots \ldots . \leq d_n$ be the vertex degrees of G. Suppose that, for each $k < {}^{n}\!/_{2}$ with $d_k \leq k$, the condition $d_{n-k} \geq n - k$ holds. Then, prove that G is Hamiltonian. [10]

P.T.O….

SECTION-II (Attempt any two questions)

5. A) State and prove Gronwall's inequality to the uniqueness of the solution of the initial value problem. [10]

   B) Obtain approximate solution to with in t^5 of the initial value problem

   $\frac{dx}{dt} = xt + t^2 y, \ \ x(0) = 1.$

   $\frac{dy}{dt} = xy + t, \quad y(0) = 2.$ [10]

6. A) If $\emptyset_1(x)$ is a solution of $L_2(y) = 0$ on an interval I and $\emptyset_1(x) \neq 0$ on I then show that the other linearly independent solution of $L_2(y) = 0$ is $\emptyset_2(x) = \emptyset_1(x) \int_{x_0}^{x} \left[\frac{1}{\emptyset_1(t)^2} e^{-\int a_1 t dt}\right] dt.$ [10]

   B)Solve the following IVP.

$$\frac{dx}{dt} = 2x + y + z, \quad x(1) = 1$$
$$\frac{dy}{dt} = 2y + 2z, \quad y(1) = 2$$
$$\frac{dz}{dt} = 2z \qquad z(1) = 3.$$ [10]

7. A)Show that the Legendre polynomial $P_n(x)$ of degree n is given by

   $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n.$ [10]

   B) Obtain solution in the form of power series of the following Differential equation:

   $\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + (4t^2 - 2)x = 0.$ [10]

8. A) Solve $\frac{\partial u}{\partial y} + c\frac{\partial u}{\partial x} = 0, \ u = u(x, y)$ with $u(x, 0) = h(x)$ for a given $h: \mathbb{R} \to \mathbb{R}$. [10]

   B) Solve $u_x . u_y = u, \ u(x, 0) = x^2.$ [10]

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