(REVISED COURSE)
(3 Hours) **[Total Marks: 80**

N.B. :

1) **Question 1 is compulsory**. Answer any **three** more from the remaining questions.
2) Assume data if necessary and **specify the assumptions** clearly.
3) Draw neat sketches wherever required.
4) Answers to the sub-questions of an individual question should be grouped and written together i.e. one below the other.

1. (a) The flow of a particular fluid is described by its velocity components as: **[05]**

$$u = -\frac{V_0}{l}x, \quad v = -\frac{V_0}{l}y$$

where V_0 *and* l are constants. Derive an expression to calculate the rate of change of the density of the fluid $\rho(x, y, z, t)$ with respect to time, along a flowing fluid particle.

(b) Consider the following equation: **[05]**

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + D\frac{\partial^2 C}{\partial x^2} = 0$$

Derive an expression to solve the equation numerically using an implicit scheme.

(c) Consider the following function: **[05]**

$$f(x, y) = exp(-x) + exp(-y) + 0.5 * \cos(xy)$$

Use backward differences to calculate:

$$\frac{\partial f(x,y)}{\partial x} \; and \; \frac{\partial f(x,y)}{\partial y}$$

at $x = 0.1,$ *and* $y = 0.1$. Also calculate the error. Take

$$\Delta x = 0.05 \; and \; \Delta y = 0.05$$

(d) Explain the following terms with examples. **[05]**

- Convergence.
- Consistency.

2. (a) An insulated metal rod has an initial temperature profile given by: **[10]**

$$T(x, 0) = 20^oC$$

At time $t = 0$, a hot reservoir at a temperature of $T = 200^oC$ is brought into contact with the left end of the rod. Simultaneously the right end is exposed to a hot reservoir at a temperature of 300^oC. The length of the rod is 1 m, and for the metal $\alpha = 1.0 \times 10^{-5}\ m^2/sec$. Using the finite difference FTCS scheme, calculate the temperature across the rod for the next 1600 sec. Assume $r = 0.2$ *and* $\Delta x = 0.2\, m$.

(b) Solve the following equation using the weighted residual method: **[10]**

$$\frac{d^2u}{dx^2} = -(x+1)$$

with $u = 0 \ at \ x = 0 \ \ and \ \frac{du}{dx} = 1 \ at \ x = 2$ Use the trial function $u = a_1 x + a_2 x^2$

3. Consider the steady, one dimensional heat conduction, in an insulated metal rod, of length 0.8 m. The ends of the rod are maintained at constant temperatures of 200^oC and 400^oC respectively. Applying the finite volume method on eight control volumes of equal length, determine the temperature profile across the rod. Thermal conductivity of the metal is $k = 1000 \ W/m.K$ and the cross-sectional area of the rod is $A = 0.01 \ m^2$ **[20]**

4. (a) A company produces a perishable product in a factory at $x = 0$, and sells it along the distribution route $x > 0$. The selling price of the product, p, is a function of the length of time after it was produced, t, and the location at which it is sold, x, i.e. $p = p(t, x)$. At a given location the price decreases in time at a rate of $-8\$/hr$. In addition, because of shipping costs, the price increases with distance from the factory at a rate of $0.1\$/km$. If the manufacturer wants to sell the product at the same price of \$100 everywhere, determine how fast he must travel along the distribution route. **[10]**

   (b) Consider the following equation: **[10]**

$$\frac{dy}{dx} + 2x + \sin x = 0$$

subject to the B.C.:

$$y(0) = 1$$

Solve the equation using the Subdomain method. Use the approximate solution:

$$y = 1 + \sum_{j=1}^{3} a_j x^j$$

5. (a) Consider the following set of data: **[10]**

i	x	y
0	0	1
1	0.08	0.9057
2	0.015	0.8327
3	0.21	0.7767
4	0.31	0.6956
5	0.38	0.6472

Using quadratic interpolation calculate the value of $y \ at \ x = 0.10 \ and \ at \ x = 0.35$.

   (b) Consider the cooling of a circular fin by convective heat transfer along its length. The cylindrical fin has a uniform cross-sectional area A, and perimeter P. The base of the fin is at a temperature of $T_B = 300\,^oC$, and the free end is insulated. The fin is exposed to an ambient temperature $T_A = 30\,^oC$. Calculate the temperature distribution along the fin, using the finite volume method, with five control volumes, and compare with the analytical solution: **[10]**

$$\frac{T - T_A}{T_B - T_A} = \frac{\cosh[n(L-x)]}{\cosh(nL)}$$

[TURN OVER

$$n^2 = \frac{hP}{kA} = 25\,m^{-2} \quad L = 1\,m$$

6. Consider the steady two-dimensional heat conduction in a metal slab of size $2m \times 2m$. The boundary conditions are given by: **[20]**

$$T(x,0) = 20^oC,\ T(0,y) = 20^oC,\ T(2,y) = 20^oC,\ and T(x,2) = 400^oC$$

Using five equally spaced grid points on each side, and ADI scheme, calculate the distribution of $T(x,y)$ across the metal slab.

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