(3 Hours) [Total Marks:75

N.B. **(1)** Answers to the **TWO** sections should be written in the **SAME** Answer book.
(2) Figures to the right indicate full marks.
(3) Use of non – programmable calculators / log tables is allowed.
(4) Symbols have their usual meaning unless otherwise stated.

Section I (Quantum Mechanics I)

1 a) Estimate the energy of an electron confined within the nucleus of radius 10^{-13} cm. Given that the electrons in the β decay have the energies ~ 10 MeV, what does it say about the possibility of electron being nuclear constituent? (1 eV = 1.6 X 10^{-12} ergs). **12**

Find the commutators:

b) 1] $[\widehat{L^2},\ \widehat{L_x}]$, where $\hat{L}$ is an angular momentum operator.

2] $[f(x), \hat{p}]$, where $f(x)$ is some function of x and $\hat{p}$ is momentum operator.

3] State the general uncertainty relation between two hermitian operators $\hat{A}$ and $\hat{B}$. From that, show, $\Delta x\ \Delta E \geq \frac{\hbar}{2m}|\langle p_x\rangle|$

OR

2 a) Find the following commutators: **12**

1] $[\hat{x}, \hat{p}]$, where p is the momentum operator.

2] $[\widehat{L_x}, \widehat{L_y}]$, where $\hat{L}$ is an angular momentum operator.

b) Show that the kinetic energy operator, $\hat{T} = \frac{-\hbar^2}{2m}\nabla^2$ is non-negative i.e. $\langle\hat{T}\rangle > 0$.

c) Consider a system described by the wave equation:

$\Psi(x,t) = Ae^{-\lambda x}e^{-i\omega t}$ if x>0,

$= Ae^{\lambda x}e^{-i\omega t}$ if x < 0

where A, λ, ω are real positive constants.

i] Normalize and sketch $\psi(x)$, ii] Calculate $\langle\hat{T}\rangle$

3 Consider the operator matrix, $A = \frac{1}{2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ **13**

1] Is A hermitian?, 2] Find its eigenvalues, 3] Obtain the eigenvectors and normalize them, 4] What is the matrix operation that diagonalizes A? Construct the unitary diagonalizing matrix S.

OR

4 a) Define Hermitian and symmetric matrices. Show that the eigenvalues of the Hermitian operator are real. **13**

b) If $\hat{A}\ and\ \hat{B}$ are two hermitian operators which commute, show that we can always find the simultaneous eigen functions of them.

c) The Hamiltonian operator of the system is

$$H = \frac{d^2}{dx^2} + x^2$$

In some system of units.

1] Show that $Ne^{-x^2/2}$ is an eigenfunction of H, 2] Also determine the eigenvalues, 3] Normalize the given wave function.

You may want to use $\int_{-\infty}^{\infty} e^{-\alpha x^2} dx = \sqrt{\frac{\pi}{\alpha}}$

5 Consider a particle in an finite potential well given by: **13**

V(x) = 0 for x < 0 and x > a

= $-V_0$ for 0 < x < a

1] Set up the Schrodinger equation in different region and solve.

2] Obtain the transcendental equation and calculate energy eigenvalues from them.

3] Sketch the ground state and the first excited state eigenfunctions

OR

6 a) For the harmonic oscillator potential, **13**

1] Define the annihilation, creation and number operators

2] Using the fact that $\hat{a}\psi_0(x) = 0$, find the ground state wave function $\psi_0(x)$.

3] Calculate the $\langle x^2 \rangle$ in $\psi_0(x)$

You may want to use $\int_{-\infty}^{\infty} e^{-\alpha x^2 + \beta x} dx = \sqrt{\frac{\pi}{\alpha}} e^{\beta^2/4\alpha}$

b) Show that if the potential is symmetric about $x = 0$, then the wave functions can be taken as either symmetric or anti-symmetric.

SECTION II (NUCLEAR PHYSICS)

7 (a) Assuming the square well potential for the deuteron, find the minimum depth of the potential , if the width of the potential is 2fm. **6**

(b) Explain why a nucleus has a zero electric dipole moment. What information does the measurement of the electric quadrupole moment of a nucleus provide? **6**

OR

8 (a) What is the reason that makes the $J = l + 1/2$ state deeper lying or more tightly bound compared to the $J = l - 1/2$ state in a nucleus ? **6**

(b) Give the expected shell-model spin and parity assignments for ground state of : (a) ${}^{7}_{3}Li$ b) ${}^{63}_{29}Cu$ c) ${}^{40}_{20}Ca$ d) ${}^{31}_{16}P$ **6**

9 (a) Why is the emission of α particle preferred over that of individual nucleons in the radioactive decay of a nucleus? State the Geiger- Nuttal law. **7**

(b) Give an account of the Nilsson model of the nucleus. What is Nilsson diagram? **6**

OR

10 Obtain the Breit Wigner single level resonance formula for S wave neutron scattering. **13**

11 (a) State which of the following reactions are allowed by the conservation laws and which are forbidden, giving the reason. In case of allowed processes state the interaction responsible for it. **6**

$$\mu^+ \rightarrow e^- + e^+ + e^+$$

$$K^+ \rightarrow \pi^+ + \pi^- + \pi^+$$

(b) Outline the working principle and construction of linear accelerator. **6**

OR

12 (a) Explain how semiconductor detectors are superior to (i) gas detectors and (ii) scintillation detectors. **6**

(b) Explain the construction and working of a Geiger –Muller counter. Explain the terms : Dead time and Recovery time. **6**

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