External (Scheme A) (3 Hours) [Total Marks: 100]
Internal (Scheme B) (2 Hours) [Total Marks: 40]

Note:
(1) External (Scheme A) students answer any five questions.
(2) Internal (Scheme B) students answer any three questions.
(3) All questions carry equal marks. Scientific calculator can be used.
(4) Write on top of your answer book the scheme under which your appearing

Q1 a) Define: absolute error, relative error and percentage error. Find absolute error, relative error and percentage error in calculation of $Z = 3x^2 + 2x$ by taking approximate value of x as 3.45, and true value of x as 3.4568.

b) Convert decimal number $(43)_{10}$ to corresponding binary, octal and hexadecimal number.

Q2 a) Explain Ramanujan Method. Using Ramanujan's method, obtain the first four convergence of $x + x^2 = 1$

b) Derive the Newton-Raphson Method iterations formula to find a root of the algebraic or transcendental equation$f(x) = 0$. Hence Use the formula to solve $f(x) = cosx - xe^x$ with initial approximation $x_0 = 1$ upto three decimal places.

Q3 a) Solve the following system by using the Crout's triangularization method.

$x_1 + x_2 + x_3 = 9$
$2x_1 - 3x_2 + x_3 = 13$
$3x_1 + 4x_2 + 5x_3 = 40$

b) Let A = $\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$. Find the smallest Eigen value in magnitude of matrix A using 4 iterations of Inverse power method.

Q4 a) Derive Newton's Divided difference formula of interpolation.

b) Derive Cubic Spline interpolation formula.

Q5 a) Derive Newton-Cotes Quadrature formula and use it to derive Trapezoidal rule for numerical integration.

b) Evaluate $\int_0^1 \int_0^1 \frac{Sin\, xy}{1+xy} dxdy$ using Trapezoidal Rule with $h = k = 0.5$

Q6 a) Using the least –squares method , obtain the normal equation to find the values of a, b and c when the curve $y = c + bx + ax^2$ is to be fitted for the data points (x_i, y_i) $i = 1,2,3, \dots \dots \dots n$.

b) Obtain the first four orthogonal polynomials $f_n(x)$ on $[-1,1]$ with respect to weight function $w(x) = 1$

Q7 a) i) Given $\frac{dy}{dx} - 1 = xy$ with $y(0) = 1$, obtain the Taylor series for $y(x)$.

ii) Using Picard's method, obtain the solution of $\frac{dy}{dx} = x(1 + x^3y)$, $y(4) = 4$.

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b) Using Milne's Method, solve the differential equation
$(1+x)\frac{dy}{dx} + y = 0$ with $y(0) = 2$, for $x = 1.5$ and $x = 2.5$

Q8 a) Derive a Jacobi Iteration formula to obtain the numerical solution of one dimensional heat equation

b) Solve the heat conduction equation subject to the boundary conditions $u(0,t) = u(1,t) = 0$, and $u(x,0) = x - x^2$. Take $h = \frac{1}{4}, k = 0.025$ and $u_t(x,0) = 0, u(x,0) = \sin^3 \pi x \; \forall \; x \in [0,1]$
