Exam:S Y B A-Semester 4
Mathematics paper 2(Revised)
Exam Date :26/04/2019
Q.P Code-66048
Answer Key

(3 Hours) [Total Marks: 100]

Note: (i) All questions are compulsory. (ii)Figures to the right indicate marks for respective parts.

Q.1	Choose correct alternative in each of the following (20)			
$i.$	Let f and g be functions such that the function $f.g$ is integrable on I , then			
	(a)	Both f and g must be integrable on I	(b)	At least one of f and g must be integrable on
	(c)	f and g may or may not be integrable on I	(d)	None of these
	Ans	(c)		
$ii.$	If $f:[0,1] \to IR$ be defined by $f(x) = \begin{cases} 1\ if\ x \in Q \\ 0\ if\ x \in IR\backslash Q \end{cases}$ Then			
	(a)	U(f, P) = 0, L(f, P) = 0		
	(b)	U(f, P) = 1, L(f, P) = 1		
	(c)	U(f, P) = 1, L(f, P) = 0		
	(d)	None of the above.		
	Ans	(a)		
$iii.$	The norm of the partition P = { −7, −5.8, , −4.8, −3.2, 3 } is			
	(a)	6.2	(b)	1
	(c)	1.6	(d)	None of the above.
	Ans	**(a)**		
$iv.$	Let f: $[a,b] \to \mathbb{R}$ be continuous function and $f(x) > 0, \forall x$.If $F(x) = \int_0^x f(t)dt$ then			
	(a)	$F(x) > 0, \forall x \in [a,b]$	(b)	$F(x)$ is strictly increasing on $[a,b]$
	(c)	$F(x)$ is convex on $[a,b]$	(d)	None of these
	Ans	(b)		
$v.$	If $F:[0,1] \to \mathbb{R}$ is defined by $F(x) = \int_0^x sin(t^2)dt$,then			
	(a)	$F'(x) = \sin(x^2)$	(b)	$F'(x) = \cos(x^2)$
	(c)	$F'(x) = \sin \mathrm{x}$	(d)	None of these
	Ans	**(a)**		
$vi.$	$The\ type\ I\ integral \int_0^\infty \frac{1}{x^2+1}\ dx$ =……..			
	(a)	π	(b)	$\frac{\pi}{2}$
	(c)	$\frac{\pi}{4}$	(d)	None of these
	Ans	(b)		

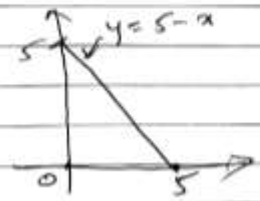
$vii.$	The expression for finding length of the curve $y = \frac{x^2}{2}$ over [0,1] is given by			
	(a)	$\int_0^1 \sqrt{1+x^2}\,dx$	(b)	$\int_0^1 \sqrt{1+x}\,dx$
	(c)	$\int_0^1 \sqrt{1+\frac{x^2}{4}}\,dx$	(d)	$\int_0^1 \sqrt{1+\frac{x^2}{2}}\,dx$
	Ans	(a)		
$viii.$	Which of the following definite integral represents the area of region bounded by the graphs of $y = x^2$ & $x + y = 2$?			
	(a)	$\int_{-2}^{1} (2 - x - x^2)\,dx$	(b)	$\int_{-2}^{1} (2 + x + x^2)\,dx$
	(c)	$\int_{-2}^{2} (2 - x^2 - x)\,dx$	(d)	$\int_{-1}^{1} (2 - x^2 - x)\,dx$
	Ans	(a)		
$ix.$	$\int_0^1 x^5(1-x)^3\,dx =$ _______			
	(a)	$\frac{1}{2}$	(b)	$\frac{1}{3}$
	(c)	$\frac{1}{4}$	(d)	None of these .
	Ans	(d)		
$x.$	$\int_0^\infty x^2 e^{-x}\,dx =$ _______			
	(a)	6	(b)	1
	(c)	2	(d)	None of these
	Ans	(c)		
Q2.	Attempt any **ONE** question from the following: (08)			
$a)$	i.	If $f: [a,b] \to \mathbb{R}$ be a bounded function and $a < c < b$, then prove that f is Riemann integrable on $[a,b]$ if and only if f is Riemann integrable on $[a,c]$ and $[c,b]$.		
	Ans	Let f is Riemann integrable on $[a,b]$ Case 1: c ∈ P then let P: a =Xo, X1,X2,…….Xr =C, Xr+1, ……..Xn=b Divide P into P1 and P2 such that P1 :{ a =Xo, X1,X2,…….Xr =C} And P2: {Xr+1, ……..Xn=b} Then since U(f,P) – L(f, P) < ∈ $implies$ U (f, P_1) – L (f, P_1) < ∈ and U (f, P_2) – L (f, P_2) < ∈ Case 2: c doesnot ∈ P then let P’ = P Ù {c} implies U(f,P_1) – L(f, P_1) < ∈ and U(f,P_2) – L(f, P_2) < ∈ Implies f is Riemann integrable on $[a,c]$ and $[c,b]$. **(4 marks)** Conversely f is Riemann integrable on $[a,c]$ and $[c,b]$. Then there exist partitions p1 and P2 of [a, c] and [c, b] such that U (f, P_1) – L(f, P_1) < ∈ and U (f, P_2) – L (f, P_2) < ∈ And U (f, P) – L (f, P) = U (f ,P1) – L(f, P1)+ U(f,P2) – L(f, P2) implies U (f, P) – L (f, P) < ∈ **(4 marks)**		

	ii.	If $f:[a,b]\to\mathbb{R}$ is a monotonic function then prove that f is Riemann integrable on $[a,b]$.
	Ans	Claim: if f is monotonic on [a, b] then f is R integrable. Let P= {x0, x1, …, xn] be a partition of [a, b] As f is monotonic there are x_i and x''_{i+1} in $[x_{i-1},xi]$ such that $Mi=f(x_i)$ and $mi=f(x_{i+1})$ where $M_i=\sup\{f(x)/x\in[x_{i-1},x_i]\}$ & $m_i=\inf\{f(x)/x\in[x_{i-1},x_i]\}$- ………..**3Marks** $U(P, f)—L(P, f)=\sum_{i=1}^{n}(Mi-mi)(xi-\mathrm{xi}-1)\le\sum_{i=1}^{n}\big(f(x'i+1)-f(x'i)\big)(x_r-x_{r-1})$ …………. **2 marks** Hence U (P, f)—L(P, f) $<\epsilon$ ……….**3marks**
Q.2	Attempt any **TWO** questions from the following: (12)	
$b)$	i.	If $f:[a,b]\to\mathbb{R}$ be a bounded function and P and Q are any two partitions of [a, b] then prove that $L(P,f)\le U(Q,f)$.
	Ans	Let P= {x0, x1, …, xn] be a partition of [a,b]. Given P is subset of Q Let y1,y2 ,….ym are extra points which are in Q but not in P. Let $P_1=P\cup\{y1\}$. let y1 $\in[xj-1,xj]$ …..2 marks L (P ,f) – L(P_1 ,f)=($m_j-m'_j$)(y1-xj-1)+ ($m_j-m''_j$)(xj- y1) ≤ 0 Where $m_j=\inf\{f(x)/x\in[x_{j-1},x_j]\}$ $m_j=\inf\{f(x)/x\in[x_{j-1},y1]\}$ $m''_j=\inf\{f(x)/x\in[y1,x_j]\}$ As $m'_j\ge m_j$ and $m''_j\ge m_j$……. 3 marks Therefore L(P_1,f) ≥L(P ,f) Similarly , L(P_2,f) ≥L(P_1 ,f) L (P_m, f) ≥ L (P_{m-1} , f) ≥ L(P_{m-2} ,f)…….. ≥L(P ,f) but P_m=Q ….1mark U (Q ,f) >L(Q ,f) ≥L(P ,f)
	ii.	If f is Riemann integrable on [a, b] then for any k ∈ IR prove that $\int_a^b kf=k\int_a^b f$
	Ans	Prove that U(kf,P) – L(kf, P) < ∈ And U(kf) = L(kf) implies $\int_a^b kf=k\int_a^b f$ OR Case1:k>0 Prove that U(k f, P) =k U(f, P) U(k f) = k U(f) Similarly L(k f)= k L(f) Since f is Riemann integrable on [a, b],U(f)=L(f)= $\int_a^b f$ U(k f)=k U(f)=k L(f)=L(k f)

		$\int_a^b kf$ = U(k f) = k U(f) = $k\int_a^b f$ (3M) Case2: k < 0 Prove that U(k f, P) =k L(f, P) U(k f) = k L(f) Similarly L(k f)= k U(f) Since f is Riemann integrable on [a, b],U(f)=L(f) U(k f)=k L(f)=k U(f)=L(k f) (2M) $$\int_a^b kf = \text{U(k f)} = \text{k L(f)} = k\int_a^b f$$ Case3: k=0…… (1M) $$\int_a^b kf = 0 = k\int_a^b f$$
	iii.	If $f:[a,b] \to \mathbb{R}$ be a bounded function then prove that $L(f) \leq U(f)$.
	Ans	Prove that U (f, P) ≤ L (f, P) Implies inf {U (f, P) for all partition P} ≤ sup {L (f, P) for all partition P} Hence $L(f) \leq U(f)$
	iv.	If f is Riemann integrable on [a, b] then prove that f^2 is Riemann integrable on [a, b].
	Ans	Since $\|f^2(x)-f^2(y)\| = \|f(x)-f(y)\|\|f(x)+f(y)\| \leq 2k\|f(x)-f(y)\|$ (Since f is bounded) U (P, f^2)-L(p,f^2)=$\sum M_j(f^2) - m_j(f^2)[x_j - x_{j-1}]$<∈ Hence f^2 is R integrable.
Q3.	Attempt any **ONE** question from the following: (08)	
a)	i.	State and prove the Fundamental Theorem of Calculus.
	Ans	Statement: Let $f:[a,b] \to \mathbb{R}$ be R-integrable on $[a,b]$ and $F(x) = \int_a^x f(t)dt, \forall x \in [a,b]$.If f is continu on $[a,b]$ then F is differentiable and $F'(x) = f(x)$. Proof: Let $h > 0$ such that $x + h \in [a,b]$. Then $\frac{F(x+h)-F(x)}{h} = \frac{1}{h}[\int_x^{x+h} f(t)\,dt]$.since f is continuous on $[x, x+h]$ hence bounded. Let sup(f)=M and inf(f)=$m \Rightarrow m \leq \frac{1}{h}\int_x^{x+h} f(t)\,dt \leq M \Rightarrow \exists c \in [x, x+h]$ such that $\frac{1}{h}\int_x^{x+h} f(t)\,dt = f(c(h))$.since $x \leq c(h) \leq x+h \Rightarrow c(h) = x\ as\ h \to 0$.hence the proof.
	ii.	State and prove comparison test for improper integrals of type-I.

	Ans	Statement If $\lvert f(x)\rvert \le k\lvert g(x)\rvert for\ all$ x ≥ x_0 then Convergence of $\int_a^\infty \lvert g(x)\rvert\, dx$ implies Convergence of $\int_a^\infty \lvert f(x)\rvert\, dx\ \ and\ divergence$ of $\int_a^\infty \lvert f(x)\rvert\, dx$ implies divergence of $\int_a^\infty \lvert g(x)\rvert\, dx$ (2M Proof : Part 1: $Given \int_a^\infty \lvert g(x)\rvert\, dx$ is Convergent By Cauchy's Criterion for any ε > 0, there exists x_1 > a such that for all y> $x \ge x_1$ > a , $\lvert \int_x^y \lvert g(x)\rvert dx \rvert < \frac{\varepsilon}{k}$ Let x_2= max { x_0, x_1 } For all y> $x \ge x_2$ > a , $\lvert \int_x^y \lvert f(x)\rvert dx\rvert \le \lvert \int_x^y k\lvert g(x)\rvert dx\rvert$ < ε By Cauchy's Criterion $\int_a^\infty \lvert f(x)\rvert\, dx$ is convergent Part 2: Given $\int_a^\infty \lvert f(x)\rvert\, dx$ is divergent. (4M) TPT $\int_a^\infty \lvert g(x)\rvert\, dx$ is divergent. Suppose $\int_a^\infty \lvert g(x)\rvert\, dx$ is convergent. But then by part1$\int_a^\infty \lvert f(x)\rvert\, dx$ is convergent ,which is not true Hence our assumption is wrong Proved (2M)
Q3.	Attempt any **TWO** questions from the following: (12)	
$b)$	i.	Let $F:\ [0,1]\ \to\ \mathbb{R}$be defined by $F(x) = \begin{cases} x^2\sin\left(\frac{\pi}{x}\right) & if\ 0 < x \le 1 \\ 0 & if\ x = 0 \end{cases}$ Show that $F$is differentiable over [0, 1].Let $f:[0,1] \to\ \mathbb{R}$be given by $f(x) = F'(x)$.find$\int_0^1 f(t)dt$.
	Ans	$F'(x) = \begin{cases} 2x\sin\left(\frac{\pi}{x}\right) - \pi\cos\left(\frac{\pi}{x}\right) & , x \ne 0 \\ 0 & , x = 0 \end{cases}$ and $\int_0^1 f(t)dt = F(1) - F(0) = 0.$
	ii.	Evaluate $\lim_{x\to\infty} \frac{1}{x^6}\int_0^x \frac{t^2}{1+t^6} dt$
	Ans	Let $F(x) = \int_0^x \frac{t^2}{1+t^6}$. Since $f(t) = \frac{t^2}{1+t^6}$ is continuous ∴ by FTC $F$is differentiable and $F'(x) = f(x)$. ∴ $\lim_{x\to\infty} \frac{1}{x^6}\int_0^x \frac{t^2}{1+t^6} dt = \lim_{x\to\infty} \frac{F(x)}{x^6} = \lim_{x\to\infty}\frac{F'(x)}{6x^5} = does\ not\ exist.$.(by L Hospitals rule)
	iii.	State Abel's and Dirichlet's Tests for the conditional convergence of type 1 improper integral and discuss convergence of $I = \int_0^\infty \cos x^2\, dx$
	Ans	Abel's Tests: If f is Reimann integrable on [a, ∞) and β is monotonic and bounded on [a,∞), then function (fβ) is Reimann integrable on [a, ∞) Dirichlet's Tests: If f is Reimann integrable on [a x) , for all x ≥ a ,if F(x)= $\int_a^x f(x)\, dx$ and if β is monotonic and if$\lim_{x\to\infty}$ β(x) = 0 then function (fβ) is Reimann integrable on [a, ∞)

		$I=\int_0^1 \cos x^2\,dx + \int_1^\infty \cos x^2\,dx = I_1 + I_2$ I_1 proper integral $I_2 = \int_1^\infty (2x\cos x^2)\frac{1}{2x}\,dx$ Let $f(x) = 2x\cos x^2$ $\beta(x) = \frac{1}{2x}$ Put $x^2 = t$ $\left\| \int_1^x (2x\cos x^2)\,dx \right\| = \left\| -\sin 1 + \sin x^2 \right\| \leq 2$ Since f is conti, f is R-integrable on [1, x] and the integral is bounded . $\lim_{x\to\infty} \beta(x) = 0$ By Dirichlet's Test I is convergent.
	iv.	Prove that $\int_a^b \frac{1}{(b-x)^p}\,dx$ converges if and only if $p < 1$.
	Ans	Standard proof: P=1… $\int_a^b \frac{1}{(b-x)^p}\,dx = \lim_{x\to b-} \int_a^x \frac{1}{b-x}\,dx = \lim_{x\to b-} -\log(b-x) + \log(b-a)$, *diverges to* ∞ p≠1 $\int_a^b \frac{1}{(b-x)^p}\,dx = \lim_{x\to b-} \int_a^x \frac{1}{(b-x)^p}\,dx = \lim_{x\to b-} \frac{(b-x)^{-p+1}}{p-1} - \frac{(b-a)^{-p+1}}{p-1}$ for p >1 ,………………..dgt for p <1 ,………………..cgt and converges to $\frac{(b-a)^{-p+1}}{-p+1}$
Q4.	Attempt any **ONE** question from the following: (08)	
a)	i.	State relation between beta and gamma function and hence show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$
	Ans	Q.4 (a) (i) Relation betⁿ beta & gamma fⁿ $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$, $m, n > 0$ test $\Gamma(\frac{1}{2}) = \sqrt{\pi}$ $\beta(\frac{1}{2},\frac{1}{2}) = \frac{\Gamma(\frac{1}{2})\Gamma(\frac{1}{2})}{\Gamma(\frac{1}{2}+\frac{1}{2})} = (\Gamma(\frac{1}{2}))^2$ $(\because \Gamma(1)=1)$ $\beta(m,n) = 2\int_0^{\pi/2} (\sin\theta)^{2m-1}(\cos\theta)^{2n-1}\,d\theta$ $\therefore \beta(\frac{1}{2},\frac{1}{2}) = 2\int_0^{\pi/2} d\theta = \pi$ $\therefore (\Gamma(\frac{1}{2}))^2 = \pi \Rightarrow \Gamma(\frac{1}{2}) = \sqrt{\pi}$
	ii.	State the disk method for finding volume of solid generated by revolving a continuous curve (i) $y = f(x)$ about X-axis between $x = a$ & $x = b$ and (ii) $x = g(y)$ about Y-axis between $y = c$ & $y = d$. Hence find volume of solid generated by revolving the line $y = 5 - x$ about X-axis between $x = 0$ & $x = 5$.

	Ans	Q.4(a)(ii) Disk method Volume of solid generated by revolving the curve $y = f(x)$ about x-axis is given by $V = \int_a^b \pi (f(x))^2 dx$ Volume of solid generated by revolving the cure $x = g(y)$ abt Y-axis is given by $V = \int_c^d \pi (g(y))^2 dy$ Line $y = 5-x$  $V = \int_0^5 \pi (5-x)^2 dx$ $= \pi \left[\frac{(5-x)^3}{-3}\right]_0^5$ $= \frac{-\pi}{3}\left[0 - 5^3\right] = \frac{125\pi}{3}$
Q4.	Attempt any **TWO** questions from the following: (12)	
$b)$	i.	Define beta function and express $\int_0^1 x^{m-1}(1-x^2)^{n-1}\,dx$ in terms of beta function.
	Ans	Q.4 (b) (i) Beta f^n def^n: $\int_0^1 x^{m-1}(1-x^2)^{n-1} dx$ sub $x^2 = t \Rightarrow 2x\,dx = dt \Rightarrow x\,dx = \frac{dt}{2}$ $\therefore \int_0^1 t^{m-2}(1-t)^{n-1} \frac{dt}{2} = \frac{1}{2}\beta(m-1, n)$
	ii.	Show that $\Gamma(n) = 2\int_0^\infty x^{2n-1}e^{-x^2}\,dx$.

	Ans	Q4(b)(ii) tst $\gamma(n) = 2\int_0^\infty x^{2n-1} e^{-x^2} dx$ $\gamma(n) = \int_0^\infty x^{n-1} e^{-x} dx$ Sub $x = t^2 \Rightarrow dx = 2t\,dt \Rightarrow x\,dx = 2t\,dt$ when $x=0, t=0$, $x \to \infty, t \to \infty$ $\therefore \gamma(n) = \int_0^\infty (t^2)^{n-1} e^{-t^2} (2t\,dt)$ $\therefore \gamma(n) = 2\int_0^\infty t^{2n-1} e^{-t^2} dt$
	iii.	Find surface area of the solid obtained by revolving the curve $x = 2t, y = t^2, 0 \le t \le 1$ about X-axis.
	Ans	Q4(b) (iii) $x = 2t, y = t^2, 0 \le t \le 1$ abt Y-axis $f(t) = 2t, \quad g(t) = t^2$ $\therefore f'(t) = 2, \quad g'(t) = 2t$ $\sqrt{(f'(t))^2 + (g'(t))^2} = \sqrt{4+4t^2} = 2\sqrt{1+t^2}$ Surface area $= \int_0^1 2\pi f(t)\sqrt{(f'(t))^2+(g'(t))^2}\,dt$ $= \int_0^1 2\pi \times 2t \times 2\sqrt{1+t^2}\,dt$ $= \cancel{\frac{4\sqrt{5}\pi}{3}}$ $= \frac{8\pi}{3}\left[8^{1/2} - 1\right]$
	iv.	The solid lies between planes perpendicular to X-axis at $x = -1$ & $x = 1$. The cross-section perpendicular to X-axis between these planes run from the parabola $y = -\sqrt{4-x^2}$ to $y = \sqrt{4-x^2}$. Using Slicing method, find volume of solid if cross-sections are circular discs with diameter in XY-plane.

	Ans	Q.4 (b)(iv) Cross-sections are circular discs with diameter in xy plane. $\therefore$ radius of $\odot$ $y = \sqrt{4-x^2}$ $\therefore$ Area $= A(x) = \pi(4-x^2)$ Volume $= \int_{-2}^{2} \pi(4-x^2)\,dx = \frac{32\pi}{3}$
Q5.	Attempt any **FOUR** questions from the following: (20)	
$a)$	If $f: [a,b] \to \mathbb{R}$ be a bounded function and P be a partition of [a, b] then define U(f, P), L(f, P),U(f),L(f) and give definition of Riemann integration.	
Ans	U(f, P)= $\sum_{r=1}^{n} M_r(x_r - x_{r-1})$ L(f, P) =$\sum_{r=1}^{n} m_r(x_r - x_{r-1})$ U(f)=Inf{ U(f, P)/ for all partitions P of [a, b]} L(f)= sup{ L(f, P)/ for all partitions P of [a, b]} If U(f)= L(f) then f is said to be Riemann integrable.	
$b)$	If $f, g : [a,b] \to \mathbb{R}$ be Riemann integrable on $[a,b]$, then prove that $f+g : [a,b] \to \mathbb{R}$ is Riemann integrable on $[a,b]$	
Ans	Let P= {x0, x1, …, xn] be a partition of [a, b] let M_i =sup {(f+ g) (x)/x∈[x_{i-1},x_i] } & m_i =inf {(f+ g)(x)/x∈[x_{i-1},x_i] } let M'_i =sup{f(x)/x∈[x_{i-1} ,x_i] } & m'_i =inf {f(x)/x∈[x_{i-1},x_i] } let M''_i =sup{g(x)/x∈[x_{i-1}, x_i] } & m''_i =inf {g(x)/x∈[x_{i-1},x_i] } then $M_i \le M'_i + M''_i$ and $m_i \ge m'_i + m''_i$ for i=1,2,….n Hence U (P, f+ g) - L(P, f+ g) ≤ U(P, f) - L(P, f) + U(P, g) - L(P, g)……(*) 3 marks But f & g are R- integrable on [a,b] hence there are partitions say P_1 and P_2 Such that U(P_1 ,f)- L(P_1 ,f) < $\frac{\epsilon}{2}$ and U(P_2 ,g)- L(P_2 ,g) < $\frac{\epsilon}{2}$ 2 marks Take P=$P_1 \cup P_2$ Then U (P ,f)- L(P ,f) < $\frac{\epsilon}{2}$ and U(P ,g)- L(P ,g) < $\frac{\epsilon}{2}$ ….2 marks Hence U (P, f +g) - L(P, f + g) < ϵ by * Therefore f +g is R-integrable on [a, b]	
$c)$	If $f : [a,b] \to \mathbb{R}$ be a continuous, then show that $\exists\, c \in (a,b)$ such that $\int_a^b f(t)dt = f(c)(b-a)$.	
Ans	Ans: Let $F(x) = \int_a^x f(t)dt, \forall x \in [a,b]$.since f is continuous ∴ by FTC , $F$is differentiable and $F'(x) = f(x)$. by LMVT $\exists\, c \in (a,b)$ such that $F'(c) = \frac{F(b)-F(a)}{b-a}$ $\Rightarrow f(c)(b-a) = \int_a^b f(t)dt - \int_a^a f(t)dt$ hence proved.	

$d)$	dentify the type and discuss the convergence of each of the following integrals $(I)\ \int_0^1 \frac{dx}{\sqrt{x^2+1}}$ (II) $\int_1^{\infty} \frac{sin^2 x}{x^2} dx$
Ans	(I) f(x)= $\frac{dx}{\sqrt{x^2+1}}$ Let g(x)= $\frac{1}{x}$ $\lim_{x\to 0+} \frac{f}{g}$ =1, finite non zero since $\int_0^1 g(x)dx$ is not cgt ,(p=1) by limit comparison Test (II) $\frac{sin^2 x}{x^2} \leq \frac{1}{x^2}$ for all x > 1 since P=2> 1, $\int_1^{\infty} g(x)dx$ is cgt. By First comparison Test $\int_1^{\infty} f(x)dx$ is cgt.
$e)$	Find the area of the region bounded by the curves $x = y^2 \& x + 2y^2 = 3$.
Ans	Q.5 (e) Area of region bdd betn the curve $x = y^2$ & $x = 3 - 2y^2$ To find interval of integra'n consider $y^2 = 3 - 2y^2 \Rightarrow y = \pm 1$ $\therefore$ Area $= \int_{-1}^{1} (3-2y^2) - y^2\, dy$ $= \int_{-1}^{1} 3 - 3y^2\, dy = 4$
$f)$	Show that $\Gamma(1) = 1$.

Ans

Q.5(f) tst $\Gamma(1) = 1$

$$\Gamma(n) = \int_0^\infty x^{n-1} e^{-x}\, dx$$

$$\therefore \Gamma(1) = \int_0^\infty e^{-x}\, dx$$

$$= \lim_{t \to \infty} \int_0^t \frac{1}{e^x}\, dx$$

$$= \lim_{t \to \infty} -1 \left[\frac{1}{e^x} \right]_0^t = \lim_{t \to \infty} - \left[\frac{1}{e^t} - 1 \right]$$

$$\therefore \boxed{\Gamma(1) = 1}$$

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