Q.P. Code :02160

[Time: $2\frac{1}{2}$ Hours] **[Marks:75]**

Please check whether you have got the right question paper.

N.B:
1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Draw neat diagrams wherever necessary.
4. Symbols have usual meaning unless otherwise stated.
5. Use of log-table and non-programmable calculator is allowed.

(a) Attempt any one:-

i) Show that the equation of motion of a particle of mass `m', moving in a central force field is given By($\frac{d^2u}{d\theta^2} + u = -\frac{m}{u^2L^2}$ F(1/ u). If F (u) is the force in inverse square field, show that, solution of the above equation represents conic section with eccentricity $\in = \sqrt{1 + \frac{2EL^2}{mk^2}}$ 10

ii) With the help of modified simple pendulum, explain rotational motion of earth about its own axis. Hence develop the necessary theory to obtain period of the rotation. 10

(b) Attempt any one:---

i) Show that the path of the particle moving under the central force lies in a single plane containing the center of force and the angular momentum is conserved 05

ii) Write equation of motion on rotating earth and hence define effective gravitational acceleration *ge* . Show that $ge \approx g - \omega^2 R_E \sin^2\theta$ 05

2. (a) Attempt any one:---

i) A double pendulum consists of two weightless rods connected to each other and a point of support. The masses m_1 and m_2 are not equal but the lengths of the rods are equal. Pendulums are free to swing only in one vertical plane. Obtain the Lagrangian for the system. 10

ii) Derive an expression for Lagrange's equations in several dimensions. 10

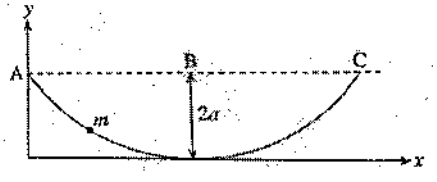
b) Attempt any one:-

i) A bead slides without friction in shape of a cycloid with equations. 5 05

$x = a(\theta - \sin\theta)$

$y = a(1 + \cos\theta)$

where $0 \leq \theta \leq 2\pi$. Write the equation of motion of the bead using Lagrange's technique.



ii) Set up the Lagrangian for a simple pendulum and obtain an equation describing its motion. 05

3. a) Attempt any one:-

   i) Derive Bernoulli's theorem and discuss how it represents the conservation law of energy for a fluid. 10

   ii) Consider a symmetric top rotating with respect to an inertial frame of reference fixed in space. Considering no external torque acting on the body, discuss its motion with respect to body fixed (rotating) frame. 10

   b) Attempt any one:-

   i) Explain the terms: streamline flow and tubes of flow. 05

   ii) What are Euler's angles? Explain the order in which the transformation of the axes is carried out. 05

4. a) Attempt any one:-

   i) State Duffing's equation for a driven damped anharmonic oscillator. Obtain its reduced form by suitable rescaling. With the help of graphical representation, discuss features of the numerical solution of the equation for the following two cases: (1) γ = 0.1 and $f = 0.5$, (2) y = 0.1 and $f = 3$. 10

   ii) Define fractal dimension and explain by applying it to a line of unit length and a square of unit area. Describe construction of Sierpinski gasket and find its fractal dimension. 10

   (b) Attempt any one:---

   i) What do you mean by a fixed point of a map? Calculate the fixed points for (1) λ= 0.6, (2) λ = 1.6 and (3) λ = 2.6. 05

   ii) What is a strange attractor? Explain using the attractor of the Henon map. 05

5. a) Attempt any one:-

   i) It takes Neptune 165 years to orbit the Sun. Find its maximum distance from the Sun in AU. (G = 6.67 $x\ 10^{-11}\ Nm^2/kg^2$ and $Mass\ of\ the\ Sun\ =$ 2.0 x $10^{30} kg$) 04

   ii) A body is dropped from a building of height 100 m in Mumbai (latitude 20°N). Find the deflection due to the coriolis force from the vertical when the body reaches the ground. 04

   b) Attempt any one:-

   i) A body is moving freely in space (no force acts on it). Write down its Lagrangian function and mention the cyclic co-ordinates considering a Cartesian axes frame. 04

   ii) Write a short note on constraints 04

c) Attempt any one:-

i) Consider a fluid flow given by $=\overline{\boldsymbol{v}} = b\ y\ \bar{\boldsymbol{i}}$ in a coordinate system S. Is the fluid incompressible? Explain. 04

ii) Show that a sphere can always rotate with a constant angular velocity about an axis passing through its centre in a torque free situation. 04

d) Attempt any one:---

i) An asymmetric tent map is defined by a function $f\ (x) = 4x$ when $0 \leq x \leq 0.25$ and $f\ (x) = (4 - 4x)/3$ when $0.25 \leq x \leq 1$. Sketch this function and find the fixed points. 03

ii) In a variation of Cantor set, a line segment is divided into five equal segments and the middle three removed. Then this process is continued on each of the remaining two segment. Find the fractal dimension of the resulting set. 03
