External (Scheme A)	(3 Hours)	Total Marks: 100
Internal (Scheme B)	(2 Hours)	Total Marks: 40

N.B.:Scheme A students should attempt any five questions.
Scheme B students should attempt any three questions.
Write the scheme under which you are appearing, on the top of the answer book.

Q1. (a) If H and K are two subgroups of a group G, Prove that H U K is a subgroup of G if and only if either $H \subset K$ or $K \subset H$.

(b) If N & M are normal sub groups of a group G, then show that $N \cap M$ is a normal sub group of G.

Q2. (a) Prove that if a group G of order 28 has a normal subgroup of order 4, then G is abelian.

(b) Let G be a finite abelian group such that p divides o(G), p being a prime number then show that there exists an element $a \neq e \in G$ such that $a^p = e$.

Q3. (a) Prove that every group of prime order is abelian.

(b) Show that every quotient group of a cyclic group is cyclic. Give an example to show that the converse need not to be true.

Q4. (a) Let $f : R \to R'$ be a homomorphism of a ring R onto a ring R'. Then show that

$$\frac{R}{Ker\ f} \approx R'$$

(b) Find the maximal ideals of Z_{12} , the ring of integers modulo 12.

Q5. (a) Prove that a finite integral domain is a field.

(b) Show that in a unique factorization domain every pair of non zero elements have a

g.c.d and l.c.m. .

Q6. (a) Let U and V be the vector spaces over the field F and let T be a linear transformation from U into V. Suppose that U is finite dimensional then prove that
Rank (T) + Nullity (T) = dim (U).

(b) Show that the mapping T: $V_2(R) \to V_3(R)$ defined as T (a, b) = (a + b , a - b , b) is a linear transformation from $V_2(R)$ into $V_3(R)$. Find the range, rank, null space and nullity of T.

Turn Over

Q7. (a) Let A and B be linear transformations on a finite dimensional vector space V and let AB=I. Then show that A and B are both invertible and $A^{-1} = B$.

Give an example to show that this is false when V is not finite dimensional.

(b) Find the Rank of the matrix.

$$\begin{bmatrix} -1 & 2 & 3 & -2 \\ 2 & -5 & 1 & 2 \\ 3 & -8 & 5 & 2 \\ 5 & -12 & -1 & 6 \end{bmatrix}$$

Q8. (a) Suppose that α and β are vectors in an inner product space. Then show that

$$\| \alpha + \beta \|^2 + \| \alpha - \beta \|^2 = 2 \| \alpha \|^2 + 2 \| \beta \|^2$$

(b) Find the minimal polynomial for the real matrix.

$$\begin{bmatrix} 7 & 4 & -1 \\ 4 & 7 & -1 \\ -4 & -4 & 4 \end{bmatrix}$$
