Q.P. Code: 27423

(3Hours) **[Total Marks: 80]**

Note:

- **Question No. 1 is compulsory.**
- Answer any **three** from the remaining five questions.
- Assume suitable data if necessary and justify the same.
- Figures to the right indicate the marks.

Q1. a Why QR factorization is important? 5

b Define eigen value and eigen vector. 5

c How subspace is different from subset? 5

d Prove that if cholesky factorization exists then the matrix is positive definite. 5

Q2. a If $A = \begin{bmatrix} 5 & 5 & 10 \\ 5 & 3 & 5 \\ 10 & 5 & 4 \end{bmatrix}$ and b= $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ Solve Ax=b using PLU factorization. Compare PLU factorization with simple LU factorization? 15

b Define Singular Value Decomposition. 5

Q3. a Explain Gauss Seidal iterative technique to solve sparse linear systems. Compare it with Jacobi method. 10

b Determine the values of **'b',** at which the following system of equations are consistent. Under such condition how many solutions exist? Justify your answer. 10

$3x_1+2x_2+5x_3=4$

$2x_1+2x_2+4x_3=\mathbf{b}$

$2x_1+4x_2+6x_3=8$

Q4. a Least square problem can not be solved uniquely if the columns of 'A' are not linearly independent. Why? If $A = \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ solve the Least square problem using QR factorization method. 10

b Define span and base vector for a vector space. Two vectors are given as V_1=(1, 2, 3) and V_2=(2, 1, 3) construct a vector V_3 such that the vectors V_1, V_2 and V_3 span the entire R^3 space. 10

Q5. a Let $A = \begin{bmatrix} 25 & 2 \\ 2 & 4 \end{bmatrix}$. Find all the eigen values of 'A' using iterative techniques. Convergence is guaranteed in this case. Why? 15

b Write Shifted QR algorithm to determine the eigen values iteratively. 5

Q6. a Determine the QR factorization of A, using Gram Schmidt process of orthogonalization where $A = \begin{bmatrix} 1 & 2 \\ 2 & 5 \\ 1 & 2 \end{bmatrix}$. 10

b Construct a '2*x2*' matrix whose determinant is very large but the condition number is the best. Justify your answer. 10

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