Q. P. Code: 27003

(03 Hours) Total Marks 80

N.B.:

(1)Question No.1 is compulsory.
(2)Attempt any three questions from remaining five questions.
(3)Assume suitable data if necessary and state it clearly.
(4)Figures to right indicates full marks.

1. Solve any five 20

   (a) Explain the concepts of column space and nullspace. For a $m \times n$ matrix of rank r, state the dimensions of column space and nullspace.

   (b) Let $y = 3x + 5$, where x is a random variable with mean 2 and variance 4. Find the mean and variance of y

   (c) State the Kalman filtering problem also state the important assumptions about the underlying state variable system.

   (d) State the CRLB (Cramer-Rao lower bound) theorem.

   (e) Write a short note on white noise process.

   (f) Explain any one method for generation of real-valued random vector $\mathbf{x}$ with zero mean using given autocorrelation matrix $\mathbf{R_x}$

2. (a) Let $\mathbf{p}_1 = [1\ 6\ 5]^T,\ \mathbf{p}_2 = [-2\ 4\ 2]^T,\ \mathbf{p}_3 = [1\ 1\ 0]^T,\ \mathbf{p}_4 = [2\ 2\ 0]^T$

      i. Check whether the set $T_1 = \{\mathbf{p}_2\ \mathbf{p}_3\ \mathbf{p}_4\}$ is independent 5

      ii. Check whether the set $T_2 = \{\mathbf{p}_1\ \mathbf{p}_2\ \mathbf{p}_3\}$ is independent. 5

   (b) Write a note on positive-definite matrices 5

   (c) Define and explain l_1, l_p and l_∞ norms. Find l_∞ norm of $\mathbf{v}$=[3 7 -8] 5

3. (a) Let $x[n] = A + w[n],\ \ n = 0, 1, ..., N-1$. It is desired to estimate the value of a DC level A in WGN $w[n]$ where $w[n]$ is zero mean and uncorrelated and each sample has variance $\sigma^2 = 1$. Consider the two estimators

      i. $\hat{A} = \frac{1}{N}\sum_{n=0}^{N-1} x[n]$

      ii. $\check{A} = x[0] + x[N-1]$

      Find mean and variance of each estimator. State whether these estimators are unbiased. Which one is better according to variance? 10

(P.T.O.)

(b) A WSS process with PSD $R_x(e^{j\omega}) = \frac{1}{1.64 + 1.6\cos\omega}$ is applied to a causal system described by the following difference equation $y[n] = 0.6\ y[n-1] + x[n] + 1.25\ x[n-1]$. Compute

i. the cross-PSD $R_{xy}(e^{j\omega})$ between the input and output 5

ii. the PSD of the output. 5

4. (a) Define and illustrate following statistical averages with the help of figures 8

i. Mean ii. Standard Deviation iii. Skewness iv. Kurtosis

(b) Consider following random processes

i. $X(t) = A\cos(\omega t + \phi)$ where ϕ is a random variable uniformly distributed in the interval $[0\ 2\pi)$

ii. $X[n] = A\ \cos(\omega n)$ where A is a Gaussian random variable with mean 0 and variance 1

Determine whether these random processes are WSS or not. 12

5. (a) Consider a stationary random process with correlation matrix

$$\mathbf{R}_x = \begin{bmatrix} 1 & a \\ a & 1 \end{bmatrix}$$

Find eigen values, eigen vectors and verify 12

i. $\det\mathbf{R}_x = \lambda_1\lambda_2$.

ii. $\mathbf{Q}^H\mathbf{Q} = \mathbf{I}$,

where $-1 < a < 1$, $\mathbf{Q}$=$[\mathbf{q_1}\ \mathbf{q_2}]$ is the eigenmatrix of $\mathbf{R}_x$, $\mathbf{q_1}$ and $\mathbf{q_2}$ are eigen vectors normalized to unit length , det and $\mathbf{I}$ denotes determinant and identity matrix respectively, λ_1 and λ_2 are eigen values.

(b) Compare and contrast orthogonal and triangular decompositions for zero-mean random vectors. 8

6. (a) Explain MVU estimator. Compute the CRLB for estimating A in the process $x[n] = A+w[n],\ \ n = 0, 1, ..., N-1$ where $w[n]$ is WGN with variance σ^2 and zero mean. 13

(b) Write a note on Kalman filter. 7