Q.P.Code:13374

(3 hours) **Total Marks : 100**

N.B. : 1) Answer any FIVE questions.

2)All questions carry EQUAL marks.

1. (a) State and prove Havel Hakimi theorem about graphic degree sequence.
   (b) Let G be a graph with vertex set $\{v_1, v_2, \ldots\ldots, v_p\}$ and adjacency matrix $A = (a_{ij})$. Show that
   (i) $(i,j)^{th}$entry of A^kdenotes the number of (v_i, v_j) walks of length k
   (ii) $\frac{1}{6}(trace\ (A^3))$ is number of triangles in G.
2. (a) Let e be any edge of a graph G, then prove that $\tau(G) = \tau(G.e) + \tau(G - e)$ where $\tau(G)$ denotes number of spanning trees of a graph G.
   (b) Show that a vertex v of a tree T is a cut vertex if and only if its degree is more than one. Show that any non trivial loopless connected graph on at least two vertices contains at least two non cut vertices.
3. (a) Define closure of a graph G. Show that closure of a graph G, is well defined. Show further that a simple graph G is Hamiltonian if and only if its closure is Hamiltonian.
   (b) Let κ, κ'denote vertex and edge connectivity of a graph G, then show that $\kappa \leq \kappa' \leq \delta$. Give an example to show that inequyality can be strict.
4. (a) Let G be a bipartite graph with a bipartition(X, Y). Show that G contains a matching that saturates every vertex of X if and only if for any subset S of V, $|N(S)| \geq |S|$.
   (b) Determine perfect matching in the graphs K_{2n} and $K_{n,n}$.
5. (a) Let $\pi_k(G)$ is number proper k colorings of G. Show that $\pi_k(G) = \pi_k(G - e) - \pi_k(G.e)$ for any edge e of G.
   (b) Define critical graph. Show that only 1critical graph is K_1, only 2critical graph is K_2 and only 3critical graphs are C_{2t+1}.
6. (a) State Kruskasl's algorithm. Prove that any spanning tree constructed by Kruskal's algorithm is optimal.
   (b) Define line graph of a graph. Show that line graph of connected graph is isomorphic to the graph if and only if it is cycle.
7. (a) State and prove Euler's theorem about planar graph. Hence deduce that K_5 and $K_{3,3}$ are not planar.
   (b) Define dual of a planar graph. Let G^*denote dual of a planar graph G. Show that if $G^* \cong G$ then $|E(G)| = 2|V(G)| - 2$. Construct such a graph on $n \geq 4$ vertices.
8. (a) Define Ramsey number $r(p,q);\ p,q \geq 2$. Show that $r(p,q) \leq \binom{p+q-2}{p-1}$.
   (b) If T is a tree on m vertices then show that $r(T, K_n) = (m-1)(n-1) + 1$.
