Exam : S.Y.BSc-Semester 4
Subject: Mathematics Paper 1(Revised)
Exam Date: 18-4-2019
Q.P.Code- 66047
ANSWER KEY

(3 Hours) [Total Marks: 100]

Note: (i) All questions are compulsory.
(ii)Figures to the right indicate marks for respective parts.

Q.1	Choose correct alternative in each of the following (20)			
$i.$	If $f:[a,b] \to IR$ be bounded function and P ,Q be partitions of [a,b] then			
	(a)	$L(P,f)) \leq U(Q,f)$	(b)	$L(P,f) \geq U(Q,f)$
	(c)	$L(P,f)) = U(Q,f)$	(d)	None of the above
Ans	**(a)**	$\boldsymbol{L(P,f)) \leq U(Q,f)}$		
$ii.$	The norm of a partition $p = \{0 < \frac{1}{2} < 1 < \frac{4}{3} < \frac{7}{3} < 3\}\ is$			
	(a)	$\frac{1}{3}$		
	(b)	$\frac{2}{3}$		
	(c)	1		
	(d)	None of the above		
Ans	**(c)**	**1**		
$iii.$	If $f:[a,b] \to IR$ is R- integrable then which of the following is true			
	(a)	f must be continuous	(b)	f must be differentiable
	(c)	f must be monotonic	(d)	None of the above
Ans	**(d)**	**None of the above**		
$iv.$	Let$f :\ \mathbb{R} \to\ \mathbb{R}$be a continuous function. Then $\int_{-a}^{a} f(t)dt = 0, \forall a > 0$if and only if			
	(a)	$f \equiv 0$	(b)	$f$is an odd function
	(c)	$f \neq 0$For only finitely many real numbers.	(d)	None of the above.
Ans	**(b)**	$\boldsymbol{f}$**is an odd function**		
$v.$	If $f,g:[a,b] \to \mathbb{R}$ are continuous functions such that $\int_a^b f(x)dx = \int_a^b g(x)dx$ then			
	(a)	$f \equiv g\ on\ [a,b]$	(b)	$f(x) = g(x)$is a constant.
	(c)	$\exists\ c \in [a,b]$such that $f(c) = g(c)$	(d)	None of the above.
Ans	**(a)**	$\boldsymbol{f \equiv g\ on\ [a,b]}$		
$vi.$	$The\ type\ 2\ integral \displaystyle\int_0^2 \frac{1}{x-1}\,dx$			

	(a)	Diverges	(b)	Converge to 0
	(c)	Converge to $\frac{1}{2}\ln 3$	(d)	Converges to $\frac{8}{9}$
Ans	**(a)**	**Diverges**		
vii.	Integral $\int_1^{\infty}\frac{1}{x^p}dx$ converges if			
	(a)	P > 1	(b)	P < 1
	(c)	P = 1	(d)	None of the above
Ans	**(a)**	**P > 1**		
viii.	Find $\int_0^{\frac{\pi}{2}} cos^{11}x\ sin^9x\ dx$			
	(a)	$\frac{1}{10!}$	(b)	$\frac{5!\,4!}{2(10!)}$
	(c)	$\frac{10!}{5!\,4!}$	(d)	0
Ans	**(b)**	$\mathbf{\frac{5!\,4!}{2(10!)}}$		
ix.	$\int_0^{\infty} x^{3/2}e^{-x}\,dx=$			
	(a)	$\frac{3\sqrt{\pi}}{4}$	(b)	$\frac{\pi}{2}$
	(c)	$\frac{\sqrt{\pi}}{5}$	(d)	None of these
Ans	**(a)**	$\boldsymbol{\frac{3\sqrt{\pi}}{4}}$		
x.	Identify the definite integral that computes the volume of the solid generated by revolving the region bounded by the graph of $y = x^3$ and the line $y = x$, between $x = 0$ and $x = 1$ about the line $x = 1$.			
	(a)	$\pi\int_0^1\left(y^{\frac{2}{3}} - y^2\right)dy$	(b)	$\pi\int_0^1\left(y^{\frac{1}{3}} - y\right)^2 dy$
	(c)	$2\pi\int_0^1\left(4-x^2)(4-x^6\right)dx$	(d)	$\pi\int_0^1\left((4-y)^2(4-y^{\frac{1}{3}})^2\right)dx$
Ans	**(a)**	$\pi\int_0^1\left(y^{\frac{2}{3}} - y^2\right)dy$		
Q2.	Attempt any **ONE** question from the following: (08)			
a)	i.	Let $f:[a,b] \to IR$ be a bounded function. Prove that f is Riemann integrable on [a, b] if and only if for any $\in > 0$ there exist a partition P of [a, b] such that $U(f,P) - L(f,P) < \in$. Ans: Proof :(4+4 marks)		
	ii.	If $f;g:[a,b] \to IR\ are\ R$- integrable then prove that $f+g$ is R- integrable & $\int_a^b f+g = \int_a^b f + \int_a^b g$		
Ans		Let P={x0, x1, …, xn] be a partition of [a,b]		

		let M_i =sup{(f+g)(x)/x∈[x_{i-1},x_i] } & m_i =inf {(f+g)(x)/x∈[x_{i-1},x_i] } let M'_i =sup{f(x)/x∈[x_{i-1},x_i] } & m'_i =inf {f(x)/x∈[x_{i-1},x_i] } let M''_i =sup{g(x)/x∈[x_{i-1},x_i] } & m''_i =inf {g(x)/x∈[x_{i-1},x_i] } then $M_i \leq M'_i + M''_i$ and $m_i \geq m'_i + m''_i$ for i=1,2,….n Hence U(P,f+g) - L(P,f+g) ≤ U(P,f) - L(P,f) + U(P,g) - L(P,g)……(*) But f & g are R- integrable on [a,b] hence there are partitions say P_1 and P_2 Such that U(P_1,f)- L(P_1,f) < $\frac{\epsilon}{2}$ and U(P_2,g)- L(P_2,g) < $\frac{\epsilon}{2}$ Take P=$P_1 \cup P_2$ Then U(P,f)- L(P,f) < $\frac{\epsilon}{2}$ and U(P,g)- L(P,g) < $\frac{\epsilon}{2}$ Hence U(P,f+g) - L(P,f+g) < ϵ by * Therefore f+g is R-integrable on [a,b] 4 marks Prove that $\int_a^b f+g < \int_a^b f + \int_a^b g + \in$ $\int_a^b f+g > \int_a^b f + \int_a^b g - \in$ for every $\in > 0$ 4 marks
Q.2	Attempt any **TWO** questions from the following: (12)	
b)	i.	Let f be a bounded function on [a, b]. Let P and P' are two partitions of [a, b] with $P \subseteq P'$. Show that L (f, P') ≥ L (f, P) Let P={x0, x1, …, xn] be a partition of [a,b]. Given P is subset of Q Let y1,y2 ,….ym are extra points which are in Q but not in P. Let P_1 = P ∪{y1}.let y1 ∈ $[xj-1, xj]$ …..2 marks L(P,f) – L(P_1,f)=(m_j –m'_j)(y1-xj-1)+ (m_j –m''_j)(xj- y1) ≤ 0 Where m_j = inf{f(x)/x∈[x_{j-1},x_j] } m'_j = inf{f(x)/x∈[x_{j-1},y1] } m''_j = inf{f(x)/x∈[y1,x_j] } As $m'_j \geq m_j$ and $m''_j \geq m_j$ ……. 3 marks Therefore L(P_1,f) ≥L(P ,f) Similarly , L(P_2,f) ≥L(P_1,f) L(P_m,f) ≥ L(P_{m-1},f) ≥ L(P_{m-2},f)…….. ≥L(P ,f) but P_m =P’ ….1mark L(P’,f) ≥L(P ,f)
	ii.	If f is an R-integrable function on $[a,b]$ then prove that $\|f\|$ is R-integrable on [a, b]. Given: f is R integrable on $[a,b]$. Claim: $\|f\|$ is R integrable on $[a,b]$. Let $P = \{x_0 = a, x_1, x_2, \ldots \ldots x_n = b\}$ be any partition of $[a,b]$ Let $M_i = \sup\{f(x) : x \in [x_{i-1}, x_i]\}$ and $M'_i = \sup\{\|f\|(x) : x \in [x_{i-1}, x_i]\}$ $m_i = \inf\{f(x) : x \in [x_{i-1}, x_i]\}$ and $m'_i = \inf\{\|f\|(x) : x \in [x_{i-1}, x_i]\}$, $i = 1,2, \ldots . n$. To show that, $$M'_i - m'_i \leq M_i - m_i, \ \ i = 1,2, \ldots . n.$$ Let, $x, y \in [x_{i-1}, x_i]$ $$m_i \leq f(x) \leq M_i$$ $$m_i \leq f(y) \leq M_i$$

		$\therefore m_i - M_i \le f(x) - f(y) \le M_i - m_i$ $\therefore -m_i - M_i \le f(x) - f(y) \le M_i - m_i$ Consider, $|f(x)| = |f(x) - f(y) + f(y)|$ $\le |f(x) - f(y)| + |f(y)|$ $\le M_i - m_i + |f(y)|$ Here, $y \in [x_{i-1}, x_i]$ $\therefore |f(x)| \le M_i - m_i + |f(y)|, \ \forall x \in [x_{i-1}, x_i]$ $\therefore M_i - m_i + |f(y)|$is an upper bound of $\{f(x) : x \in [x_{i-1}, x_i]\}$. $\therefore M_i' \le M_i - m_i + |f(y)|$, ($\because M_i'$ is least of upper bound) $\therefore M_i' - M_i + m_i \le |f(y)|, \forall\, y \in [x_{i-1}, x_i]$ $\therefore M_i' - M_i - m_i$is lower bound of $\{f(x) : x \in [x_{i-1}, x_i]\}$. $\therefore M_i' - M_i + m_i \le m_i'$($\because m_i'$ is greatest lower bound) $\therefore\ M_i' - m_i' \le M_i - m_i,\ \ i = 1,2, \dots .n.$ …… 3 marks Multiplying above relation by $(x_i - x_{i-1})$ and adding above n relations we have, $U(|f|, P) - L(|f|, P) \le U(f, P) - L(f, P)$ $(*)$ As, f is R integrableon $[a, b]$. Hence, for given $\epsilon > 0, \exists$ partition P_ϵ of $[a, b]$ such that, $U(f, P_\epsilon) - L(f, P_\epsilon) < \epsilon$ $\therefore$ by $(*)$ $U(|f|, P_\epsilon) - L(|f|, P_\epsilon) < \epsilon$ $\therefore\ |f\,|$ is R integrable on $[a, b]$. ……… 3 marks
	iii.	Using Riemann Criterion, prove that the function $f : [0, 1] \longrightarrow \mathbb{R}$ defined by $f(x) = x$ is Riemann integrable. Ans: define the partition 0<1/n < 2/n < 3/n……….. Find U(f,P) L(f,P) and prove that U(f,P)-L(f, P) < ∈
	iv.	If f , g : [a, b]→ ℝ are integrable functions such that $f(x) \le g(x)$,∀ x ∈ [a, b] then prove that $\int_a^b f(x)\,dx \le \int_a^b g(x)\,dx$.
		Prove that if $g(x) \ge 0$ $then$ $\int_a^b g(x)dx \ge 0$ Using this prove that $f(x) \le g(x)$,∀ x ∈ [a, b] implies $\int_a^b f(x)\,dx \le \int_a^b g(x)\,dx$.
Q3.	Attempt any **ONE** question from the following: (08)	
$a)$	i.	State and prove the Fundamental Theorem of Calculus.
Ans		Statement: Let$f: [a, b] \to \mathbb{R}$ be R-integrable on $[a, b]$ and $F(x) = \int_a^x f(t)dt, \forall x \in [a, b]$.If f is continuous on $[a, b]$ then F is differentiable and $F'(x) = f(x)$. Proof: Let $h > 0$ such that $x + h \in [a, b]$. Then $\frac{F(x+h)-F(x)}{h} = \frac{1}{h}[\int_x^{x+h} f(t)\,dt]$.since f is continuous on $[x, x + h]$ hence bounded. Let sup(f)=M and inf(f)=$m \Rightarrow m \le \frac{1}{h}\int_x^{x+h} f(t)\,dt \le M \Rightarrow \exists c \in [x, x + h]$ such that $\frac{1}{h}\int_x^{x+h} f(t)\,dt = f(c(h))$.since $x \le c(h) \le x + h \Rightarrow c(h) = x\ as\ h \to 0$.hence the proof.

	ii.	State and prove comparison test for improper integrals of type-I.
Ans		Statement If $\lvert f(x)\rvert \le k\lvert g(x)\rvert \text{ for all } x \ge x_0$ then Convergence of $\int_a^\infty \lvert g(x)\rvert\, dx$ implies Convergence of $\int_a^\infty \lvert f(x)\rvert\, dx$ *and di*vergence of $\int_a^\infty \lvert f(x)\rvert\, dx$ implies divergence of $\int_a^\infty \lvert g(x)\rvert\, dx$ (2M) Proof : Part 1: *Given* $\int_a^\infty \lvert g(x)\rvert\, dx$ is Convergent By Cauchy's Criterion for any $\varepsilon > 0$, there exists $x_1 > a$ such that for all $y > x \ge x_1 > a$, $\lvert \int_x^y \lvert g(x)\rvert dx\rvert < \frac{\varepsilon}{k}$ Let $x_2 = \max\{x_0, x_1\}$ For all $y > x \ge x_2 > a$, $\lvert \int_x^y \lvert f(x)\rvert dx\rvert \le \lvert \int_x^y k\lvert g(x)\rvert dx\rvert < \varepsilon$ By Cauchy's Criterion $\int_a^\infty \lvert f(x)\rvert\, dx$ is convergent Part 2: Given $\int_a^\infty \lvert f(x)\rvert\, dx$ is divergent. (4M) TPT $\int_a^\infty \lvert g(x)\rvert\, dx$ is divergent. Suppose $\int_a^\infty \lvert g(x)\rvert\, dx$ is convergent. But then by part1 $\int_a^\infty \lvert f(x)\rvert\, dx$ is convergent ,which is not true Hence our assumption is wrong Proved (2M)
Q3.	Attempt any **TWO** questions from the following: (12)	
b)	i.	Let $F : [0,1] \to \mathbb{R}$ be defined by $F(x) = \begin{cases} x^2 \sin\left(\frac{\pi}{x}\right) & if\ 0 < x \le 1 \\ 0 & if\ x = 0 \end{cases}$ Show that is differentiable over [0, 1].Let $f : [0,1] \to \mathbb{R}$ be given by $f(x) = F'(x)$.find $\int_0^1 f(t)dt$.
Ans		$F'(x) = \begin{cases} 2x \sin\left(\frac{\pi}{x}\right) - \pi\cos\left(\frac{\pi}{x}\right) & , x \ne 0 \\ 0 & , x = 0 \end{cases}$ and $\int_0^1 f(t)dt = F(1) - F(0) = 0$.
	ii.	Evaluate $\lim_{x\to\infty} \frac{1}{x^3}\int_0^x \frac{t^2}{1+t^4} dt$
Ans		Let $F(x) = \int_0^x \frac{t^2}{1+t^4}$. Since $f(t) = \frac{t^2}{1+t^4}$ is continuous $\therefore$ by FTC F is differentiable and $\Rightarrow F'(x) = f(x)$. $\therefore \lim_{x\to\infty} \frac{1}{x^3}\int_0^x \frac{t^2}{1+t^4} dt = \lim_{x\to\infty} \frac{F(x)}{x^3} = \lim_{x\to\infty} \frac{F'(x)}{3x^2} = 0$.(by L Hospitals rule)
	iii.	Prove that $\int_a^b \frac{1}{(b-x)^p}\, dx$ converges if and only if $p < 1$.
Ans		Standard proof:

		P=1… $\int_a^b \frac{1}{(b-x)^p}\,dx = \lim_{x\to b-}\int_a^x \frac{1}{b-x}\,dx$ $= \lim\limits_{x\to b-} -\log(b-x) + \log(b-a)\,, diverges\ to\ \infty$ (2) p≠1 $\int_a^b \frac{1}{(b-x)^p}\,dx = \lim_{x\to b-}\int_a^x \frac{1}{(b-x)^p}\,dx = \lim\limits_{x\to b-} \frac{(b-x)^{-p+1}}{p-1} - \frac{(b-a)^{-p+1}}{p-1}$ (1) for p >1 ,………………..divergent for p <1 ,………………..convergent and converges to $\frac{(b-a)^{-p+1}}{-p+1}$ (3)
	iv.	State Abel's and Dirichlet's Tests for the conditional convergence of type 1 improper integral and discuss convergence of $I = \int_0^\infty \sin x^2\,dx$
Ans		Abel's Tests: If f is Reimann integrable on [a,∞) and β is monotonic and bounded on [a,∞), then function (fβ) is Reimann integrable on [a,∞) Dirichlet's Tests: If f is Reimann integrable on [a,x) ,for all x ≥ a,if F(x)= $\int_a^x f(x)\,dx$ and ifβ is monotonic and if$\lim_{x\to\infty}$ β(x) $= 0$ then function (fβ) is Reimann integrable on [a,∞) I=$\int_0^1 \sin x^2\,dx$ +$\int_1^\infty \sin x^2\,dx$ = $I_1 + I_2$ I_1 proper integral I_2 =$\int_1^\infty$ (2xsin x^2) $\frac{1}{2x}dx$ Let f(x)=2xsin x^2β(x) = $\frac{1}{2x}$ Put $x^2 = t$ $\lvert \int_1^x (2x\sin x^2)\,dx\rvert = \lvert\cos 1 - \cos x^2\rvert \le 2$ Since f is conti, f is R-integrable on [1,x] and the integral is bounded . $\lim_{x\to\infty}$ β(x) =0 ByDirichlet's Test I is convergent
Q4.	Attempt any **ONE** question from the following: (08)	
a)	i.	Prove that $\int_0^1 x^{m-1}(1-x)^{n-1}dx$ converges if and only if m and n are both positive.
Ans		For $m \ge 0, n \ge 0$ the integral is proper. When $m \le 1$, infinite discontinuity at 0 and when $n \le 1$, infinite discontinuity at 1. $\int_0^1 x^{m-1}(1-x)^{n-1}dx = \int_0^{1/2} x^{m-1}(1-x)^{n-1}dx + \int_{1/2}^1 x^{m-1}(1-x)^{n-1}dx$=$I_1$+$I_2$ For I_1, $f(x) = \frac{(1-x)^{n-1}}{x^{1-m}}, g(x) = \frac{1}{x^{1-m}}$. $\lim_{x\to 0}\frac{f(x)}{g(x)} = 1$. By comparison test $\int_0^{\frac{1}{2}} f(x)dx, \int_0^{\frac{1}{2}} g(x)dx$converge and diverge together. $\int_0^{\frac{1}{2}} g(x)dx = \int_0^{\frac{1}{2}} \frac{1}{x^{1-m}}\,dx$converges iff $1 - m < 1\ i.e. m > 0$. For $I_2(x) = \frac{x^{m-1}}{(1-x)^{1-n}}, g(x) = \frac{1}{(1-x)^{1-n}}$. Same approach as above.

	ii.	With usual notations for beta and gamma functions prove that (p) $\beta(m,n) = \beta(n,m)$ (q) $\frac{\beta(m,n+1)}{n} = \frac{\beta(m+1,n)}{m} = \frac{\beta(m,n)}{m+n}$.
Ans		$$\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx = -\int_1^0 (1-y)^{m-1}y^{n-1}\,dx = \beta(n,m)$$ $$\beta(m+1,n)+\beta(m,n+1) = \int_0^1 [x^m(1-x)^{n-1}+x^{m-1}(1-x)^n]dx = \beta(m,n)$$ By integration by parts $\beta(m,n+1) = \frac{n}{m}\beta(m+1.n)$ (Student must show) $\beta(m,n) = \beta(m+1,n)+\beta(m,n+1) = \beta(m+1,n)+\frac{n}{m}\beta(m+1.n) =$ $\frac{m+n}{m}\beta(m+1.n)$. Hence the proof.
Q4.	Attempt any **TWO** questions from the following: (12)	
b)	i.	Prove that β(m,n)=$\int_0^\infty \frac{t^{m-1}}{(1+t)^{m+n}}dy$.
Ans		$$\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$$ Substitute $x = \frac{t}{1+t}$. $\beta(m,n) = \int_0^\infty \left(\frac{t}{1+t}\right)^{m-1}\left(1-\frac{t}{1+t}\right)^{n-1}\frac{dt}{(1+t)^2} = \int_0^\infty \frac{t^{m-1}}{(1+t)^{m+n}}dy$
	ii.	Show that : $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.
Ans		$$\beta\left(\frac{1}{2},\frac{1}{2}\right) = \left(\Gamma\left(\frac{1}{2}\right)\right)^2$$ $$\beta\left(\frac{1}{2},\frac{1}{2}\right) = 2\int_0^{\frac{\pi}{2}} \sin^{\frac{2.1}{2}-1}\theta\, cos^{\frac{2.1}{2}-1}\theta d\theta = \pi$$
	iii.	Find the volume of the solid whose base is the disk $x^2+y^2 \leq 1$ and the cross sections by the planes perpendicular to the $y-axis$ between $y=-1$ and $y=1$ are isosceles right triangles with one leg in disk by the method of slicing.
Ans		Length of the base =$2\sqrt{1-y^2}$. Area of the triangle=$\left(\frac{1}{2}\right)\left(2\sqrt{1-y^2}\right)^2 = 2(1-y^2)$. Volume=$2\int_{-1}^1(1-y^2)dy = \frac{8}{3}$.

	iv.	Find the volume of the solid generated by revolving the regions bounded by the lines $y = 2x$, $y = x$, $x = 1$ and about $x - axis$ by the washer method.
Ans		$r(x) = x, R(x) = 2x.$ Area by washer method= $\int_0^1 \pi[4x^2 - x^2]dx = \pi.$
Q5.	Attempt any **FOUR** questions from the following: (20)	
$a)$	If f(x) = 1+ 2x, x ∈ IR and P be a partition such that 0 < 0.25< 0.5 < 0.75 <1 Then find U(P, f). Ans:U(P, f) = 0.25(1.5+2+2.5+3)=3.6	
$b)$	If f is Riemann integrable on [a, b] then for any k ∈ IR prove that kf is also Riemann integrable on [a, b]. Ans: Prove that U(P, kf) – L(P, kf) < ∈	
$c)$	Show that if $F'(x) = 0$, $\forall x \in [a, b]$ then f is a constant function.	
Ans	since f is differentiable hence continuous on $[a, x]$ By LMVT $\exists c \in (a, x)$ such that $f'(c) = \frac{f(x)-f(a)}{x-a} \Rightarrow f(x) - f(a) = 0 \Rightarrow f(x) = f(a), \forall x$	
$d)$	Identify the type and discuss the convergence of each of the following integrals $(I)\ \int_0^1 \frac{dx}{x^2(1+x)^3}$ (II)$\int_1^\infty \frac{sin^2x}{x^2} dx$	
Ans	(I) f(x)=$\frac{dx}{x^2(1+x)^3}$ Let g(x)=$\frac{1}{x^2}$ $\lim_{x\to 0+} \frac{f}{g}$ =1,finitenonzero since$\int_0^1 g(x)dx$ is not cgt ,(p=2) by limit comparison Test (II)$\frac{sin^2x}{x^2} \leq \frac{1}{x^2}$ for all x > 1 since P=2> 1,$\int_1^\infty g(x)dx$ is cgt. By comparison Test $\int_1^\infty f(x)dx$ is cgt.	
$e)$	Prove that $\int_0^{\frac{\pi}{2}} \sqrt{\sin x}\ dx \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{\cos x}}\ dx = \pi.$	
Ans	$\int_0^{\frac{\pi}{2}} \sqrt{\sin x}\ dx = \frac{1}{2}\beta\left(\frac{3}{4}, \frac{1}{2}\right)$ $\int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{\cos x}}\ dx = \frac{1}{2}\beta\left(\frac{1}{2}, \frac{1}{4}\right)$ Using beta gamma relationship get the result.	
$f)$	Find the areaof the surfacegenerated by revolving the curves about $x = 2\sqrt{4-y}, 0 \leq y \leq \frac{15}{4}$about y –axis.	
Ans	Surface area = = $\int_0^{\frac{15}{4}} 2\pi x\sqrt{1+\left(\frac{dx}{dy}\right)^2}\ dy = 2\pi \int_0^{\frac{15}{4}} \sqrt{5-y}\ dy = \frac{35\sqrt{5}}{6}\pi.$	

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