Q. P. CODE: 26811

Duration:[$2\frac{1}{2}$ Hours] [Marks: 60]

N.B.
1) All questions are compulsory.
2) Attempt any **TWO** subquestions **from First Four questions** .
3) Attempt any **FOUR** subquestions **from Fifth question** .
4) Figures to the right indicate full marks.

1. (a) Prove that any $n \times n$ real matrix A is orthogonal if and only if multiplication by A preserves the inner product of column vectors. (6)
   (b) i. Show that every isometry is a composition of an orthogonal linear operator and a translation. (3)
   ii. Find eigenvalues of a 3×3 orthogonal matrix A with $|A| = 1$. (3)
   (c) i. Is orthogonal linear operator an isometry? Justify. (3)
   ii. Find the vector parallel to the line of intersection of the planes $3x - 6y - 2z = 7$ and $2x + y - 2z = 5$. (3)

2. (a) Prove that a space curve lies in some plane in $\mathbb{R}^3$ if and only if its torsion is zero. (6)
   (b) i. Prove that any reparametrization of a regular curve is regular. (3)
   ii. Compute the signed curvature of the curve $\gamma(t) = (t, \cosh t)$ (3)
   (c) i. Show that the tangent line to the regular parametrized curve $\gamma(t) = (3t, 3t^2, 2t^3)$ makes constant angle with the lines $z = x$ and $y = 0$. (3)
   ii. Let γ be the helix in $\mathbb{R}^3$ defined by $\gamma(t) = (3\cos t, 3\sin t, 4t)$. Find the arc length function of γ starting at origin and parametrize γ by arc length. (3)

3. (a) Let S be a regular surface and $p \in S$. Prove that there exists a neighborhood V of p in S such that V is the graph of a differentiable function which has the form $z = f(x, y)$ or $y = g(x, z)$ or $x = h(y, z)$. (6)
   (b) i. Show that the vector subspace of dimension two coincides with the set of tangent vectors. (3)
   ii. Is the set $S = \{(x, y, z) \in \mathbb{R}^3 : z^2 = x^2 + y^2\}$ a regular surface? Justify. (3)
   (c) Decsribe Mobius Band as a non-orientable surface. (6)

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4. (a) Show that a diffeomorphism $f : S_1 \to S_2$ is an isometry if and only if for any surface patch σ_1 of S_1, the patches σ_1 and $f o \sigma_1$ of S_1 and S_2 respectively have the same first fundamental form. (6)

   (b) Calculate the Gaussian curvature, Mean curvature and Principal curvature of $\sigma(u, v) = ((3 + 2\cos u)\cos v, (3 + 2\cos u)\sin v, 2\sin u), 0 < u < 2\pi, 0 < v < 2\pi$. (6)

   (c) i. Using geodesic equations find the geodesics on the circular cylinder $\sigma(u, v) = (\cos u, \sin u, v)$. (3)

   ii. Prove or disprove: A geodesic has constant speed. (3)

5. (a) Prove that a linear operator on $I\!R^2$ is a reflection if its eigenvalues are 1 and -1 and the eigenvectors with these eigenvalues are orthogonal. (3)

   (b) Define an isometry of $\mathbb{R}^n$. Prove that composition of two isometries is an isometry. (3)

   (c) Find the curvature of the curve $\gamma(t) = (\cos t, \sin t, 2t)$. (3)

   (d) Prove that $T = \{(x, y, z) \in \mathbb{R}^3 : z^2 = r^2 - (\sqrt{x^2 + y^2} - a)^2, 0, r < a\}$ is a regular surface. (3)

   (e) Find the Second Fundamental Form of the helicoid $\sigma(u, v) = (v\cos u, v\sin u, \lambda u)$, where λ is a constant. (3)

   (f) For the hyperbolic paraboloid $S = \{(x, y, z) \in \mathbb{R}^3 : z = y^2 - x^2\}$, find the differential $DN(p)$ at point $p = (0, 0, 0)$. (3)

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