

**[Time: Three Hours]**

**[ Marks: 80]**

Please check whether you have got the right question paper.

- N.B:
1. Question No.1 is compulsory.
  2. Answer any three from the remaining.
  3. Figures to the right indicate marks.

- Q.1. a.** Separate into real part and imaginary of  $\text{Cos}^{-1}\left(\frac{3i}{4}\right)$  **03**
- b.** Show that the matrix A is unitary where  $A = \begin{bmatrix} \alpha + i\gamma & \beta + i\delta \\ \beta + i\delta & \alpha + i\gamma \end{bmatrix}$  is unitary if  $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 1$  **03**
- c.** If  $z = \tan(y + ax) + (y - ax)^{3/2}$  then show that  $\frac{\partial^2 z}{\partial x^2} = a^2 \frac{\partial^2 z}{\partial y^2}$  **03**
- d.** If  $x = uv$      $y = \frac{u}{v}$  Prove that  $JJ^T = 1$  **03**
- e.** Find the  $n^{\text{th}}$  derivative of  $\frac{x^3}{(x+1)(x-2)}$  **04**
- f.** Using the matrix  $A = \begin{bmatrix} -1 & 2 \\ -1 & 1 \end{bmatrix}$  decode the message matrix  $C = \begin{bmatrix} 4 & 11 & 12 & -2 \\ -4 & 4 & 9 & -2 \end{bmatrix}$  **04**
- Q.2. a.** If  $\sin^4\theta \cos^3\theta = a \cos\theta + b \cos 3\theta + C \cos 5\theta + d \cos 7\theta$  then find a, b, c, d. **06**
- b.** Using Newton Raphson method Solve  $3x - \text{Cos}x - 1 = 0$  Correct to 3 decimal places. **06**
- c.** Find the stationary points of the function  $x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$  & also find maximum and minimum values of the function. **08**
- Q.3. a.** Show that **06**  
 $x \operatorname{cosec} x = 1 + \frac{x^2}{6} + \frac{7}{360} x^4 + \dots$
- b.** Reduce matrix to PAQ normal form and find 2 non Singular matrices P & Q **06**  
 $\begin{bmatrix} 1 & 2 & -1 & 2 \\ 2 & 5 & -2 & 3 \\ 1 & 2 & 1 & 2 \end{bmatrix}$
- c.** If  $y = \cos(m \sin^{-1}x)$  Prove that  $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0$  **08**

**Q.4. a.** State and prove Euler's theorem for three Variables. **06**

**b.** Show that all the roots of  $(x + 1)^6 + (x - 1)^6 = 0$  are given by  $-\text{icot} \frac{(2k+1)\pi}{12}$  where  $k= 0,1,2,3,4,5$  **06**

**c.** Show that the equations **08**

$$-2x + y + z = a$$

$$x - 2y + z = b$$

$$x + y - 2z = c$$

have no solutions unless  $a + b + c = 0$  in which case they have infinitely many solutions.

Find these Solutions when  $a = 1$      $b = 1$      $c = -2$

**Q.5. a.** If  $z = f(x, y)$      $x = r \cos \theta$  **06**

$y = r \sin \theta$  Prove that

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2} \left(\frac{\partial z}{\partial \theta}\right)^2$$

**b.** If  $\cos hx = \sec \theta$  Prove that **06**

i)  $x = \log(\sec \theta + \tan \theta)$

ii)  $\theta = \frac{\pi}{2} - 2 \tan^{-1}(e^{-x})$

**c.** Solve by Gauss Jacobi **08**

Iteration method

$$5x - y + z = 10$$

$$2x + 4y = 12$$

$$x + y + 5z = -1$$

**Q.6. a.** Prove that **06**

$$\cos^{-1}[\tan h(\log x)] = \pi - 2 \left( x - \frac{x^3}{3} + \frac{x^5}{5} - \dots \right)$$

**b.** If  $y = e^{2x} \sin \frac{x}{2} \cos \frac{x}{2} \sin 3x$  Find  $y_n$  **06**

**c.** (i) Evaluate  $\lim_{x \rightarrow 0} (\cot x)^{\sin x}$  **04**

(ii) Prove that  $\log \left[ \frac{\sin(x+iy)}{\sin(x-iy)} \right] = 2i \tan^{-1}(\cot x \tan hy)$  **04**

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