QP Code : 50116

(3 Hours) [Total Marks: 100]

N.B.: (1) **All** questions are **compulsory**.
(2) **Figures** to the **right** indicate marks.

Section I

(20X2=40marks)

All questions are compulsory.

1) The Binomial distribution $B(n, p)$ is symmetric if.
(A) n is even (B) $p = 0.5$ (C) n is odd (D) $p = 0.3$

2) In a single factor ANOVA problem involving five populations, with a random sample of four observations from each one, it is found that SST_r =16.1408 and $SSE =$ 37.3801. Then the value of the test statistic is
(A) 0.432 (B) 0.812 (C) 1.619 (E) 2.316

3) The rv X have Bernoulli distribution defined by $P[X = 1] = 1 - P[X = 0] = \theta$, where $0 < \theta < 1$. The mle of θ based on single observation is
(A) $\frac{1-X}{3}$ (B) $1 - X$ (C) $\frac{3-x}{3}$ (D) X

4) Let X follows Poisson distribution with parameter λ, the estimate of $e^{-\lambda}$ is defined as
$T = 1 \quad if \quad X = 0$
= 0 otherwise
which of the following statement is false?
(A) T is unbiased for $e^{-\lambda}$.
(B) Variance of T attains Crammer-Rao lower bound.
(C) T is not UMVUE of $e^{-\lambda}$.
(D) Variance of T is greater than Crammer-Rao lower bound.

5) Consider the event $A = \Omega$, the entire sample space and $B = \phi$, then the $P(A|B)$ is
(A) 0 (B) 1 (C) P(A) (D) not defined

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6) The maximum and minimum values of the quadratic form $4x_1^2 + 4x_2^2 + 6x_1x_2$ for all points $x' = (x_1 \quad x_2)$ such that $x'x = 1$ are

(A) (4,4) (B) (5,3) (C)(7,1) (D)(6,2)

7) Which of the following test is used to test equality of variances in ANOVA?

(A) Kolmogorov Smirnov test (B) Sign test
(C) Levene's test (D) Wilcoxon test

8) If $P = ((p_{ij}))$ denotes the prediction matrix then which of the following is not true?

(A) P is symmetric (B) P is idempotent
(C) $0 \le p_{ii} \le 1, \forall_i$ (D) P is Nonsingular

9) y_1, y_2, y_3, y_4 are independent random variables such that $E(y_1) = E(y_2) = \theta_2 + \theta_3$ and $E(y_3) = E(y_4) = \theta_1 + \theta_2$. Which of the following function is estimable?

(A) $\theta_1 - \theta_3$ (B) θ_1 (C) θ_2 (D) $\theta_1 + \theta_2 + \theta_3$

10) The cumulative distribution function of random variable X is given by

$$F_X(x) = \begin{cases} 0 & x < -1 \\ \frac{1}{4} & -1 \le x < 0 \\ \frac{1}{2} & 0 \le x < 1 \\ \frac{1}{2} & 1 \le x < 2 \\ 1 & x \ge 2 \end{cases}$$

If M denotes the median of the distribution of X then

(A) M=0 (B) 0<M<1 (C) M=1 (D) 1<M<2

11) Let $A = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{pmatrix}$, the distinct eigenvalues of A are

(A) 3 and 1 (B) 5 and 2 (C) 3 and 2 (D) 5 and 3

12) A simple random sample of 10 units is drawn without replacement from a serially numbered population of 100 units. The probability that the i^{th} unit is included in the sample is

A) 0.11 B) 0.1 C) 0.12 D) 0.01

13) The total possible number of samples of size 4 that can be drawn with replacement from a population of 20 units is

(A) 4^{20} (B) 20^4 (C) 480 (D) ${}^{20}C_4$

14) In a 2^4 factorial experiment with two blocks of eight plots each in a replication it was decided to confound BCD and blocks were constructed as given below

Block 1 : (1), a, x_1, x_2, ad, abc, abd, acd.

Block 2: b, c, d, x_3, x_4, ad, bcd, abcd.

Identify treatment combination x_1, x_2 from block 1 and x_3, x_4 from block 2.

(A) $x_1 = bc$ $x_2 = bd$ $x_3 = ab$ $x_4 = ac$

(B) $x_1 = ac$ $x_2 = bc$ $x_3 = bd$ $x_4 = ab$

(C) $x_1 = ab$ $x_2 = ac$ $x_3 = bc$ $x_4 = bd$

(D) $x_1 = bd$ $x_2 = ab$ $x_3 = ac$ $x_4 = bc$

15) If X, Y, Z are independent Poisson Variables each having mean 2 then $P[X+Y+Z=0]$ is

(A) e^{-2} (B) e^{-4} (C) e^{-6} (D) $6e^{-6}$

16) To test $H_0 : F_X(x) = F_0(x)$ against $H_1 : F_X(x) \geq F_0(x)$ for all x using Kolmogorov-Smirnov test and sample of size n, if $D_n^+ = \sup_x [S_n(X) - F_0(x)]$ reject H_0 if

(A) $|D_n^+| > D_n, \alpha$ (B) $D_n^+ > D_n, \alpha$ (C) $|D_n^+| < D_n, \alpha$ (D) $D_n^+ < D_n, \alpha$

17) If a hypothesis is rejected at 1% level of significance then

(A) It is rejected at 5% level.

(B) It can not be rejected at 5% level.

(C) It is rejected at 0.1% level.

(D) It is rejected at 0.01% level.

18) Which of the following statement is false?
(A) A block design is connected if and only if rank of C matrix is $v-1$
(B) A connected block design is balanced if there exists $\theta > 0$ such that $\theta C = C^2$
(C) A block design is orthogonal if all the elements of incidence matrix are non zero.
(D) The C-matrix has $(v-1)$ equal non zero eigen values then block design is balanced.

19) For the design given below which of the statement is false?
B1: A, B, C, D B 2: A, B, C, E.
B3: A, B, D, E B 4: A, C, D, E.

(A) rank of C matrix is four
(B) block design is non orthogonal
(C) block design is equiblock sized.
(D) block design is equireplicated

20) Which of the following statement is false in connection with construction of 2^{6-3} Fractional factorial design (FFD)?
(A) design generators are D=AB, E=AC, F=BC
(B) defining relation is I = ABD = ACB = BCF
(C) ade, bdf, cdf treatment combination can be used to generate the FFD
(D) Resolution IV design.

Section II

(3X10 =30 marks)

Attempt any three (03) questions out of five (05).

1) Find the mean vector and the covariance matrix of random vector $X' = (X_1, X_2)$ with p.d.f. $f(\underline{x})$ which is bivariate normal with mean vector μ and covariance matrix Σ.

$$f(x) = \frac{1}{2\pi}\exp\left(-\frac{1}{2}\left(X_1^2 + 2X_2^2 + 2X_1X_2 - 14x_1 - 22x_2 + 65\right)\right)$$

Also obtain marginal p.d.f. of X_1 and X_2.

2) Prove that empirical distribution function is unbiased estimator of $F(x)$.

3) A layout of the block design with five treatments A, B, C, D, E in four blocks is given below

Block 1:	A, B, C, E	Block 2:	A,B, D,E
Block 3:	A,C, D, E	Block 4 :	B,C,D,E

(a) Find the incidence matrix and C matrix of this design.
(b) Obtain bounds for variance of best linear unbiased estimate of any elementary treatment contrast given that C matrix has eigenvalues $11/4$ with multiplicity 3.

4) State the assumptions required in the analysis of variance. Derive Bartlett's test for testing equality of variances for one way analysis of variance model.

5) It is claimed that a given sample of size n comes from uniform distribution over (0,1). How would you verify the claim using Kolmogorov- Smirnov test?

Section III

(2X15=30 marks)

Attempt any two (02) questions out of three (03)

1) (a) Define population principal components. What is scree plot?

(b) Let $X' = (x_1 \quad x_2 \quad x_3)$ has mean vector $(0 \quad 0 \quad 0)'$ and covariance matrix

$$\Sigma = \begin{pmatrix} 9+\delta & 3 & 3 \\ 3 & 1+\delta & 1 \\ 3 & 1 & 1+\delta \end{pmatrix} \quad , \quad \delta > 0$$

Obtain the principal components Y_1, Y_2, Y_3 for Σ. [hint: δ is one of the eigenvalue of Σ with multiplicity 2].

2) (a) A coin is tossed n times and let p denotes the probability that head is observed. How do you test the hypothesis that the coin is a fair coin?
(b) How do you test whether a given sequence of two symbols is a random sequence? Is the sequence given below a random sequence? Find the numerical value of the statistic and indicate the testing procedure.
MFFMMFFFMFMMFMMFFFMF.

3) Consider 3^3 factorial experiment in r replicates having 3 quantitative factors. Write down the regression model using coded variables x_1, x_2, x_3 and state the assumptions.

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Let

$$A_1 = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & -2 & 1 \end{bmatrix} \quad F_1 = \{1, \rho_1, \rho_1^2\}$$

$$A_i = \begin{bmatrix} A_{i-1} & A_{i-1} & A_{i-1} \\ -A_{i-1} & 0 & A_{i-1} \\ A_{i-1} & -2A_{i-1} & A_{i-1} \end{bmatrix}$$

$$F_i = \{F_{i-1}, \rho_i F_{i-1}, \rho_i^2 F_{i-1}\} \quad i = 2,3,...m$$

Show that elements of $A_i F_i$ represents the contrasts belonging to main effects and interaction obtained from the total yield of treatment combination in 3^m factorial experiments. Explain the use of this result in preparing ANOVA for factorial experiment.

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