REVISED **Time: 3 Hours** **Total Marks: 80**

Instructions:

- Attempt **any two** questions from **each section**
- **All questions** carry **equal marks**
- **Answer to section I and II** should be written on the **same answer book**.

SECTION I (Attempt any two Questions)

Q.1. (a) State and prove Lebesgue covering lemma.
(b) State and prove Heine Borel theorem.

Q.2. (a) Define a compact set. Show that the continuous image of a compact set is compact.
(b) Define a connected set. If A and B are connected sets is $A\cup B$ *and* $A \cap B$ are connected? Justify your answer.

Q.3. (a) Let S be an open subset of $\mathbb{R}^2$. If $a \in S$, and the partial derivatives $D_1 f,\ D_2 f$ exist in some open ball $B(a, r)$ and are continuous at a, then show that f is differentiable at a.
(b) Use chain rule and find $\frac{\partial z}{\partial u}, \frac{\partial z}{\partial v}$ where $z = x^2 - 3x^2y^3, x(u, v) = ve^u, y(u, v) = ve^{-u}$.

Q.4. (a) State and prove Inverse function theorem.
(b) State and prove mean value theorem for scalar fields.

SECTION II (Attempt any two Questions)

Q.5. (a) Prove that a subset of a topological space is open if and only if it is a neighbourhood of each of its points.
(b) Let X and Y be topological spaces. Then show that a mapping f: X →Y is continuous if and only if the inverse image under f of every closed set in Y is also closed in X.

Q.6. (a) Show that a topological space X is disconnected if and only if there exists a non-empty proper subset of X which is both open and closed in X.
(b) Show that continuous image of a connected space is connected.

Q.7. (a) Show that closed subsets of a compact sets are compact.
(b) Show that a Hausdorff space X is locally compact if and only if each of its points is an interior point of some compact subspace of X.

Q.8. (a) Show that a compact subset of a metric space is closed and bounded.
(b) Show that all completions of a metric space are isometric.
