(3 Hours) [Total Marks:75

N.B. **(1)** Answers to the TWO sections should be written in the SAME Answer book.

(2) Figures to the right indicate full marks.

(3) Use of non – programmable calculators / log tables is allowed.

(4) Symbols have their usual meaning unless otherwise stated.

SECTION I

MATHEMATICAL METHODS

1. (a) Find the Fourier series of the function 13

   f(x) = x(x+1)+2 if $-\pi < x < \pi$.

   (b) Using the method of Laplace transform, solve the differential equation

   $y'' + 2y' + 2y = 0$ subject to initial conditions $y(0) = 4, y'(0) = -12$.

OR

2. (a) Find the fourier transforms of 13

   (i) $3xe^{-x^2}$

   (ii) e^{-bx} $x > 0, b > 0$

   (b) Find the eigen-values and eigenvectors of

   $A = \begin{pmatrix} 2 & 2 \\ 2 & -1 \end{pmatrix}$

3. (a) Check whether the following functions are analytic : 12

   (i) f(z)= $\bar{z}$

   (ii) f(z)= sin x cosh y + i cosx sinh y

   (b) State and prove Cauchy's theorem for an analytic function f(z) on a closed contour C.

OR

4. (a) Evaluate $\int_0^{+\infty} \frac{cosx\, dx}{1+x^2}$ 12

   (b) Evaluate (i) $\oint \frac{e^z}{z-2} dz$ (ii) $\oint \frac{z^3-6}{2z-i}\, dz$

5. (a) Assuming $u(x,t) = \phi(x)\eta(t)$, solve the Differential equation — 12

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$$

Where 'a' is constant subject to condition $\eta(t = 0) = 1$

(b) Using Frobenius method determine the solution of- $(x^2 - x)y'' - xy' + y = 0$

OR

6. (a) Solve the differential equation : $y'' + y' - 2y = e^{2x}$ — 12

(b) Solve the differential equation $y'' + (1 + x^2)y = 0$ by power series method.

SECTION II

CLASSICAL MECHANICS

7 (a) Using D'Alembert's principle, obtain the Lagrange's equations of motion. — 12

(b) Obtain the Lagrnage's equations for a spherical pendulum i.e. mass point suspended by a rigid weightless rod.

OR

8 (a) Use the Hamilton's principle to obtain Lagrnage's equations of motion. — 12

(b) A particle is moving in a plane. Obtain an expression for the time derivative of angular momentum.

9 (a) For a general system of point masses with position vectors $\vec{r_i}$ and applied forces $\vec{f_i}$ (including any forces of constraints) show that — 13

$$\bar{T} = -\overline{\frac{1}{2}\sum_{i} \overrightarrow{f_i} \cdot \vec{r_i}}$$

(b) Assuming $u = \frac{1}{r}$, for a central force motion , prove :

$$\frac{d^2u}{d\theta^2} + u = -\frac{m}{l^2}\frac{d}{du}V\left(\frac{1}{u}\right)$$

Where V is potential and $l = mr^2\dot{\theta}$ is angular momentum.

OR

10 (a) Obtain the equations of motion for small oscillations of a system around the point of equilibrium. — 13

(b) Determine the eigen frequencies and the eigen coordinates of a system with two degrees of freedom whose lagrangian is

$$L = \frac{m}{2}\left(\dot{x}^2 + \dot{y}^2\right) - (\xi_1 x^2 + \xi_2 y^2) + \omega^2 xy \ \ldots.\omega^2 > 0$$

11 (a) Derive Hamilton's equations of motion from variation principle. 13

(b) Obtain the condition for F_2 (q,Q,t) to be the generating function of canonical transformations.

OR

12 (a) If $[\varphi, \psi]$ be the poison bracket of φ and ψ. Then prove that 13

$$\frac{\partial}{\partial t}[\varphi, \psi] = \left[\frac{\partial \varphi}{\partial t}, \psi\right] + \left[\varphi, \frac{\partial \psi}{\partial t}\right]$$

(b) Show that $\frac{dF}{dt} = \frac{\partial F}{\partial t} + [F, H]$
