OLD COURSE [Max Marks:75]

Duration: $2^1/_2$ Hours

N.B. 1. All questions are compulsory.

2. From Question 1, 2 and 3, Attempt any one from part(a) and any two from part(b).

3. From Question 4, Attempt any THREE

4. Figures to the right indicate marks for the respective parts.

Q.1 a i Define atoms in a field F. Show that different atoms in a field are disjoint. Show that if F is a finite field, then every nonempty event A of F contains an atom. (8)

ii Define $\lim_{n\to\infty} sup A_n$ and $\lim_{n\to\infty} \inf A_n$

Show that $\lim_{n\to\infty} \inf A_n \subset \lim_{n\to\infty} \sup A_n$.

Further show that

Show that $\lim_{n\to\infty} \sup A_n$ = {w/w$\in A_n$ for infinitely many n}

b i State and prove Total Probability theorem. (12)

ii Define a σ field. Show that every σ field is a field. Is the converse true? Justify your answer.

iii Ω={1,2,3,4}, F={ϕ,{1,2},{3,4}, Ω }. P(ϕ)=0, P(Ω)=1, P({1,2})=$\frac{4}{8}$, P({3,4})=$\frac{5}{8}$. Is P a finitely additive probability measure? Justify your answer.

iv Let F be σ-field on Ω=[0,1], such that $[\frac{1}{n+1},\frac{1}{n}]\in$F for n=1,2,3,4………
Show that {0} ∈F,{1/n/ n=2,3,4……} ∈F for all n $\epsilon\mathbb{N}$.

Q.2 a i Define a random variable.Let X be a random variable. Show that the map $P_X(B)$=P({X ∈B}) is a probability measure on the σ-field B of Borel subsets of ℝ.Show that if X is a random variable with respect to a σ-field F and F⊆G for some σ-field G, then X is a random variable with respect to G. (8)

ii Define the distribution function F_X of a random variable X. Show that $\lim_{y\to-\infty} Fx(y) = 0$

b i For the function X(w)=w, find the smallest σ-field on Ω = {-4, 0, 4} with respect to which the function is a random variable. (12)

ii Find the distribution function of X(x)=x on Ω=[0,2] with Lebesgue measure.

iii Check if $f(x) = \begin{cases} [x]\ for\ x \in [0,2] \\ 0\ \ otherwise \end{cases}$ is a density function on ℝ ? Justify.

iv If $A_1\subset A_2\subset A_3\subset \ldots\subset A_n \subset\ldots$ is an increasing sequence of events then show that $P(U_{k=1}^{\infty} A_k) = \lim_{k\to\infty} P(A_k)$.

TURN OVER

Q.3 a i Suppose that X and Y are random variables defined on the same probability space(Ω,F,P). Show that $\{(X,Y) \in B\}$ belongs to F for any Borel set B in $\mathbb{R}^2$. Further show that if X and Y are jointly continuous random variables, then X is absolutely continuous. (8)

ii Let X and Y be random variables defined on the same probability space. Define Joint distribution of random variables X and Y as a probability measure $P_{X,Y}$ on $\mathbb{R}^2$. Show that the distribution P_X of X and P_Y of Y can be obtained from the joint distribution as follows

$P_Y(B) = P_{X,Y}(\mathbb{R} \times B)$, for any Borel set B in $\mathbb{R}$.

b i A die is outcome for the first rolled twice; X is the sum of the outcomes and Y is the outcome for the first roll. Find $E(X|Y)$. (12)

ii Compute the expectation of a binomial distribution.

iii If X is a simple random variable with values in an interval (a,b) and $h:(a, b) \rightarrow \mathbb{R}$ is a convex function, then prove that $h(E(X)) \leq E(h(X))$.

iv Suppose that X and Y are jointly continuous random variables with joint density $f_{X,Y}(x,y)=ce^{x+y}$ for $x,y \in (-\infty, 0]$ and $f_{X,Y}(x,y)=0$ otherwise. Find the value of c and the probability that X<Y.

Q.4 i Show that (15)

$\lim_{n\rightarrow\infty} \inf A_n = \{w / w \in A_n \text{ for all but finitely many } n\}$

ii Romeo and Juliet have a date at a given time, and each will arrive at the meeting place with a delay between 0 and 1 hour, with all pairs of delays being equally likely. The first to arrive will wait for 15 minutes and will leave if the other has not yet arrived. What is the probability that they will meet?

iii Let $\Omega=\{1,2,3\}$ and $F\{\emptyset,\Omega,\{1\},\{2,3\}\}$ .Is $X(w)=1+w$ a random variable with respect to the σ-field F ? If not, give an example of non-constant function which is a random variable.

iv Find the distribution function of X , where X(w)=2 for $w \in A$ and X(w)=3 otherwise, and P(A)=1/5 .

v Compute the variance of a binomial distribution.

vi Define $E(X|B)$ for any arbitrary random variable X and any event B. Also define $E(X|Y)$ for any arbitrary random variable X and a discrete random variable Y.
