Scheme A(External)	(3 Hours)	Total marks: 100
Scheme B(Internal/External)	(2 Hours)	Total marks: 40

N.B: 1) Scheme A students answer **any five** questions.
2) Scheme B students answer **any three** questions.
3) All questions carry equal marks.
4) Write on the top of your answer book the scheme under which you are appearing.

1. (a) How many integers strictly between0 and 10,000 have exactly one digit equal to 5?
   (b) Determine number of 8-permutations of the multiset T = {3.a, 2.b, 4.c}

2. (a) Prove the recurrence relation using combinatorial argument for D_n, derangement of n objects; $D_n = (n-1)(D_{n-1} + D_{n-2})$, $n \geq 3$.
   (b) If S(n,k) denotes Stirling numbers of second kind then show that
   (i) S(n,1) = 1 = S(n,n), (ii) $S(n,2) = 2^{n-1} - 1$,
   (iii) $S(n,n-1) = \binom{n}{2}$, for $n \geq 2$.

3. (a) What is circular permutation?
   Ten people including two who do not wish to sit next to one another are to be seated at a round table. How many circular sitting arrangements are there?
   (b) State and prove strong form of Pigeon hole principle. Give example.

4. (a) Find the sum of all coefficients in $(4x -3y +z)^5$.
   (b) State and prove Baye's theorem.

5. (a) Solve the recurrence relation $a_n = 2a_{n-1} + 3^n$ subject to the initial condition $a_0 = 2$; $n \geq 1$.
   (b) Compute the Möbius function of linearly ordered set $(X_n, \leq)$ where $X_n = \{1, 2, 3, \ldots\ldots., n\}$.

6. (a) Define SDR, system of distinct representatives. Find the number of SDR's for the family {1},{1,2},…………..,{1,2,3,…,n}.
   (b) There are three groups of children containing 3 girls and 1 boy, 2 girls and 2 boys, 1 girl and 3 boys. One child is selected at random from each group. Show that the chance that the three selected consists of 1 girl and 2 boys is 13/32.

7. (a) Show that every sequence of n^2+1 distinct real numbers contains a subsequence of length n+1 that is either strictly increasing or strictly decreasing.
   (b) Determine number of regions that are created by n mutually overlapping circles in general position in the plane

Turn Over

8. (a) Show that $\sum_{k=1}^{n} k \binom{n}{k}^2 = n \binom{2n-1}{n-1}$.

   (b) What is variance of discrete random variable? Compute variance of random variable with normal distribution.

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