

(3 hours)

Total Marks: 80

N.B. (1) Question no. 1 is Compulsory

(2) Solve any three from the remaining.

Q.(1)(a) If  $5 \sinh x - \cosh x = 5$  find  $\tanh x$ . (3)

(b) If  $u = e^{x^2+y^2+z^2}$  prove that  $\frac{\partial^3 u}{\partial x \partial y \partial z} = 8xyz u$ . (3)

(c) If  $u = \frac{x+y}{1-xy}$ ,  $v = \tan^{-1} x + \tan^{-1} y$  find  $\frac{\partial(u, v)}{\partial(x, y)}$ . (3)

(d) By Maclaurins series expand  $\log(1 + e^x)$  in powers of  $x$  upto  $x^4$ . (3)

(e) Show that the matrix  $A = \frac{1}{2} \begin{bmatrix} \sqrt{2} & -i\sqrt{2} & 0 \\ i\sqrt{2} & -\sqrt{2} & 0 \\ 0 & 0 & 2 \end{bmatrix}$  is unitary and hence find  $A^{-1}$  (4)

(f) Find the  $n^{th}$  derivative of  $y = \frac{x^2}{(x+2)(2x+3)}$  of (4)

Q.2) (a) Solve  $x^5 = 1 + i$  and find the continued product of the roots. (6)

(b) Find the nonsingular matrices P and Q such that PAQ is in normal form also (6)

find the rank of A, where  $A = \begin{bmatrix} 2 & 1 & 4 \\ 3 & 2 & 2 \\ 7 & 4 & 10 \\ 1 & 0 & 6 \end{bmatrix}$

(c) State and prove Euler's theorem for homogeneous functions on three variables. (8)

Q.3) (a) Investigate for what values of  $\lambda$  and  $\mu$  the equations (6)

$x + y + z = 6, x + 2y + 3z = 10, x + 2y + \lambda z = \mu$  have i) no solutions. (6)

ii) a unique solution. iii) infinite number of solutions.

(b) Find the stationary values of  $f(x, y) = x^3 + xy^2 + 21x - 12x^2 - 2y^2$  (6)

(c) If  $\sin(\theta + i\phi) = \cos \alpha + i \sin \alpha$  Prove that  $\cos^4 \theta = \sin^2 \alpha = \sinh^4 \phi$  (8)

Q.4) (a) If  $z = e^{x/y} + \log(x^3 + y^3 - x^2y - xy^2)$  find the value of (6)

$$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} + x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2}.$$

(b) Show that  $\tan^{-1} i \left( \frac{x-a}{x+a} \right) = \frac{i}{2} \log \frac{x}{a}$  (6)

(c) Solve the following equations by Gauss Jordan Method

$$\begin{aligned} 2x + 3y + 4z &= 11 \\ x + 5y + 7z &= 1 \\ 3x + 11y + 13z &= 25 \end{aligned}$$

(8)

Q.5) (a) Find the value of a,b,c so that  $\lim_{x \rightarrow 0} \frac{ae^x - b \cos x + ce^{-x}}{x \sin x} = 2$  (6)

(b) Expand  $\log(1 + x + x^2 + x^3)$  upto  $x^8$  (6)

(c) If  $y = \cos(m \sin^{-1} x)$  Prove that  $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0$  (8)

Q.6) (a) Find a,b,c if A is orthogonal where  $A = \frac{1}{9} \begin{bmatrix} -8 & 4 & a \\ 1 & 4 & b \\ 4 & 7 & c \end{bmatrix}$  (6)

(b) Fit a second degree curve to the following data (6)

x	0	1	2	3	4
y	1.0	1.8	1.3	2.5	6.3

(c) If  $x^x y^y z^z = c$  show that the value of  $\frac{\partial^2 z}{\partial x^2} - 2xy \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = \frac{2(x^2 - 2)}{x(1 + \log x)}$ , at  $x=y=z$  (8)