Q. P. Code: 11732

REVISED **Time: 3 Hours** **Total Marks: 80**

Instructions:

- Attempt **any two** questions from **each section**
- **All questions** carry **equal marks**
- **Answer to section I and II** should be written on the **same answer book**.

SECTION I (Attempt any two Questions)

Q1. (a) Prove that every n-dimensional vector space V(F) is isomorphic to V_n (F).

(b) Let F be a field of complex numbers and let T be the function from F^3 into F^3 defined by

$T(X_1, X_2, X_3) = (X_1 - X_2 + 2X_3,\ 2X_1 + X_2 - X_3,\ -X_1 - 2X_2)$

Verify that T is a linear transformation. Describe the null space of T.

Q2. (a) Let A is a linear transformation on a vector space V such that $A^2 - A + I = 0$. Then show that A is invertible.

(b) Find the Rank of the matrix.

$$\begin{bmatrix} 3 & -4 & -1 & 2 \\ 1 & 7 & 3 & 1 \\ 5 & -2 & 5 & 4 \\ 9 & -3 & 7 & 7 \end{bmatrix}$$

Q3. (a) Show that similar matrices have the same minimal polynomial.

(b) Find all (complex) characteristic values and characteristic vectors of the following matrix.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Q4. (a) If α and β are vectors in an inner product space V(F) and a,b ϵ F, then prove that

$\mathrm{Re}\,(\alpha, \beta) = \frac{1}{4} \| \alpha + \beta \|^2 - \frac{1}{4} \| \alpha - \beta \|^2$

(b) To every self-adjoint operator T on a finite dimensional inner product space V there corresponding distinct real numbers $C_1, C_2, C_3 \ldots C_k$ and perpendicular projections $E_1, E_2, E_3 \ldots E_k$. (where k is strictly positive integer, and greater than the dimension of the space)

Show that

a) The E_i are pairwise orthogonal and different from 0.
b) $E_1 + E_2 + E_3 + \ldots + E_k = I$
c) $T = C_1E_1 + C_2E_2 + C_3 E_3 + \ldots + C_kE_k$

SECTION II (Attempt any two Questions)

Q5. (a) Show that every quotient group of a group is a homomorphic image of the group.

(b) If N & M are normal sub groups of a group G, then show that $N \cap M$ is a normal sub group of G.

Q6. (a) Prove if a group G of order 28 has a normal subgroup of order 4, then G is abelian.

(b) If G is any finite group such that p / o(G) , where p is a prime number then show that G has an element of order p.

Q7. (a) Prove that a finite integral domain is a field.

(b) Show that the intersection of two ideals of a ring R is an ideal of R. Give one example to show that the union of two ideals of R need not to be an ideal of R.

Q8. (a) Show that in a unique factorization domain every pair of non-zero elements have a g.c.d and l.c.m.

(b) Show that every ring R can be embedded in the polynomial ring R[x].
