QP Code : 78965

(OLD COURSE)

Duration:[2½Hours] [Total Marks: 75]

N.B. 1) All questions are compulsory.
2) Figures to the right indcate full marks.

1. (a) Attempt any **ONE** question: (8)

i. If $(A^n) = (a_{ij}^n)$ is the n^{th} power of adjacency matrix A of a graph G with $V(G) = \{v_1, v_2, \ldots v_n\}$, then prove that

(1) a_{ij}^2, $i \neq j$ is the number of $v_i - v_j$ path of length 2.

(2) $a_{ii}^2 = deg(v_i)$.

(3) $\frac{1}{6}$trace of A^3 is the number of triangles in G.

ii. State and prove $Havel - Hakimi$ theorem for degree sequence of a graph.

(b) Attempt any **TWO** questions: (12)

i. Define cut edge of a graph G. Prove that an edge e of a graph G is a cut edge of G if and only if e is acyclic and hence prove that every edge in a tree is a cut edge.

ii. If G is a simple graph on at least six vertices, then prove that either $K_3 \subseteq G$ or $K_3 \subseteq G^c$

iii. If G is graph of order n with $\delta(G) \geq (n-1)/2$, then show that G is connected where $\delta(G)$ denotes the minimum degree of G. Give an example of a graph with $\delta(G) \geq (n-2)/2$ which is not connected.

iv. Prove that every (p, q) graph with $q \geq p$ contains a cycle. Is it true if $q \geq p - 1$? Justify.

2. (a) Attempt any **ONE** question: (8)

i. Let G be (p, q) graph. Show that the following statements are equivalent.

1) G is tree.

2) G is acyclic and $p = q + 1$.

3) G is connected and $p = q + 1$.

ii. State and prove Cayley's formula for spanning trees.

(b) Attempt any **TWO** questions: (12)

i. Define vertex connectivity and edge connectivity of a graph G. Prove that vertex connectivity of a graph is less than or equal to edge connectivity of a graph G.

ii. If T is spanning tree of a connected graph G and e is an edge of G that is not in T, then prove that $T + e$ contains a unique cycle that contains the edge e.

iii. Use Huffman coding to encode these symbols with the given frequencies:
a : 0.20, b : 0.10, c : 0.15, d : 0.25, e : 0.30. what is average number of bits required to encode a character?

iv. Let $\tau(G)$ denote the number of spanning trees of a graph G. If $e \in E(G)$ is not a loop then prove that $\tau(G) = \tau(G - e) + \tau(G.e)$

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3. (a) Attempt any **ONE** question: (8)
   i. Prove that the cube graph Q_k is connected bipartite $k-$regular graph with 2^k vertices.
   ii. If G is a graph on p vertices with $p \geq 3$ such that $deg(u) + deg(v) \geq p$ for every pair of non adjacent vertices u and v in G, then prove that G is Hamiltonian.

   (b) Attempt any **TWO** questions: (12)
   i. Define closure of a graph $C(G)$. Show that if the closure of graph G is complete then G is Hamiltonian.
   ii. Show that the cube graph Q_k, $k \geq 2$ is a Hamiltonian graph.
   iii. If G is a graph on p vertices with $p \geq 3$ such that $deg(u) + deg(v) \geq p - 1$ for every pair of non adjacent vertices u and v in G, then show that G contains a Hamiltonian path.
   iv. Let G be a simple graph with p vertices and q edges with $p \geq 3$. If $q \geq \frac{p^2-3p+6}{2}$ then prove that G is Hamiltonian.

4. Attempt any **THREE** questions: (15)
   (a) If G is a graph of order p and size q, then prove that $\sum_{v \in v(G)} degv = 2q$. Hence prove that every graph has an even number of odd vertices.
   (b) Show that every nontrivial graph contains at least two vertices which are non cut vertices.
   (c) Show that a vertex v in a tree T is a cut vertex of T if and only if $deg(v) > 1$.
   (d) If T is tree with p vertices whose degree sequences is $(d_1, d_2, ...d_p)$, then prove that $\sum_{i=1}^{p} d_i = 2(p-1)$
   (e) If G is Hamiltonian graph then for every nonempty proper subset S of $V(G)$, prove that $\omega(G - S) \leq |S|$. Is converse true? Justify.
   (f) Prove that $K_{m,n}$ is Hamiltonian if and only if $m = n$.

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