Rev. Course **(2½ Hours)** **Total Marks: 75**

N.B.: (1) All questions are compulsory.
(2) Figures to the right indicate marks for respective sub questions.

1. (a) Attempt any ***one*** of the following: **[8]**

(i) Derive the Muller's iteration formula

$$x_{k+1} = x_k - \frac{2a_2}{a_1 \pm \sqrt{a_1^2 - 4a_1a_2}}$$ for finding the roots of the polynomial

$f(x)= a_0x^2+a_1x+a_2$

(ii) Prove that the iteration method $x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$ has quadratic rate of convergence.

Hence find the double root near x=1.1 of $x^3-4x^2+5x-2=0$.

(b) Attempt any ***two*** of the following: **[12]**

(i) Find the smallest positive root correct to 4 decimal places by False position method for the equation $x^4-x-10=0$.

(ii) Find the smallest positive root correct to 3 decimal places by Secant method for the equation $x- e^{-x} =0$.

(iii) Perform three iterations of Newton-Raphson method correct to 4 decimal places to obtain approximate value of $(17)^{1/3}$ taking initial approximation as $x_0=2$.

(iv) Perform one iteration using Muller method for the equation $x^3- (1/2) = 0$, $x_0=0$, $x_1=1$, $x_2= (1/2)$.

2. (a) Attempt any ***one*** of the following: **[8]**

(i) Discuss Birge-Vieta process for determining the roots of an algebraic equation.

(ii) Discuss Bairstow process for determining the roots of an algebraic equation.

(b) Attempt any ***two*** of the following: **[12]**

(i) Using synthetic deviation, find the value of P(2), $P'(2)$, $P''(2)$ for the polynomial $x^5+x^4-3x^2+2x-7=0$.

(ii) Use Birge-Vieta method to find a real root correct to 3 decimal places of the equation $x^3-11x^2+32x-22=0$, $P_0=0.5$

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(iii) Perform one iteration using Bairstow method for the polynomial $x^4+x^3+2x^2+x+1=0$, using initial approximation $p_0=0.5$, $q_0=0.5$.

(iv) Find all the roots of the polynomial $x^3-6x^2+11x-6=0$ by Graffe's root squaring method. Do 3 iterations.

3. (a) Attempt any **one** of the following : [8]

i) Describe Jacobi iterative method to solve a system of linear equations.

ii) Let L and U denote lower and upper triangular matrices obtained by triangular decomposition and consider the process $A = A_0 = L_0U_0$, $A_1 = U_0L_0 = L_1U_1$, … , $A_k = U_{k-1}L_{k-1} = L_kU_k$, …
Show that A and A_k have the same eigenvalues.

(b) Attempt any **two** of the following : [12]

i) Find Crout's factorization of the matrix, $A = \begin{bmatrix} 3 & 6 & 9 \\ 2 & 6 & 10 \\ 1 & 3 & 6 \end{bmatrix}$
Hence solve a system of equations :
$3x + 6y + 9z = 0$, $2x + 6y + 10z = 2$ and $x + 3y + 6z = 2$

ii) Consider a system of equations :
$10x + y - 2z = 17$, $x+10y + 2z = -6$ and $x - 2y + 10z = 14$
Check whether Jacobi's iteration method to solve a system of linear equations is applicable to this system? Justify your answer.
If so, apply it by taking $(x^{(0)}, y^{(0)}, z^{(0)}) = (1, 0, 0)$.
Perform two iterations.

iii) Using Jacobi iteration method find the eigenvalues and eigenvectors of a matrix $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$.

iv) Determine the smallest eigenvalue in magnitude and corresponding eigenvector for a matrix, $A = \begin{bmatrix} 4 & 5 \\ 6 & 5 \end{bmatrix}$ using power method. Perform two iterations by taking an initial approximation to the eigenvector as $v^{(0)} = [2, -3]^t$.

4. Attempt any **three** of the following : **[15]**

(a) Apply Regula - Falsi method to find $\sqrt[3]{20}$ in the interval [2, 3]. Perform two iterations. Find absolute error in the calculations.

(b) Prove that $f(x) = x^3 - x^2 - x + 1$ has a double root at $x = 1$. Use modified Newton – Raphson method to obtain this root by taking $x_0 = 0.5$. Perform two iterations.

(c) Perform two iterations of Chebyshev method to find an approximate root of $f(x) = x^3 - 5x + 1$, by taking $x_0 = 0.5$.

(d) Use the Birge – Vieta method to find a root of $P_5(x) = x^5 - x + 1$. Perform one iteration by taking $x_0 = -1$.

(e) (e) Check whether Cholesky factorization is possible for a matrix, $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 5 & 5 \\ 1 & 5 & 14 \end{bmatrix}$. Justify your answer. If so, find its Cholesky factorization.

(f) Determine the largest eigenvalue in magnitude and corresponding eigenvector for a matrix, $A = \begin{bmatrix} 1 & 3 \\ 3 & 5 \end{bmatrix}$ using power method. Perform three iterations by taking an initial approximation to the eigenvector as $v^{(0)} = [1, 2]^t$.
