Q.P. Code :02748

[Time: $2\frac{1}{2}$ hours] **[Marks:75]**

Please check whether you have got the right question paper.

N.B: 1. All questions are compulsory.
2. Figures to the right indicates full marks.
3. Use of calculator is allowed.

Q 1 (a) Define the following terms with one illustration each. (10)
i) Null hypothesis
ii) Alternative hypothesis
iii) Critical region
iv) Type I error
v) Type II error

(b) Suppose that a single observation X is taken from a population with probability density function (p.d.f) (05)

f $(x,\theta) = \frac{1}{\theta}$ $\quad 0 \le x \le \theta$
= 0 $\quad$ otherwise

for testing H_o : θ =1 against H_1 : θ =2. We reject H_o when X > 0.57. Calculate probabilities of type I error and type II error

OR

Q 1 (p) State and prove Neyman Pearson lemma to test simple null hypothesis against simple alternative hypothesis. (10)

(q) Let X_1 X_2, …… X_n, be a random sample of size n from Binomial distribution with parameters (n,p).Obtain Best Critical Region (BCR) of size α for testing H_o: $p=p_0$ against H_1 : $p=p_1$ ($p_1 > p_0$) (05)

Q 2 (a) Explain uniformly Most powerful (UMP) test. Also explain the procedure for obtaining a UMP test. (05)

(b) Obtain a Likelihood Ratio Test (LRT) of size α to test H_0 : $\mu = \mu_0$ (specified) against H_1 : $\mu \neq \mu_0$ based on a random sample of size n drawn from normally distributed population with unknown mean μ and known variance σ^2 (10)

OR

Q 2 (p) Explain the procedure for obtaining a Likelihood Ratio Text. (05)

(q) Let X_1 X_2, …….. X_n, be a random sample from a population with the probability density function (p.d.f) (10)

f $(x,\theta) = \theta e^{-\theta x}$ $\quad x>0, \theta>0$
= 0 $\quad$ otherwise

Obtain uniformly Most Powerful test of size α, based on the given sample of size n, to test H_o: $\theta = \theta_0$ against H_1: $\theta < \theta_0$ where θ_0 is specified positive constant.

Q 3 (a) Explain the procedure for Sequential Probability Ratio Test (SPRT). (05)

(b) Construct SPRT of strength (α, β) for testing H_o: $\lambda = \lambda_0$ against H_1: $\lambda = \lambda_1$ ($\lambda_1 > \lambda_0$) when the random variable X follows Poisson distribution with mean λ. (10)

TURN OVER

OR

Q 3 (p) What is SPRT? What are the differences between SPRT and Neyman Pearsonian procedure? (05)

(q) Construct SPRT of strength (α, β) to test H_o: $p=p_0$ against H_1 : $p=p_1$ ($p_1 < p_0$) for the following probability distribution. (10)

$f(x, p) = p^x (1-p)^{1-x}$ $\quad x = 0, 1 ; 0 \leq p \leq 1$

$= 0$ $\quad$ otherwise

Q 4 (a) What are Non parametric (NP) Test? When are they useful? (05)

(b) Stating clearly the assumptions made, describe the procedure (with appropriate justification) for Median test based on two independent samples. (10)

OR

Q 4 Stating clearly the assumption made ,describe the procedure (with proper justification) for (15)

i) Sign test based on paired observations

ii) Wilcoxon signed rank test for one sample.

Q 5 (a) Define:- (06)

i) Simple hypothesis

ii) Composite hypothesis

iii) Power function

(b) Let a random variable X have p.d.f. (09)

$f(x, \theta) = \theta e^{-\theta x}$ $\quad x>0, \theta>0$

$= 0$ $\quad$ otherwise

Construct SPRT of strength (α, β) for testing H_o: $\theta = \theta_0$ Against H_1: $\theta = \theta_1$ $(\theta_1 > \theta_0)$

OR

Q 5 (p) Explain the procedure for the Run test for randomness. State clearly the assumptions made and give appropriate justification for the test procedure. (06)

(q) Obtain the UMP test of size α based on a random sample of size n from a normally distributed population with mean μ and variance σ^2 to test $H_0 : \mu = \mu_0$ against $H_1 : \mu < \mu_0$ where μ_0 is a specified constant. (09)

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