Q.P.Code : 07084

.Duration: 2 hrs 30 min Revised Course Max Marks:75

N.B:1) All questions are compulsory.
2) From questions 1,2 and 3 attempt any one from part(a) and any two from part(b)
3) From question 4 attempt any three.
4) Figures to the right indicate marks for the sub-parts

1.a) i) State and prove Baye's Theorem **8**

ii) Show that for any events A and B, the following conditions are equivalent: **8**

- A and B are independent,
- Ω \ A and B are independent,
- Ω \ A and Ω \ B are independent

b) i) Let A, B, C be independent events. Show that A, B ∩ C and A, B \ C are independent **6**

ii) You randomly throw a dart at a circular dartboard with radius *R*. It is assumed that the dart is infinitely sharp and lands on a completely random point on the dartboard. How do you calculate the probability of the dart hitting the bull's-eye having radius b? **6**

iii) Let Ω={1,2,3,4,5,6} with uniform probability . Show that if A,B⊂ Ω are independent and A has 4 elements, then B must have 0,3 or 6 elements. **6**

iv) Let Ω =[0,1]. Adding as few sets as possible, complete the family of sets $\{\phi, [0, \frac{1}{2}), \{1\}\}$ to obtain a field **6**

2.a) i) If $R_1 \ldots R_n$ are discrete random variables on a given probability space with probability functions $p_1, \ldots p_n$. Let $p_{1,2,\ldots n}$ be the joint probability function of $R_1, \ldots R_n$ defined by $p_{1,2,\ldots n}(x_1, x_2, \ldots, x_n) = P\{R_1 = x_1, R_2 = x_2, \ldots R_n = x_n\}$. Then *prove that* $R_1 \ldots R_n$ are independent iff $p_{1,2,\ldots n}(x_1, x_2, \ldots, x_n) = p_1(x_1) \ldots \ldots p_n(x_n)$. **8**

ii) Let R'=g(R_1,R_2) , R''= h(R_1,R_2). Show that **8**

- E(R'+R'')= E(R')+E(R'')
- E(aR)= a E(R)
- If $R_1 \leq R_2$ then E(R_1) $\leq E(R_2)$

b) i) If $R_1 \ldots R_n$ are discrete random variables on a given probability space with probability functions $p_1, \ldots p_n$. Let $p_{1,2,\ldots n}$ be the joint probability function of $R_1, \ldots R_n$ defined by $p_{1,2,\ldots n}(x_1, x_2, \ldots, x_n) = P\{R_1 = x_1, R_2 = x_2, \ldots R_n = x_n\}$. Then $R_1 \ldots R_n$ are independent iff $p_{1,2,\ldots n}(x_1, x_2, \ldots, x_n) = p_1(x_1) \ldots \ldots p_n(x_n)$. **6**

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ii) Find the distribution function of X(w)=c (a constant random variable identically equal to c). **6**

iii) Let R_1 be absolutely continuous with density $f_1(x) = e^{-x}, x\geq 0$ **6**

$= 0, \quad x<0.$

Define $$R_2 = R_1 \;\; if \; R_1 \leq 1$$
$$= \frac{1}{R_1} \; if \;\; R_1 > 1$$

Show that R_2 is absolutely continuous and find its density

iv) A biased coin with probability of heads p and tails 1-p is tossed repeatedly. Let X be the number of tosses until heads appears for the first time. Compute the expectation of X. **6**

3.a) i) State and prove Schwarz inequality. **8**

ii) State and prove Chebyshev's inequality **8**

b) i) Prove cov$(R_1, R_2\ ) = E(R_1 R_2\ )$-E$(R_1)E(R_2)$ **6**

ii) A fair coin is tossed independently n times. Let S_n be the number of heads obtained. Use Chebyshev'sinequality to find a lower bound of the probability then S_n/n differ from ½ by less then 0.1 when n=100. **6**

iii) Three coins 5\$; 10\$ and 25\$ are tossed; X is the total amount shown and Y is the number of heads. Find the explicit formula for E(X|Y). How many different values does E(X|Y) take? **6**

iv) Let Y be a discrete random variable. Show that E(E(X|Y)) = E(X). **6**

4) a) A monkey hits a computer keyboard three times at random. What is the chance of getting a three letter word with a consonant followed by two vowels? The word does not have to make sense. For simplicity, assume that there are 100 keys. **5**

b) Two identical coins are flipped simultaneously. Let X be the number of heads and Y be the number of tails shown. What is the joint distribution of X and Y? What are the marginal distributions? **5**

c) Find the lower bound for the probability that the average number of heads in 100 tosses of a coin differs from ½ by less then 0.1. **5**

d) A coin is tossed. If it shows heads, you pay 2 dollars. If it shows tails, you spin a which gives the amount you win distributed with uniform probability between 0 and 10 dollars. You gain (or loss) is a random variable X. Find the distribution function and use it to compute the probability that you will not win at least 5 dollars. **5**

e) Let R_1 be normally distributed with mean m and variance σ^2. Let R_2=aR_1+b. Show that is normally distributed. Find the density function of R_2 , $E(R_2)$ *and* $Var(R_2)$ **5**

f) Prove: $P(A\backslash B) = P(A) - P(B)\; if \;\; B \subseteq A.$ **5**

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
