Exam : S.Y.B.A-Semester 4
Subject: Mathematics Paper 2(Revised)
Exam Date: 27-4-2019
Q.P.Code- 66045
ANSWER KEY

(3 Hours) [Total Marks: 100]

Note: (i) All questions are compulsory. (ii)
Figures to the right indicate marks for respective parts.

Q.1	Choose correct alternative in each of the following (20)			
$i.$	If $F:[a,b]\to IR$ be Riemann Integrable function then which of the following is true			
	(a)	F must be continuous	(b)	F must be monotonic
	(c)	F must be constant	(d)	F must be bounded
	Ans	(d)		
$ii.$	If $f:[0,1]\to IR$ be defined by $f(x)=\begin{cases} 1\ if\ x\in Q \\ 0\ if\ x\in IR\backslash Q\end{cases}$ then			
	(a)	U(f, P) = 0, L(f, P) = 0		
	(b)	U(f, P) = 1, L(f, P) = 1		
	(c)	U(f, P) = 1, L(f, P) = 0		
	(d)	None of the above.		
	Ans	(c)		
$iii.$	If $f:[0,2]\to IR$ be a function such that $f(x)=4x-3$. Let P be a partition with P: 0, 05, 1, 1.5, 2. Then L(f, P) is			
	(a)	-3	(b)	0
	(c)	2	(d)	None of the above.
	Ans	(d)		
$iv.$	Let f:$[a,b]\to\mathbb{R}$ be continuous function and $f(x)>0,\forall x$.If $F(x)=\int_0^x f(t)dt$ then			
	(a)	$F(x)>0,\forall x\in[a,b]$	(b)	$F(x)$ is strictly increasing on $[a,b]$
	(c)	$F(x)$ is convex on $[a,b]$	(d)	None of these
	Ans	(b)		
$v.$	If $f:\left[0,\frac{\pi}{2}\right]\to\mathbb{R}$ is defined by $f(x)=\int_{sinx}^{cosx} e^{-t^2}dt$,then $f'\left(\pi/4\right)$ equals			
	(a)	$\sqrt{\frac{1}{e}}$	(b)	$-\sqrt{\frac{2}{e}}$

	(c)	$\sqrt{\frac{2}{e}}$	(d)	$-\sqrt{\frac{1}{e}}$
	Ans	(b)		
vi.	$The\ type\ 2\ integral \int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx$			
	(a)	Diverges	(b)	Converge to 0
	(c)	Converge to $\frac{\pi}{2}$	(d)	None of these
	ANS	(c)		
vii.	$\int_0^1 x^7(1-x)^3 dx =$ ________			
	(a)	$\frac{3!2!}{5!}$	(b)	$\frac{4!2!}{6!}$
	(c)	$\frac{3!1!}{5!}$	(d)	None of these
	Ans	(c)		
viii.	$\int_0^{\frac{\pi}{2}} cos^3 x\,dx =$ __________			
	(a)	1	(b)	2
	(c)	4	(d)	None of these
	Ans	(a)		
ix.	The expression for finding length of the curve $y = \frac{x^4}{4}$ over [0,2] is given by			
	(a)	$\int_0^2 \sqrt{1+x^6}dx$	(b)	$\int_0^1 \sqrt{1+x}dx$
	(c)	$\int_0^1 \sqrt{1+x^4}dx$	(d)	$\int_0^1 \sqrt{1+\frac{x^2}{2}}dx$
	Ans	**(a)**		
x.	The expression for the volume enclosed by revolving $y = f(x)$ about X-axis between $x = a$ & $x = b$ is given by			
	(a)	$\pi\int_a^b (f(x))^2 dx$	(b)	$\int_a^b (f'(x))^2 dx$
	(c)	$\int_a^b f'(x)dx$	(d)	$\pi\int_a^b (f'(x))^2 dx$
	Ans	(a)		
Q2.	Attempt any **ONE** question from the following: (08)			

$a)$	i.	Let f:$[a,b]\rightarrow IR$ be a bounded function. Prove that f is Riemann integrable on [a, b] if and only if for any $\in > 0$ there exist a partition P of [a, b] such that $U(f,P) - L(f,P) < \in$.
	Ans	f is Riemann integrable on [a, b] if for any $\in > 0$ there exist a partition P of [a, b] such that $U(f,P) - L(f,P) < \in$. (4 marks) f is Riemann integrable on [a, b] only if for any $\in > 0$ there exist a partition P of [a, b] such that $U(f,P) - L(f,P) < \in$. (4 marks) Let f be R-integrable then U(f)=L(f) Since U(f)= inf{U(f,P) for all partition P of [a,b]} There exist a partition P1 of [a,b] such that U(f,P1) < U(f)+ $\in /2$ Also L(f) =sup{ L(f,P) for all partition of [a,b]} There exist a partition P2 of [a,b] such that L(f,P2) > L(f)- $\in /2$ Let p be the union of P1 and P2 Then U(f,P) – L(f, P) < U(f) – L(f) + $\in /2 + \in /2 < \in$ Conversely Let U(f,P)-L(f, P) < $\in$ Since U(f)-L(f) $\leq$ U(f,P)- L(f,P) < $\in$ Implies 0 $\leq$ U(f)-L(f) < $\in$ Hence u(f)=L(f) (4 marks)
	ii.	Let f:$[a,b]\rightarrow IR$ be continuous function then prove that f is Riemann integrable on [a, b].
	Ans	Claim: if f is continuous on [a,b] then f is R-integrable. Let P={x0, x1, …, xn] be a partition of [a,b] As f is continuous by boundedness theorem there are x'_i and x''_i in $[x_{i-1}, x_i]$ such that $M_i=f(x'_i)$ and $m_i=f(x''_i)$ where $M_i=\sup\{f(x)/x \in [x_{i-1},x_i]\}$ & $m_i=\inf\{f(x)/x \in [x_{i-1},x_i]\}$- ……….3Marks $U(P,f)—L(P,f)=\sum_{i=1}^{n}(Mi - mi)(xi - xi - 1) \leq \sum_{i=1}^{n}(f(x'i) - f(x''i))\lVert P\rVert$ but f is continuous hence is uniformly continuous on [a,b] For $\epsilon > 0$ *there exist* $\delta > 0$ such that $\lvert x - y\rvert < \delta \Rightarrow \lvert f(x) - f(y)\rvert < \frac{\epsilon}{b-a}$ …………. 2 marks Hence $U(P,f)—L(P,f) < \sum_{i=1}^{n}\lVert P\rVert\frac{\epsilon}{b-a} = \frac{\epsilon}{b-a}(b-a)=\epsilon$ if $\lVert P\rVert < \delta$ ……….3marks
Q.2	Attempt any **TWO** questions from the following: (12)	
$b)$	i.	If f is Riemann integrable on [a, b] then prove that $\lvert f \rvert$ is also Riemann integrable on $[a,b]$.

Ans

Given: f is R integrable on $[a,b]$.

Claim: $|f|$ is R integrable on $[a,b]$.

Let $P = \{x_0 = a, x_1, x_{2,} .. ..x_n = b\}$ be any partition of $[a, b]$

Let $M_i = \sup \{f(x) : x \in [x_{i-1}, x_i]\}$ and $M_i^{'} = \sup \{|f|(x) : x \in [x_{i-1}, x_i]\}$

$m_i = \inf\{f(x) : x \in [x_{i-1}, x_i]\}$ and $m_i^{'} = \inf\{|f|(x) : x \in [x_{i-1}, x_i]\}$,

$i = 1,2, ... n.$

To show that,

$$M_i^{'} - m_i^{'} \leq M_i - m_i, \quad i = 1,2,....n.$$

Let, $x,y \in [x_{i-1}, x_i]$

$$m_i \leq f(x) \leq M_i$$
$$m_i \leq f(y) \leq M_i$$
$$\therefore m_i - M_i \leq f(x) - f(y) \leq M_i - m_i$$
$$\therefore - m_i - M_i \leq f(x) - f(y) \leq M_i - m_i$$

Consider,

$$|f(x)| = |f(x) - f(y) + f(y)|$$
$$\leq |f(x) - f(y)| + |f(y)|$$
$$\leq M_i - m_i + |f(y)|$$

Here, $y \in [x_{i-1}, x_i]$

$\therefore |f(x)| \leq M_i - m_i + |f(y)|, \ \forall x \in [x_{i-1}, x_i]$

$\therefore M_i - m_i + |f(y)|$is an upper bound of $\{f(x) : x \in \left[x_{i-1}, x_i\right]\}$.

$\therefore M_i^{'} \leq M_i - m_i + |f(y)|$, ($\because M_i'$ is least of upper bound)

$$\therefore M_i^{'} - M_i + m_i \leq |f(y)|, \forall y \in [x_{i-1}, x_i]$$

$\therefore M_i^{'} - M_i - m_i$is lower bound of $\{f(x) : x \in \left[x_{i-1}, x_i\right]\}$.

$\therefore M_i^{'} - M_i + m_i \leq m_i'$($\because m_i'$ is greatest lower bound)

$\therefore M_i^{'} - m_i^{'} \leq M_i - m_i, \ \ i = 1,2,....n.$ ……3 marks

Multiplying above relation by $(x_i - x_{i-1})$ and adding above n relations we have,

$$U(|f|, P) - L(|f|, P) \leq U(f, P) - L(f, P) \qquad (*)$$

As, f is R integrableon $[a, b]$.

Hence, for given $\epsilon > 0$, $\exists$partition$P_\epsilon$of $[a, b]$ such that,

$U\left(f, P_\epsilon\right) - L\left(f, P_\epsilon\right) < \epsilon$

$\therefore$ by ($*$)

$$U\left(|f|, P_\epsilon\right) - L\left(|f|, P_\epsilon\right) < \epsilon$$

$\therefore$ $|f|$ is R integrable on $[a,b]$. ……… 3 marks

ii.

If f is a bounded function on [a, b] then prove that $L(f) \leq U(f)$.

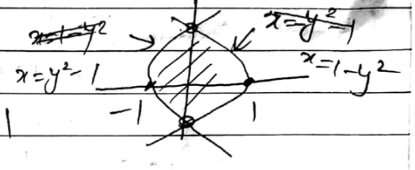
	Ans	Prove that U(f, P) $\le$ L(f, P) Implies inf{U(f, P) for all partition P } $\le$ sup{L(f, P) for all partition P } Hence $L(f) \le U(f)$
	iii.	If f and g are Riemann integrable on [a, b] then prove that f + g is also Riemann integrable on [a, b].
	Ans	Let P={x0, x1, …, xn] be a partition of [a,b] let M_i =sup{(f+g)(x)/x ∈ [x_{i-1},x_i] } &m_i =inf {(f+g)(x)/x ∈ [x_{i-1},x_i] } let M'_i =sup{f(x)/x ∈ [x_{i-1},x_i] } &m'_i =inf {f(x)/x ∈ [x_{i-1},x_i] } let M''_i =sup{g(x)/x ∈ [x_{i-1},x_i] } &m''_i =inf {g(x)/x ∈ [x_{i-1},x_i] } then $M_i \le M'_i + M''_i$ and $m_i \ge m'_i + m''_i$ for i=1,2,….n Hence U(P,f+g) - L(P,f+g) $\le$U(P,f) - L(P,f) + U(P,g) - L(P,g)……(*) 3 marks But f & g are R- integrable on [a,b] hence there are partitions say P_1 and P_2 Such that U(P_1 ,f)- L(P_1 ,f) $<\frac{\epsilon}{2}$ and U(P_2 ,g)- L(P_2 ,g) $<\frac{\epsilon}{2}$ 2 marks Take P=$P_1 \cup P_2$ Then U(P,f)- L(P,f) $<\frac{\epsilon}{2}$ and U(P,g)- L(P,g) $<\frac{\epsilon}{2}$ ….2 marks Hence U(P,f+g) - L(P,f+g) $<\epsilon$ by * Therefore f+g is R-integrable on [a,b]
	iv.	For any two Riemann integrable functions f and g, prove that $$\int_a^b f+g = \int_a^b f + \int_a^b g$$
	Ans	Prove that $\int_a^b f+g < \int_a^b f + \int_a^b g + \in$ And $\int_a^b f+g > \int_a^b f + \int_a^b g - \in$
Q3.	Attempt any **ONE** question from the following: (08)	
$a)$	i.	Let f:[a,b]→ℝ be a continuous and let g be Riemann integrable on [a,b]such that $g(x) \ge 0, \forall x \in [a,b]$.Show that $\exists c \in (a,b)$ such that $\int_a^b f(x)g(x)dx = f(c)\int_a^b g(x)dx$.
	Ans	Ans: Since f is continuous ⇒f is Riemann integrable on [a,b] ⇒f is bounded also ⇒ m $$\le f(x) \le M \Rightarrow m\int_a^b g(x)dx \le \int_a^b f(x)g(x)dx \le M\int_a^b g(x)dx \Rightarrow \int_a^b f(x$$ $$\int_a^b g(x)dx$$ ∴ $\exists c \in (a,b)$ such that $f(c) = \mu$ hence the proof.

	ii.	State and prove comparison test for improper integrals of type-II, $\int_a^b f(x)dx$, $\lvert f(x)\rvert \to \infty$ as $x \to a+$
	Ans	State and prove comparison test for improper integrals of type-II, $\int_a^b f(x)dx$, $\lvert f(x)\rvert \to \infty$ $as\ \ x \to a+$ Statement: Suppose f , g are two functions defined on (a , b] and if $\lvert f(x)\rvert \to \infty$, $\lvert g(x)\rvert \to \infty\ \ as$ $x \to a+$ If $\lvert f(x)\rvert \leq k\lvert g(x)\rvert for\ all$ b≥ x ≥ x_0 >a ,for some k > 0, then Convergence of $\int_a^b \lvert g(x)\rvert\ dx$ implies Convergence of $\int_a^b \lvert f(x)\rvert\ dx\ and\ d$ivergence of $\int_a^b \lvert f(x)\rvert\ dx$ implies divergence of $\int_a^b \lvert g(x)\rvert\ dx$ (2M) Proof: Consider any Ɛ > 0 Given $\int_a^b \lvert g(x)\rvert dx$ is Convergent at a. Hence by Cauchy's Criterion for Ɛ > 0 there exists δ_1>0 such that for all x ,y ∈ (a ,a + δ_1) , $\lvert \int_x^y \lvert g(x)\rvert dx\rvert < \frac{\varepsilon}{k}$ Let 0 < δ < min { $x_0 - a$, δ_1 } for all x ,y ∈ (a ,a + δ) , $\lvert \int_x^y \lvert f(x)\rvert dx\rvert \leq$ k =$\lvert \int_x^y \lvert g(x)\rvert dx\rvert < k\frac{\varepsilon}{k}$ = □ By Cauchy's Criterion $\int_a^b \lvert f(x)\rvert\ dx$ is convergent . (4M) Part 2: Given $\int_a^b \lvert f(x)\rvert\ dx$ is divergent. TPT $\int_a^b \lvert g(x)\rvert\ dx$ is divergent. Suppose $\int_a^b \lvert g(x)\rvert\ dx$ is convergent. But then by part1 $\int_a^b \lvert f(x)\rvert\ dx$ is convergent ,which is not true Hence our assumption is wrong Proved (2M)

Q3.	Attempt any **TWO** questions from the following: (12)	
$b)$	i.	Let f:[0,1]→ℝ be defined by $f(x) = \begin{cases} 0 \;\; if\; 0 \le x \le {}^{1}/_{2} \\ 1 \;\; if\; {}^{1}/_{2} < x \le 1 \end{cases}$ And F:[0,1]→ℝ be defined by $F(x) = \int_0^x f(t)dt$ then discuss the continuity of f at $x = \frac{1}{2}$ and differentiability of $Fx = \frac{1}{2}$..
	Ans	$\lim_{x \to \frac{1}{2}^+} f(x) = 1 \neq \lim_{x \to \frac{1}{2}^-} f(x) = 0$⇒f is not continuous. $F'\left(\frac{1}{2}\right) = \lim_{h \to 0} \frac{1}{h} \int_{\frac{1}{2}}^{\frac{1}{2}+h} 1dx. \Rightarrow F'\left(\frac{1}{2}\right) = 1.$
	ii.	If $x = \int_0^y \frac{dt}{\sqrt{1+t^2}}$,$y \ge 0$.Find $\frac{d^2y}{dx^2}$.
	Ans	Differentiate equation by chain rule ,we get $1 = \frac{1}{\sqrt{1+y^2}} \frac{dy}{dx} \Rightarrow \frac{d^2y}{dx^2} = y.$
	iii.	(I)Identify the type and discuss the convergence of the following integral $\int_1^\infty \frac{dx}{\sqrt{x^3+1}}$ (II)Find $\int_0^1 \frac{1}{\sqrt{1-x}}$
	ans	(1)g(x)=$\frac{1}{x^{\frac{3}{2}}}$ *by limit comparison test*cgt Ans(II)$\int_0^1 \frac{1}{\sqrt{1-x}} = \lim_{x \to 0+} \int_x^1 \frac{1}{\sqrt{1-t}}$ dt =…………..=2
	iv)	State Abel’s and Dirichlet’s Tests for the conditional convergence of type 1 improper integral and discuss convergence of $I = \int_a^\infty \frac{\sin x}{\sqrt{x}} dx$ for a > 0
	Ans	Abel’s Tests: If f is Reimann integrable on [a,∞) and βis monotonic and bounded on [a,∞), then function (fβ) is Reimann integrable on [a,∞) (1M) Dirichlet’s Tests: If f is Reimann integrable on [a,x) ,for all x ≥a,if F(x)=$\int_a^x f(x)\, dx$and ifβ is monotonic and if $\lim_{x \to \infty} \beta(x) = 0$ then function (fβ) is Reimann integrable on [a,∞) (1M) Let f(x)=sin x β(x) = $\frac{1}{\sqrt{x}}$

		Put $x^2 = t$ $\lvert\int_a^x \sin x\,dx\rvert = \lvert\cos a - \cos x\rvert \leq 2$ for all x (2M) Since f is conti, f is R-integrable on [a,,x] and the integral is bounded . $\lim_{x\to\infty} \beta(x) = 0$ By Dirichlet's Test I is convergent (2M)
Q4.	Attempt any **ONE** question from the following: (08)	
a)	i.	State and prove duplication formula for gamma function.
	Ans	Q.4 (a)(i) Duplication formula $\sqrt{\pi}\,\Gamma(2m) = 2^{2m-1}\Gamma(m)\Gamma(m+\frac{1}{2})$ proof: $\beta(m,m) = 2\int_0^{\pi/2}(\sin\theta)^{2m-1}(\cos\theta)^{2m-1}d\theta$ $= \frac{2}{2^{2m-1}}\int_0^{\pi/2}(\sin 2\theta)^{2m-1}d\theta$ sub $t = 2\theta \Rightarrow dt = 2d\theta \Rightarrow d\theta = \frac{1}{2}dt$ when $\theta = 0,\ t = 0$ & when $\theta = \pi/2,\ t = \pi$ $\therefore \beta(m,m) = \frac{2}{2^{2m-1}}\int_0^{\pi}(\sin t)^{2m-1}\frac{dt}{2}$ $= \frac{1}{2^{2m-1}}\left[\int_0^{\pi/2}(\sin t)^{2m-1}dt + \int_0^{\pi/2}(\sin(\pi-t))^{2m-1}dt\right]$ $\therefore \beta(m,m) = \frac{2}{2^{2m-1}}\int_0^{\pi/2}(\sin t)^{2m-1}dt$ consider $\beta(m,\frac{1}{2}) = 2\int_0^{\pi/2}(\sin\theta)^{2m-1}d\theta$ $= 2\int_0^{\pi/2}(\sin t)^{2m-1}dt$ $\therefore \beta(m,m) = \frac{2}{2^{2m-1}} \times \frac{1}{2}\beta(m,\frac{1}{2})$ $\therefore \frac{\Gamma(m)\Gamma(m)}{\Gamma(m+m)} = \frac{1}{2^{2m-1}}\beta(m,\frac{1}{2}) = \frac{1}{2^{2m-1}}\frac{\Gamma(m)\Gamma(\frac{1}{2})}{\Gamma(m+\frac{1}{2})}$ $\therefore \sqrt{\pi}\,\Gamma(2m) = 2^{2m-1}\Gamma(m)\Gamma(m+\frac{1}{2})$
	ii.	Give formula for finding volume of a solid using method of slicing and hence show that the volume of a sphere of radius 'r' is $\frac{4}{3}\pi r^3$.

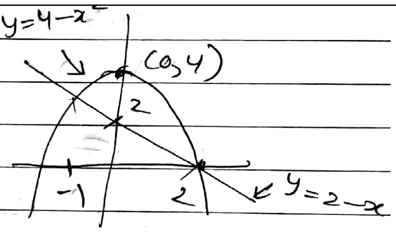
	Ans	Q.4(a)(ii) Slicing method: If $A(x)$ denote the cross-sectional area of a solid def on $[a, b]$ then volume of solid betⁿ $x = a$ & $x = b$ is given by $V = \int_a^b A(x)\,dx$ tst volume of a sphere with radius 'r' is $\frac{4}{3}\pi r^3$ Intersect the sphere with plane perpendicular to X-axis at $x \in [-r, r]$. ∴ Radius of circle $y = \sqrt{r^2 - x^2}$ ∴ Area of cross secⁿ $A(x) = \pi y^2 = \pi(r^2 - x^2)$ Volume $= \int_{-r}^{r} \pi(r^2 - x^2)\,dx = \frac{4}{3}\pi r^3$.
Q4.	Attempt any **TWO** questions from the following: (12)	
b)	i.	With usual notation of gamma function, show that $n\Gamma(n) = \Gamma(n+1)$, for $n > 0$
		Q.4(b)(i) tst $n\Gamma(n) = \Gamma(n+1)$, $n > 0$ $\Gamma(n+1) = \int_0^\infty x^n e^{-x} dx = \lim_{x\to\infty} \int_0^x t^n e^{-t} dt$ $= \lim_{x\to\infty} \left[t^n \frac{e^{-t}}{-1}\right]_0^x - \int_0^x \frac{e^{-t}}{-1} n t^{n-1} dt$ $= \lim_{x\to\infty} \left[\frac{x^n}{e^x} + n\int_0^x t^{n-1} e^{-t} dt\right]$ $= \lim_{x\to\infty} \left[\frac{n x^{n-1}}{e^x} + n\int_0^x t^{n-1} e^{-t} dt\right]$ ($\frac{\infty}{\infty}$ form) $= \lim_{x\to\infty} \left[\frac{n!}{e^x} + n\int_0^x t^{n-1} e^{-t} dt\right]$ 0 $\therefore \Gamma(n+1) = n\int_0^\infty t^{n-1} e^{-t} dt = n\Gamma(n)$
	ii.	Show that $\int_0^{\frac{\pi}{2}} \frac{dx}{\sqrt{\cos x}} \int_0^{\frac{\pi}{2}} \sqrt{\sin x}\,dx = \pi$.

<table>
<tr><td></td><td>Ans</td><td>Q-4 (b)(ii) tst $\int_0^{\pi/2} \frac{dx}{\sqrt{\cos x}} \int_0^{\pi/2} \sqrt{\sin x}\, dx = \pi$
$\beta(m,n) = 2\int_0^{\pi/2} (\sin x)^{2m-1} (\cos x)^{2n-1}\, dx$ — (*)
sub $m = \frac{1}{2}, n = \frac{1}{4}$ in (*)
$\therefore \beta(\frac{1}{2}, \frac{1}{4}) = 2\int_0^{\pi/2} \frac{1}{\sqrt{\cos x}}\, dx$
$\therefore \frac{\Gamma(\frac{1}{2})\Gamma(\frac{1}{4})}{\Gamma(\frac{3}{4})} = 2\int_0^{\pi/2} \frac{1}{\sqrt{\cos x}}\, dx$ — (1)
sub $m = \frac{3}{4}, n = \frac{1}{2}$ in (*)
$\therefore \beta(\frac{3}{4}, \frac{1}{2}) = 2\int_0^{\pi/2} \sqrt{\sin x}\, dx$
$\therefore \frac{\Gamma(\frac{3}{4})\Gamma(\frac{1}{2})}{\Gamma(\frac{5}{4})} = 2\int_0^{\pi/2} \sqrt{\sin x}\, dx$ — (2)
multiply (1) & (2)
$\therefore \frac{\Gamma(\frac{1}{2})\Gamma(\frac{1}{4})}{\Gamma(\frac{3}{4})} \times \frac{\Gamma(\frac{3}{4})\Gamma(\frac{1}{2})}{\Gamma(\frac{5}{4})} = 4\int_0^{\pi/2} \frac{dx}{\sqrt{\cos x}} \int_0^{\pi/2} \sqrt{\sin x}\, dx$
$\therefore \int_0^{\pi/2} \frac{dx}{\sqrt{\cos x}} \int_0^{\pi/2} \sqrt{\sin x}\, dx = \pi$</td></tr>
<tr><td></td><td>iii.</td><td>Find the area of region bounded by the curves $x = 1 - y^2$ & $x = y^2 - 1$.</td></tr>
<tr><td></td><td></td><td>Q.4(b)(iii) Area of region bdd betn $x = 1-y^2$ & $x = y^2-1$
To find interval of integrn, consider $1-y^2 = y^2-1$
$\Rightarrow 2y^2 = 2 \Rightarrow y = \pm 1$


$\therefore$ Area $= \int_{-1}^{1} (1-y^2)-(y^2-1)\, dy$
$= \int_{-1}^{1} 2-2y^2\, dy = 4\int_0^1 1-y^2\, dy$
$= 4(1-\frac{1}{3}) = \frac{8}{3}$
$y=4-x^2$</td></tr>
<tr><td></td><td>iv.</td><td>Find volume of solid generated by revolving the region bounded by $y = 4 - x^2$ & $y = 2 - x$ about X-axis using disk method.</td></tr>
</table>

Ans

Q.4 (b)(iv) $y=4-x^2$, $y=2-x$



To find interval of integrⁿ

consider $4-x^2 = 2-x$

$\Rightarrow x^2-x-2=0$

$\Rightarrow x=2, \; x=-1$

Outer radius $= 4-x^2$ & Inner radius $= 2-x$

$$\text{Volume} = \int_{-1}^{2} \pi\left[(4-x^2)^2 - (2-x)^2\right] dx$$

$$= \int_{-1}^{2} \pi \left\{16 - 8x^2 + x^4 - 4 + 4x - x^2\right\} dx$$

$$= \pi \int_{-1}^{2} x^4 - 9x^2 + 4x + 12 \, dx$$

$$= \frac{108\pi}{5}$$

Q5. Attempt any **FOUR** questions from the following: (20)

a) If $f:[0,1]\rightarrow IR$ be a function such that $f(x) = 2x + 1$. Let P be a partition with P: 0, 0.25, 0.5, 1. Then find U(*f,* P).

Ans 0.25(1+2 (0.25)+2+2.5+3)= 3.6

b) Let $f:[a,b]\rightarrow\mathbb{R}$ be a bounded function. If P and Q are partitions of $[a,b]$ with Q is refinement of P then prove that $L(P,f) \leq L(Q,f)$.

Ans Let P={x0, x1, …, xn] be a partition of [a,b]. Given P is subset of Q

Let y1,y2 ,….ym are extra points which are in Q but not in P.

Let $P_1 = P \cup \{y1\}$.let y1 $\in [xj-1, xj]$ …..2 marks

$L(P,f) - L(P_1,f) = (m_j - m'_j)(y1 - xj-1) + (m_j - m''_j)(xj - y1) \leq 0$

Where $m_j = \inf\{f(x)/x \in [x_{j-1}, x_j]\}$

$m'_j = \inf\{f(x)/x \in [x_{j-1}, y1]\}$

$m''_j = \inf\{f(x)/x \in [y1, x_j]\}$

As $m'_j \geq m_j$ and $m''_j \geq m_j$ ……. 3 marks

Therefore $L(P_1,f) \geq L(P,f)$

Similarly , $L(P_2,f) \geq L(P_1,f)$

$L(P_m,f) \geq L(P_{m-1},f) \geq L(P_{m-2},f) \ldots\ldots \geq L(P,f)$

but $P_m = Q$ ….1mark

$L(Q,f) \geq L(P,f)$

c) If $f,g:[a,b]\rightarrow\mathbb{R}$ be Riemann integrable such that f' and g' exist then prove that

	$\int_a^b fg' = f(b)g(b) - f(a)g(a) - \int_a^b f'g$.
Ans	since f', g' exist$\Rightarrow f, g$ are continuous hence R integrable $\Rightarrow fg$ is R integrable Since $(fg)' = f'g + fg'$ then integrate from a to b we get the result.
d)	Prove that convergence of $\int_a^\infty \lvert f(x)\rvert\, dx$ implies convergence of $\int_a^\infty f(x)dx$, a > 0
Ans	Consider any ε > 0 Given $that \int_a^\infty \lvert f(x)\rvert\, dx\ is$ convergent By Cauchy's general Principle of convergence there exists some $X_0 > 0$ such that $\lvert \int_x^y \lvert f(x)\rvert dx\rvert <$ ε for all x, y ≥ x_0 for all x, y ≥ x_0 ,$\lvert\int_x^y f(x)dx\rvert \leq \lvert\int_x^y \lvert f(x)\rvert dx\rvert <$ ε Hence By Cauchy's general Principle of convergence $\int_a^\infty f(x)dx\ is\ convergent,$
e)	Find the value of $\int_0^\infty e^{-k^2x^2}\, dx$.
Ans	Q.5 (e) $\int_0^\infty e^{-k^2x^2}\, dx = ?$ Gamma f^n is $\Gamma(n) = \int_0^\infty x^{n-1} e^{-x}\, dx$ sub $x = k^2t^2$ $\Rightarrow dx = 2k^2t\, dt$ when $x=0$, $t=0$, when $x \to \infty$, $t \to \infty$ $\therefore \Gamma(n) = \int_0^\infty (k^2t^2)^{n-1} e^{-k^2t^2} \times 2k^2t\,dt$ $\therefore \Gamma(n) = \int_0^\infty 2k^{2n} \times t^{2n-1} \times e^{-k^2t^2}\, dt$ sub $n = 1/2$ $\therefore \Gamma(\frac{1}{2}) = \int_0^\infty 2k\, e^{-k^2t^2}\, dt$ $\therefore \sqrt{\pi} = 2k\int_0^\infty e^{-k^2t^2}\, dt$ $\therefore \int_0^\infty e^{-k^2x^2} = \frac{\sqrt{\pi}}{2k}$
f)	Find the surface area of solid generated by revolving the curve $x = 3cos\theta, y = 3sin\theta$, $0 \leq \theta \leq \pi$ about X-axis.

Ans	(f) $x = 3\cos\theta$, $y = 3\sin\theta$ Let $f(\theta) = 3\cos\theta$, $g(\theta) = 3\sin\theta$ $\therefore f'(\theta) = -3\sin\theta$, $g'(\theta) = 3\cos\theta$ $\sqrt{(f'(\theta))^2 + (g'(\theta))^2} = \sqrt{9\sin^2\theta + 9\cos^2\theta} = 3$ surface area $= \int_0^\pi 2\pi(3\sin\theta)(3)\,d\theta$ $= 18\pi\left(-\cos\theta\right)_0^\pi$ $= 36\pi$

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