

## Q. P. Coder 59962

[Total Marks: 100]

Note: (i) All questions are compulsory.

(ii) Figures to the right indicate marks for respective parts.

(i) Figures to the right indicate marks for respective parts.		
i)Figures to the right material		(20)
Choose correct alternative in each of the following		
Choose correct alternative in each of the line i. If $T: U \to V$ is a linear transformation such that ker i. (for the linear transformation such that ker is a linear transformation such transformation	$r T = \{ 0 \} U$	Always injective
	<u>b)</u>	None the above
Always bijective	<u>~)</u> _	
(c) Always offective  Ans b  Which of the following is a linear transformation from the following is a linear tran	m R <sup>2</sup> to R	R <sup>2</sup>
ii. Which of the following is a linear transformation it	(b)	T(x,y)=(x,y+1)
$T(\mathbf{r}, \mathbf{v}) = (\mathbf{x}, \mathbf{v})$	(d)	All the above
\"\" <del>- +                             </del>		Lesthe following is
	s V and W.	. Then, which of the following
iii. Let $f: V \to W$ be an isomorphism of vocation		
	(b)	f is dijection
(a) Dim V = dim W	(d)	All the above
$(c)$ $f^1$ exist		i La non zero column
(c) $f^1$ exist  Ans $\mathbf{d}$ iv. Consider the system $AX = b$ , Where $A \in M_n(\mathbb{R})$	an inverti	ible matrix and b a non zero
iv. Consider the system $AX = b$ , Where $A \in M_n(\mathbb{R})$ vector. The solution of above system obtained by	using cran	May not be unique
vector. The solution of above system  (a) Always unique	(b)	
(a)	(d)	None of these.
(c) Always Zero	1	
Ans (a)  Output  Ans (b)  Ans (a) $v$ . Dimension of Solution space of a $m \times n$ homogement	meaus syst	em of linear equations $AX = 0$ is
Dimension of Solution space of a $m \times n$ homoge	(b)	n
(2)	(d)	n – Rank A
m - Rank A		
Ans (d)		
vi. $\int_{1 \text{ et } F} \frac{1}{2\pi} \left( \begin{array}{cc} 1 & 0 & 0 \\ 0 & 0 & 1 \end{array} \right)$ then $E^{-1}$ is		
0 1 0	(b)	$(1 \ 0 \ 0)$
(-1) $(-1)$ $(0)$	(b)	$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$
$ \begin{array}{c cccc} \hline  & (a) & $		$(0 \ 0 \ 0)$
$\frac{0}{(1 \ 0 \ 0)}$	(d)	$\begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$
$ \begin{array}{c c} \hline  & (c) \\ \hline  & (0 & 0 & 1) \end{array} $		0 1 0
0 1 0/		
Ans (c)	he rank of	A is
Ans (c)  vii. If A is a non-singular matrix of order n, then t	(b)	0
(a) n	(d)	None of these
n-1		
Ans (a)	10 n, U(n)	is
Ans (a)  viii. Order of group of prime residue classes modu	(b)	n!
(a) n	(d)	None of these
(c) Euler function $\phi(n)$	1_\	
Ans (c)		
TANO		

0	
2	

ix.	Let (	$G_1 = \{\overline{1}, \overline{2}, \overline{3}\} \mod 4$ under multiplication of under multiplication of residue classes m	of residue	olagon	
		3 under multiplication of residue classes m	odulo 3 T	classes modulo 4 and $G_2 = \{\overline{1}, \overline{2}\}$	
<u></u>	(a)_	G <sub>1</sub> and G <sub>2</sub> are groups	(b)	<del> </del>	
	(c)	only G <sub>2</sub> is group		only G <sub>1</sub> is group	
	Ans	(c)	(d)	None of these	
x.	Ident	Identity element of group $\mathbb{Q}^*$ under binary operation defined as $a*b = \frac{ab}{a}$ is			
<u>-</u>	(a)	1 group & under binary opera	tion define	ed as $a * b = \frac{ab}{a}$ is	
		<u> </u>	(b)	2	
	(c)	1	(d)		
· <u> </u>	<del></del>	$\frac{\overline{2}}{2}$	(-)	4	
	Ans	(b)	<u> </u>		
Q2.	Atten	opt any ONE question from the following:		(08)	
a)	i.	Let $V, W$ be vector spaces over $\mathbb{R}$ and $T: V$ -dimensional then show that dim $V$ = dim Ker $T$ + dim Im $T$ .	<i>W</i> be a lir	near transformation and if $V$ is finite	

Ans	<b>Proof:</b> We have $T: V \to W$ , be a linear transformation, ker $T \subseteq V$ is a subspace of $V$ .	
	Let $\dim V = n$ , $\dim \ker T = r$ , $\dim W = m$	
	Let $B = \{u_1, u_2, \dots, u_r\}$ be basis of ker T As ker T is subspace of V, B is a linearly independent subset of V and hence can be extended to a basis of V.	
:	Let $B_1 = \{u_1, u_2, \dots, u_r, u_{r+1}, \dots, u_n\}$ be a basis of V, obtained by extension of B.	1
	Let $w_i = T(V_{r+i}), \forall i = 1, \dots, n-r$ .	.1
	Claim: $B_2 = \{w_1, w_2, \dots, w_{n-r}\}$ forms a basis of $I_mT$	
	Let us prove first, B <sub>2</sub> is linearly independent	1
	Let $a_1, a_2, \dots a_{n-r}$ be scalars such that	
	$a_1w_1 + a_2w_2 + \cdots + a_{n-1}w_{n-1} = 0$	ı
	But $T(u_{r+1}) = w_1, T(u_{r+2}) = w_2, \cdots T(u_n) = w_{n-r}$	
	$\therefore a_1 T(u_{r+1}) + a_2 T(u_{r+2}) + \cdots + a_{n-r} T(u_n) = 0$	
	$T(a_1u_{r+1} + a_2u_{r+2} + \cdots + a_{n-r}u_n) = 0$	1
	$\left( n-r\right)$	
	$\Rightarrow T\left(\sum_{i=1}^{n-r} a_i u_{r+i}\right) = 0$	
	n-r	
	$\Rightarrow \sum a_i u_{r+i} \in \ker T$	
	i = 1	
	$\Rightarrow \exists b_1, b_2, \dots, b_r \text{ scalar s.t.}$	1
	n-r r	
	$\sum a_i u_{r+i} = \sum b_j u \cdot \cdots as B \text{ is basis of ker } T$	1
	i=1 $i=1$	
	$\Rightarrow b_1 u_1 + b_2 u_2 + \cdots + b_r u_r - (a_1 u_{r+1} + \cdots + a_{n-r} u_n) = 0$	
	As $B_1 = \{u_1, u_2, \dots, u_r, u_{r+1}, \dots u_n\}$ is lin independent	1
	$\Rightarrow$ $b_1 = b_2 = b_r = 0$ and	

(4)

	$a_1 = a_2 = a_{n-r} = 0$	1
	$\Rightarrow (w_1, w_2, \dots, w_{n-r})$ is linearly independent $\dots (1)$	'
	Claim: $\{w_1, w_2, \dots, w_{n-r}\}$ spans $l_m(T)$	
	Let $w \in I_mT$	
	$\Rightarrow \exists v \in V \text{ such that } T(v) = w.$	
<u> </u>	As $B_1 = \{u_1, u_2, \dots, u_r, u_{r+1}, u_{r+2}, \dots u_n\}$ is a basis of V.	
	$\Rightarrow \exists b_1, b_2, \dots, b_n \in \mathbf{1R}$ such that	
	$v = b_1 u_1 + b_2 u_2 + \cdots + b_r u_r + b_{r+1} u_{r+1} + \cdots + b_n u_n$ As T is linear,	
	$T(v) = T(b_1u_1 + \cdots + b_ru_r) + T(b_{r+1}, u_{r+1} + \cdots + b_nu_n)$	
	$= b_1 T(u_1) + b_2 T(u_2) + \cdots + b_r T(u_r) + b_r + 1 T(u_{r+1}) + \cdots + b_n T(u_n) + \cdots$ as T is linear	
	$T(v) = b_{r+1}T(u_{r+1}) + b_{r+2}T(u_{r+2}) + \cdots + b_nT(u_n)$	
	as $u_1, u_2, \dots, u_r \in \ker T$	
	$\Rightarrow T(\mathbf{v}) = \mathbf{b}_{r+1} \mathbf{w}_1 + \mathbf{b}_{r+2} \mathbf{w}_2 + \cdots + \mathbf{b}_n \mathbf{w}_{n \leftarrow Y}$	
	$\Rightarrow \mathbf{w} = \mathbf{b_{r+1}} \mathbf{w_i} + \mathbf{b_{r+2}} \mathbf{w_2} \cdot \cdots + \mathbf{b_n} \mathbf{w_n}.$	
	$\Rightarrow$ w $\in$ span $\{w_1, \dots, w_{n-r}\}$	
	$\Rightarrow$ If $w \in I_m T \Rightarrow w \in \text{span} \{w_1, w_2, \cdots, w_{n-r}\}$	
	$\Rightarrow \{w_1, w_2, \dots, w_{n-r}\} \text{ spans } I_m T_{n-1} $	
	$\Rightarrow$ $\{w_1, w_2, \dots, w_{n-r}\}$ forms a basis of $ImT \dots From (I)$ and $(II)$	
	$\therefore \dim (I_m T) = n - r$	
	$\dim (\ker T) = r$	
	$\dim v = n = \dim (I_m T) + \dim (\ker T) = r + n - r,$	·
	$\therefore \left[ \dim (V) = \dim (\ker T) + \dim (I_m T) \right]$	
	as $n = r + n - r$	
	Aunk nullity theorem is verified.	
ii	Let $T: V \to V$ be a linear transformation where V is a finite dimensional vector space	over
<u> </u>	then prove that $T$ is injective if and only if $T$ is surjective.	

		1
1		~ I
	S	• 1
	_	
•		

1	Ans	Let $T: V \to V$ . Suppose T is 1-1.	
	. 215	$\therefore \ker T = \{0\}$	1
		$\therefore \dim (\ker T) = 0.$	$  {}_1  $
		By rank nullity theorem.	
		$\dim V = \text{nullity } T + \text{Rank of } T$	
	!	$\dim V = 0 + \text{Rank of } T$	1
		$\dim V = \dim (I_m T)$	
		As I <sub>n</sub> T ⊆ V subspace of V such that	
		$\dim V = \dim I_m T$	
		$\Rightarrow I_m T = V$	1
		 ∴ T(V) = V	
		T is onto.	
		Conversly, suppose T is onto.	$ _{1} $
		$I_{m}T = V$	
		T(V) = V.	1
		$\therefore \dim (I_m T) = \dim V$	1
		Rank nullity theorem,	1
		$din(V) = dim(ker T) + dim I_m T$	$\begin{vmatrix} 1 \end{vmatrix}$
		$\Rightarrow$ dim V = dim (ker T) + dim V	
		$\Rightarrow$ dim ker T = 0.	1
		$\Rightarrow \ker T = \{0\}$	
	<u> </u>	⇒ T is 1 – 1	
		∴ T is 1 – 1 if and only if T is onto.	
Q.2	Atten	apt any TWO questions from the following: (12)	
b)	i.		
		If $T: \mathbb{R}^3 \to \mathbb{R}^2$ such that $T(x,y,z) = (x,y)$ , show that T is linear. Find ker T, basis	of
		ker T and nullity T.	
	Ans	Let $(x, y, z) \in ker T$ so that	1
		x=0, y=0	1
		Thus we put $z = t$ ,	2
		$\Rightarrow ker T = \{t(0,0,1) / t \in \mathbb{R}\}$ . Therefore the basis of $ker F$ is $(0,0,1)$ and nullity	2
		T=1.	<u></u>
	ii.	Check whether $T: \mathbb{R}^3 \to \mathbb{R}^3$ such that	
		T(x,y,z) = (x+y,x-z,y+2z) is an isomorphism?	

(b)

Ans Let $(x, y, z) \in ker T$ so that	11
The matrix corresponding to this system $\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 2 \end{pmatrix}$ whose row reduced form is	
(1 1 0)	1
whose row reduced form is $0 - 1$ whose row reduced form is	
	2
$\begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \qquad \qquad \text{for } F = \{0\}$	
$\begin{vmatrix} 1 & 0 & 1 & 0 \end{vmatrix}$ . Thus $x = y = z = 0 \implies \text{Ref } Y = \{0\}$	2
$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \text{ Thus } x = y = z = 0 \Rightarrow \ker F = \{0\}$	
Since Dim $V = 3 = \dim \mathbb{R}^3$ , T is onto. Therefore T is invertible. Therefore T is an	
isomorphism $\mathbb{R}^n$ .	
vi -i viector space is isolated	
Ans Let $x_1 x_2 \dots x_n$ be a basis of	2
$ \mathcal{L}_{-}$ $\mathcal{L}_{-}$ $\mathcal{L}_{-}$	2
	2
Thus a LT is defined by $T(x) = (c_1,, c_n)$ is since Dim $V = n = \dim \mathbb{R}^n$ , T is onto. Therefore T is invertible. Therefore $V \cong \mathbb{R}^n$ .	-
Since Diffi v = 1	f(x) =
iv. $P_3[\mathbb{R}]$ denote the vector space of all polynomials over $\mathbb{R}$ of degree 3 or less an $D(\mathbb{R})$	-
$P_3[\mathbb{R}]$ denote the vector spanning. Let $\frac{df(x)}{dx}$ , $\forall f(x) \in P_3[\mathbb{R}]$ denote the differentiation mapping. Let	
$\frac{d(x)}{dx}, \forall f(x) \in P_3[\mathbb{R}] \text{ denote an}$	
$B = \{1, 1+x, 1+x^2, 1+x^3\} \text{ be the basis. Find} \left[m(D)\right]_B^B.$	
	\
Ans $Dv_1 = 0 = 0v_1 + 0v_2 + 0v_3 + 0v_4$	
$D_{v_1} = 1 - 1v_1 + 0v_2 + 0v_3 + 0v_4$	4
$\int_{Dv_1}^{2} = 2x = -2v_1 + 2v_2 + 0v_3 + 0v_4$	
$Dv_3 = 2x = -2v_1 + 2v_2 + 0v_3 + 0v_4$	
$Dv_3 = 2x = -2v_1 + 2v_2 + 0v_3 + 0v_4$	
$Dv_3 = 2x = -2v_1 + 2v_2 + 0v_3 + 0v_4$	2
$Dv_3 = 2x = -2v_1 + 2v_2 + 0v_3 + 0v_4$	2
$Dv_3 = 2x = -2v_1 + 2v_2 + 0v_3 + 0v_4$	2
$Dv_3 = 2x = -2v_1 + 2v_2 + 0v_3 + 0v_4$	2
$Dv_3 = 2x = -2v_1 + 2v_2 + 0v_3 + 0v_4$ $Dv_4 = 3x^2 = -3v_1 + 0v_2 + 3v_3 + 0v_4$ $\Rightarrow \left[ m(D) \right]_B^B = \begin{pmatrix} 0 & 1 & -2 & -3 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$	
$Dv_3 = 2x = -2v_1 + 2v_2 + 0v_3 + 0v_4$ $Dv_4 = 3x^2 = -3v_1 + 0v_2 + 3v_3 + 0v_4$ $\Rightarrow \left[ m(D) \right]_B^B = \begin{pmatrix} 0 & 1 & -2 & -3 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$	2
$Dv_3 = 2x = -2v_1 + 2v_2 + 0v_3 + 0v_4$ $Dv_4 = 3x^2 = -3v_1 + 0v_2 + 3v_3 + 0v_4$ $\Rightarrow \left[ m(D) \right]_B^B = \begin{pmatrix} 0 & 1 & -2 & -3 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$	2
$Dv_{3} = 2x = -2v_{1} + 2v_{2} + 0v_{3} + 0v_{4}$ $Dv_{4} = 3x^{2} = -3v_{1} + 0v_{2} + 3v_{3} + 0v_{4}$ $\Rightarrow \left[ m(D) \right]_{B}^{B} = \begin{pmatrix} 0 & 1 & -2 & -3 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ $Q3. \text{ Attempt any ONE question from the following:} $ $(08)$	
$Dv_{3} = 2x = -2v_{1} + 2v_{2} + 0v_{3} + 0v_{4}$ $Dv_{4} = 3x^{2} = -3v_{1} + 0v_{2} + 3v_{3} + 0v_{4}$ $\Rightarrow \left[m(D)\right]_{B}^{B} = \begin{pmatrix} 0 & 1 & -2 & -3 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ $\Rightarrow DNE \text{ question from the following:} $ $(08)$	



Αn	(=	$\Rightarrow$ ) iven: $det(A^1, A^2) = 0$ iven: $\{A^1, A^2\}$ is linearly dependent.	
	S	uppose $\{A^1, A^2\}$ is linearly independent $\{A^1, A^2\}$ is the basis of $\mathbb{R}^2$ set $E^1 = \alpha_1 A^1 + \alpha_2 A^2$ and $E^2 = \beta_1 A^1 + \beta_2 A^2$ set $E^1 = \alpha_1 A^1 + \alpha_2 A^2$ and $E^2 = \alpha_1 A^1 + \beta_2 A^2$ set $E^1 = (\alpha_1 A^1 + \alpha_2 A^2, \beta_1 A^1 + \beta_2 A^2)$ set $E^1 = (\alpha_1 \beta_2 - \alpha_2 \beta_1)$ det $E^1 = (\alpha_1 \beta_1 - \alpha_2 \beta_1)$	3
		( $\Leftarrow$ ) Given: $\{A^1, A^2\}$ is linearly dependent. T.P.T: $\det(A^1, A^2) = 0$	
		As $\{A^1, A^2\}$ is linearly dependent $\therefore \text{Let } A^2 = cA^1, c \neq 0, c \in \mathbb{R}$ $\det(A^1, A^2) = \det(A^1, cA^1)$ $= \cot(A^1, A^1)$	3
			2
	ii.	Prove that the general solution of the non homogeneous system is sum of a particular solution of the system and the solutions of the associated homogeneous system.  Let $AX = b$ be a non-homogeneous system of linear equations.	
	Ans	And $AX = 0$ be its associated homogeneous system $AX = b$ . Let $x_0$ be the particular solution of non-homogeneous system $AX = b$ is given by, T.P.T: Set of solutions of the non-homogeneous system $AX = b$ is given by, $\{x_0 + x \mid x \text{ is a solution of associated homogeneous system } AX = b$ . Claim 1: Every element of $(*)$ is a solution of non-homogeneous system $AX = b$ . Let $x$ is be a solution of associated homogeneous system.	
		$AX = 0$ $\therefore Ax = 0$ Then, $A(x_0 + x) = Ax_0 + Ax = b + 0 = b$ $\therefore x_0 + x$ is the solution of non-homogeneous system $AX = b$ Claim 2: Every solution of non-homogeneous system $AX = b$ Let $x'$ be the solution of non-homogeneous system $AX = b$ $x' = x_0 + (x' - x_0)$	
		Hence, x' is an element of (*) By claim 1 and claim 2  Set of solution of the non homogeneous system is precisely the sum of a particular solution of the system and the solutions of the associated homogeneous system.	



)	(12)	
/ Q3.	resto questions from the following:	
b)	i. Define adjoint of a matrix. Find $A^{-1}$ for $A = \begin{pmatrix} 3 & 0 & 2 \\ 2 & 0 & -2 \\ 0 & 1 & 1 \end{pmatrix}$ using adjoint.	
	Ans   For $A \in M_n(\mathbb{R})$ , Let $A_{ij}$ be the matrix obtained from $A$ by deleting its ith row and jth column   Let $c_{ij} = (-1)^{i+j} \det A_{ij}$   $C = (c_{ij})$ is called matrix of cofactors   $adj(A) := C^t$   Given matrix is $A = \begin{pmatrix} 3 & 0 & 2 \\ 2 & 0 & -2 \\ 0 & 1 & 1 \end{pmatrix}$   Matrix of cofactors is $C = \begin{pmatrix} 2 & 2 & 0 \\ -2 & 3 & 10 \\ 2 & -3 & 0 \end{pmatrix}$   Adj $(A) = C^t = \begin{pmatrix} 2 & 2 & 0 \\ -2 & 3 & 10 \\ 2 & -3 & 0 \end{pmatrix}$   $A^{-1} = \frac{1}{\det A} Adj(A) = \frac{1}{10} \begin{pmatrix} 2 & 2 & 3 & 10 \\ 2 & -3 & 0 \end{pmatrix}$   ii.   Solve the following system using Cramer's rule.   $2x + y + z = 1, x - y + 4z = 0, x + 2y - 2z = 3$   Ans   $2x + y + z = 1, x - y + 4z = 0, x + 2y - 2z = 3$   The corresponding nonhomogeneous system is   $\begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & 4 \\ 1 & 2 & -2 \end{pmatrix} \begin{pmatrix} x & 1 \\ y & x & 1 \end{pmatrix} \begin{pmatrix} x & 1 & 1 \\ 3 & 2 & -2 \\ 2 & 1 & 1 \\ 4 & 2 & 2 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 3 & 2 & -2 \\ 2 & 1 & 1 \\ 4 & 2 & -2 \end{pmatrix}$   $\det \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 4 \\ 1 & 2 & -2 \end{pmatrix}$   $\det \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 4 \\ 1 & 2 & -2 \end{pmatrix}$   $\det \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 4 \\ 1 & 2 & -2 \end{pmatrix}$   $\det \begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & 0 \\ 2 & 1 & 1 \\ 1 & -1 & 4 \\ 1 & 2 & -2 \end{pmatrix}$   $\det \begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & 0 \\ 2 & 1 & 1 \\ 1 & -1 & 4 \\ 1 & 2 & -2 \end{pmatrix}$   $\det \begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & 0 \\ 2 & 1 & 1 \\ 1 & -1 & 4 \\ 1 & 2 & -2 \end{pmatrix}$   $\det \begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & 0 \\ 2 & 1 & 1 \\ 1 & -1 & 4 \\ 1 & 2 & -2 \end{pmatrix}$   $\det \begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & 0 \\ 2 & 1 & 1 \\ 1 & -1 & 4 \\ 1 & 2 & -2 \end{pmatrix}$   $\det \begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & 4 \\ 1 & 2 & -2 \end{pmatrix}$   $\det \begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & 4 \\ 1 & 2 & -2 \end{pmatrix}$   $\det \begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & 4 \\ 1 & 2 & -2 \end{pmatrix}$   $\det \begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & 4 \\ 1 & 2 & -2 \end{pmatrix}$   $\det \begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & 4 \\ 1 & 2 & -2 \end{pmatrix}$   $\det \begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & 4 \\ 1 & 2 & -2 \end{pmatrix}$   $\det \begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & 4 \\ 1 & 2 & -2 \end{pmatrix}$   $\det \begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & 4 \\ 1 & 2 & -2 \end{pmatrix}$   $\det \begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & 4 \\ 1 & 2 & -2 \end{pmatrix}$   $\det \begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & 4 \\ 1 & 2 & -2 \end{pmatrix}$   $\det \begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & 4 \\ 1 & 2 & -2 \end{pmatrix}$   $\det \begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & 4 \\ 1 & 2 & -2 \end{pmatrix}$   $\det \begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & 4 \\ 1 & 2 & -2 \end{pmatrix}$   $\det \begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & 4 \\ 1 & 2 & -2 \end{pmatrix}$   $\det \begin{pmatrix} 2 & 1 & 1 \\ 2 & -1 & 1 \end{pmatrix}$   $\det \begin{pmatrix} 2 & 1 &$	2 2 2

iii. Define Elementary matrix. Which of the following are elementary matrices? Justify. $ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} C = \begin{pmatrix} 1 & 3 \\ 0 & 2 \end{pmatrix} $
watrix. Which of the following are elementary
iii. Define Elementary matrix. Without $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ , $B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ , $C = \begin{pmatrix} 1 & 3 \\ 0 & 2 \end{pmatrix}$
0 0 1/ (0 1 or dentity matrix by applying any of 2
$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, B = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ Ans Elementary matrix is the matrix obtained from the identity matrix by applying any of 2  Ans I be a property row operations.
Ans Elementary matrix is the elementary row operations, the elementary row operations, the elementary row operation $R_1 \rightarrow R_1 + 2R_3$
the elementary by applying row operation $R_1 \rightarrow R_1$
Ans Elementary matrix.  Elementary row operations, the elementary row operations, and the elementary matrix by applying row operation $R_1 \rightarrow R_1 + 2R_3$ .  A is obtained from identity matrix.
A is obtained from identity matrix.  A is elementary matrix.  B is obtained from identity matrix by applying row operation $R_2 \rightarrow R_3$ B is obtained from identity matrix.
n is obtained from identity matrix by applying
B is obtained from $B$ is elementary matrix. $B$ is elementary matrix.
Lead from identity matrix by applying any of
B is obtained from A.  ∴ B is elementary matrix.  C can not be obtained from identity matrix by applying any of the row operation.  C is not elementary matrix.
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
AX = B has a system of linear equations $AX = B$ has a
iv. $A \in M_n(\mathbb{R})$ . Prove that the non-homogeneous system of linear equations $AX = b$ has a lation if and only if rank $A$ =rank $(A, b)$
iv. $A \in M_n(\mathbb{R})$ . Flow $A = \operatorname{rank}(A, B)$ solution if and only if rank $A = \operatorname{rank}(A, B)$ and $A = \operatorname{rank}(A, B)$ The non-homogeneous system of linear equations $AX = b$ has a solution $A = \operatorname{rank}(C_1)$ (2)
Solution if the solution of the system of linear equations $AX = b$ has the solution if the non-homogeneous system of linear equations $AX = b$ has the s
$\begin{pmatrix} c_2 \\ \pm \end{pmatrix} \begin{pmatrix} 0 \\ \text{such that } Ac = b \end{pmatrix}$
$\Rightarrow \exists c = \left(\begin{array}{c} \vdots \\ \vdots \\ \end{array}\right)^{T} \left(\begin{array}{c} \vdots \\ \vdots \\ \end{array}\right)$
$C_n \in \mathbb{R}$ (not all zero) such that $C_1$ ?
$\Leftrightarrow \exists c_1, c_2, \dots \circ n$ A is linearly dependent with 'n' columns of A
$\Leftrightarrow \exists c_1, c_2, \dots c_n \in \mathbb{R} \text{ (not any alpha and a second of } A$ $\Leftrightarrow \text{Column vector } b \text{ is linearly dependent with 'n' columns of } A$
$\Leftrightarrow \operatorname{rank} A = \operatorname{rank} (A, b) \tag{08}$
⇔ rank A-rank (00)
Q4. Attempt any ONE question from the following:
his not a square. Identify
a) i. Discuss the group of symmetries of a rectangle which is not a square and let O denote  Ans Let 1,2,3,4 denote the vertices of a rectangle which is not a square and let O denote
Ans Let 1,2,3,4 denote the vertices of a rectangle which is not a square and the centroid of the rectangle. The group of symmetries of G consists of 2 rotations
Ans Let 1,2,3,4 denote the verse. The group of symmetries of G consists of
the rectangle.
and 2 reflections. $(1,2,3,4)$ and $(1,2,3,4)$ and $(1,2,3,4)$ .
the centroid of the and 2 reflections.  (1) $e = \text{Rotation of } 0^{\circ} \text{ about O in the anticlockwise direction} = \begin{pmatrix} 1,2,3,4 \\ 1,2,3,4 \end{pmatrix}$ . (draw
(1) e = Rotation of
figure) figure direction = $\binom{1,2,3,4}{3,4,1,2}$ =
figure)  (2) $\sigma_1$ = Rotation of 180° about O in the anticlockwise direction = $\binom{1,2,3,4}{3,4,1,2}$ =
$(2) \sigma_1 = \text{Kolation of }$
(1,3)(2,4). (draw figure)
9
· · · · · · · · · · · · · · · · · · ·

	(3) $\sigma_2$ = Reflection about the line passing through O which is the perpendicular bisector of side $(1, 2) = \binom{1,2,3,4}{2,1,4,3} = (1,2)(3,4)$ . (draw figure)  (4) $\sigma_3$ = Reflection about the line passing through O which is the perpendicular bisector of side $(1,4) = \binom{1,2,3,4}{2,3,2,1} = (1,4)(2,3)$ . (draw figure)  Thus $G = \{e, \sigma_1, \sigma_2, \sigma_3\}$ is a group of order 4. Further, since the square of every element of $G$ gives the identity element $e$ (this can also be seen geometrically). Therefore $G$ is actaually the Klein-4 group $V_4$ .	6
ii.	Define Subgroup. Let $G$ be a group and $H, K$ be subgroups of $G$ . Prove that $H \cap K$ is a subgroup of $G$ but $H \cup K$ may not be a subgroup of $G$ .  Let $H$ be a non-empty subset of a group $G$ . We say, $H$ is a subgroup of $G$ if $H$ itself is a group under the same binary operation as of $G$ .  Let $H, K$ be subgroups of $G$ . $\therefore e \in H$ and $e \in K \Rightarrow e \in H \cap K \Rightarrow H \cap K \neq \emptyset$ Consider any $a, b \in H \cap K \Rightarrow a, b \in H$ and $a, b \in K$ Since $H$ , $K$ are subgroups of $G$ , we get $ab^{-1} \in H$ and $ab^{-1} \in K$ . $\therefore ab^{-1} \in H \cap K$ $\therefore H \cap K$ is a subgroup of $G$ Further, $H \cup K$ may not be a subgroup of $G$ .  For example, Consider the group $U(8) = \{\overline{1}, \overline{3}, \overline{5}, \overline{7}\}$ under multiplication modulo $g$ .  Let $g$ and $g$ and $g$ are subgroups of $g$ which follows from the following composition tables: $g$ and $g$ are subgroups of $g$ which follows from $g$ and $g$ are subgroups of $g$ which follows from $g$ and $g$ are subgroups of $g$ and $g$ are subgroups of $g$ which follows from $g$ and $g$ are subgroups of $g$ and	3
Q4.	But $H \cup K = \{\overline{1}, \overline{3}, \overline{5}\}$ is not a subgroup of $G$ since $\overline{3}, \overline{5} \in H \cup K$ and $\overline{3} \cdot \overline{5} = \overline{7} \notin H \cup K$ .  Attempt any TWO questions from the following: (12)	
b)	i. Let G be a group with identity e. (p) Show that: $(ab)^{-1} = b^{-1}a^{-1}, \forall a, b \in G$ . (q) If $a^2 = e, \forall a \in G$ , then G is abelian.	

11	_ \
11	- )

iii. Let $G$ be a group and $a \in G$ . Show that $H = \{a^n   n \in \mathbb{Z} \text{ is the smallest subgroup of } G$ containing $a$ .  Ans $H \neq \phi$ , since $e = a^0 \in H$ . Also, for $a^n, a^m \in H$ , $a^n(a^m)^{-1} = a^{n-m} \in H$ .  Therefore, $H$ is a subgroup of $G$ .  Also, if $K$ is a subgroup of $G$ containing $a$ , then $a, a^{-1} \in K$ . Now, since $K$ is a group, $a^n, a^{-n} = (a^{-1})^n \in K$ , $\forall n \in \mathbb{N}$ . Whence $a^n \in K$ , $\forall n \in \mathbb{Z}$ . $\forall n \in \mathbb{Z}$ is the smallest subgroup of $G$ convaining $a$ .  iv. Prove that $G = \{\begin{pmatrix} a & b \\ -b & a \end{pmatrix} : a, b \in \mathbb{R} \}$ is a group under matrix addition.  Ans Closure: $\begin{pmatrix} a & b \\ -b & a \end{pmatrix} + \begin{pmatrix} c & d \\ -d & c \end{pmatrix} = \begin{pmatrix} a+c & b+d \\ -b-d & a+c \end{pmatrix} \in G$ Prove Associative  Prove identity is $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ Prove inverse of $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ is $\begin{pmatrix} -a & -b \\ b & -a \end{pmatrix}$ .  Q5. Attempt any FOUR questions from the following:  (20)  Ans Proof: Let $C_1 \cap C_1 \cap C_2 \cap C_2 \cap C_3 \cap C_4 \cap C_4 \cap C_4 \cap C_5 \cap C_4 \cap C_5 \cap C_4 \cap C_5 \cap C_$		$-\frac{1}{2} - \frac{1}{2} - \frac{1}{2} - \frac{1}{2} - \frac{1}{2} - \frac{1}{2}$	
We have $(ab)(ab)$ used by the sides of this equation first by $a$ and then by Left-multiplying both sides of this equation first by $a$ and then by then by $b^{-1}$ yields $(ab)^{-1} = b^{-1}a^{-1}$ . (a) Let $a, b \in G$ , then $a^2 = e$ and $b^2 = e$ as also $(ab)^2 = e$ , which implies $(ab)(ab) = aa \Rightarrow bab = a \Rightarrow bab^2 = ab \Rightarrow ba = ab$ . $\therefore G$ is abelian  ii. Construct composition table of $\mathbb{Z}_5^*$ under multiplication modulo 5. Also find order of its each element.  Ans $\mathbb{Z}_5^* = \{\overline{1}, \overline{2}, \overline{3}, \overline{4}\}$ Composition table $\overline{1}$ is the identity. $\therefore o(\overline{1}) = 1$ $\overline{2} = 2 + 4$ , $\overline{2} = 3 = 3$ , $\overline{2}^4 = \overline{1}$ $\therefore o(\overline{3}) = 4$ $\overline{3}^2 = \overline{4}$ , $3^3 = \overline{2}$ , $3^4 = \overline{1}$ $\therefore o(\overline{3}) = 4$ $\overline{3}^2 = \overline{4}$ , $3^3 = \overline{2}$ , $3^4 = \overline{1}$ $\therefore o(\overline{3}) = 4$ $\overline{3}^2 = \overline{4}$ , $a = \overline{3}^3 = \overline{2}$ , $a = \overline{1}$ $\therefore o(\overline{3}) = 4$ $\overline{3}^2 = \overline{4}$ , $a = \overline{3}^3 = \overline{2}$ , $a = \overline{1}$ $\therefore o(\overline{3}) = 4$ $\overline{3}^2 = \overline{4}$ , $a = \overline{3}^3 = \overline{2}$ , $a = \overline{1}$ $\therefore o(\overline{3}) = 4$ $\overline{3}^2 = \overline{4}$ , $a = \overline{3}$ , $a = \overline{1}$ $\Rightarrow o(\overline{3}) = 4$ $\overline{3}^2 = \overline{4}$ , $a = \overline{3}$ , $a = \overline{1}$ $\Rightarrow o(\overline{3}) = 4$ $\overline{3}^2 = \overline{4}$ , $a = \overline{3}$ , $a = \overline{1}$ , $a = 1$		(a) To show that for any group $G$ , $(ab)^{-1} = b^{-1}a$	
thenbyb <sup>-1</sup> yields( $ab$ ) <sup>-1</sup> = $b^{-1}a^{-1}$ .  (a) Let $a,b \in G$ , then $a^2 = e$ and $b^2 = e$ as also ( $ab$ ) <sup>2</sup> = $e$ , which implies ( $ab$ )( $ab$ ) = $aa \underset{\longrightarrow}{\Longrightarrow} bab = a \Rightarrow bab^2 = ab \Rightarrow ba = ab$ .  3  ii. Construct composition table of $\mathbb{Z}_5^*$ under multiplication modulo 5. Also find order of its each element.  Ans $\mathbb{Z}_5^* = \{\overline{1}, \overline{2}, \overline{3}, \overline{4}\}$ Composition table $\overline{1}$ is the identity. $b$ :	Ans	We have $(ab)(ab)^{-1} = e$ .	3
thenbyb 'yetcus(ut)' $(q)$ Let $a,b \in G$ , then $a^2 = e$ and $b^2 = e$ as also $(ab)^2 = e$ , which improve $(ab)(ab) = aa \underset{\square G}{\Rightarrow} bab = a \Rightarrow bab^2 = ab \Rightarrow ba = ab$ .  ii. Construct composition table of $\mathbb{Z}_5^e$ under multiplication modulo 5. Also find order of its each element.  Ans $\mathbb{Z}_5^e = \{\overline{1}, \overline{2}, \overline{3}, \overline{4}\}$ Composition table $\overline{1}$ is the identity. $\cdots o(\overline{1}) = 1$ $\overline{2}^2 = \overline{4}, \overline{3}^3 = \overline{3}, \overline{2}^4 = \overline{1} \cdots o(\overline{3}) = 4$ $\overline{4}^2 = \overline{1} \cdots o(\overline{4}) = 2$ iii. Let $G$ be a group and $a \in G$ . Show that $H = \{a^n \mid n \in \mathbb{Z} \text{ is the smallest subgroup of } G$ containing $a$ .  Ans $H \neq \phi$ , since $e = a^0 \in H$ . Also, for $a^n, a^m \in H$ , $a^n(a^m)^{-1} = a^{n-m} \in H$ . Therefore, $H$ is a subgroup of $G$ . Also, if $K$ is a subgroup of $G$ containing $a$ , then $a, a^{-1} \in K$ . Now, since $K$ is a group, $A$ and $A$ is a subgroup of $A$ containing $A$ .  iv. Prove that $G = \{(a  b) : a, b \in \mathbb{R}\}$ is a group under matrix addition.  Ans Closure: $(a  b) + (c  d) = (a + c  b + d) \in G$ Prove Associative Prove Identity is $(a  b) = (a  b) = (a $		1 - O' 1-3 MOTH SILIES OF MIN TH	1
<ul> <li>(a) Let a, b ∈ c, then a = a ⇒ bab = a ⇒ ba = ab.</li> <li>(ab)(ab) = aa ⇒ bab = a ⇒ bab² = ab ⇒ ba = ab.</li> <li>(b) (ab) = aa ⇒ bab = a ⇒ bab² = ab ⇒ ba = ab.</li> <li>(c) is abelian</li> <li>(d) Construct composition table of Z*s under multiplication modulo 5. Also find order of its each element.</li> <li>(d) Composition table 1</li></ul>	1	then by $h^{-1}$ yields $(ab)^{-1} = b^{-1}a^{-1}$ .	1
(ab)(ab) = ad	1	then $a^2 = e$ and $b^2 = e$ as also (ab)	1 1
<ul> <li>ii. Construct composition table of Z<sub>5</sub>* under multiplication modulo 5. Also find order of its each element.</li> <li>Ans Z<sub>5</sub>* = (1, 2, 3, 4) Composition table 1 is the identity. ∴ o(1) = 1</li></ul>	1	$(ab)(ab) = aa \Rightarrow bab = a \Rightarrow bab^2 = ab \Rightarrow ba = ab$	13
Ans   $\mathbb{Z}_5^* = \{\overline{1}, \overline{2}, \overline{3}, \overline{4}\}$   Composition table   $\overline{1}$ is the identity. $\therefore$ $o(\overline{1}) = 1$   $\overline{2^2 = \overline{4}}$ , $\overline{2^3} = \overline{3}$ , $\overline{2^4 = \overline{1}}$ $\therefore$ $o(\overline{2}) = 4$   $\overline{3^2 = \overline{4}}$ , $\overline{3^3 = \overline{2}}$ , $\overline{3^4 = \overline{1}}$ $\therefore$ $o(\overline{3}) = 4$   $\overline{4^2 = \overline{1}}$ $\therefore$ $o(\overline{4}) = 2$   iii.   Let $G$ be a group and $a \in G$ . Show that $H = \{a^n   n \in \mathbb{Z} \text{ is the smallest subgroup of } G$ of Goontaining $a$ .   H $\neq \phi$ , since $e = a^0 \in H$ . Also, for $a^n, a^m \in H$ , $a^n(a^m)^{-1} = a^{n-m} \in H$ .   Therefore, $H$ is a subgroup of $G$ .   Also, if $K$ is a subgroup of $G$ containing $a$ , then $a, a^{-1} \in K$ . Now, since $K$ is a group, Also, if $K$ is a subgroup of $G$ containing $a$ , then $a, a^{-1} \in K$ , so that $H$ is the smallest subgroup of $G$ convaining $a$ .   iv.   Prove that $G = \{\begin{pmatrix} a & b \\ -b & a \end{pmatrix} + \begin{pmatrix} c & d \\ -d & c \end{pmatrix} = \begin{pmatrix} a + c & b + d \\ -b - d & a + c \end{pmatrix} \in G$   Prove Associative   Prove Identity is $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$   Prove inverse of $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ is a group under matrix addition.   Q0   Prove inverse of $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ is $\begin{pmatrix} 0 & 0 \\ -b & a \end{pmatrix}$   Q5.   Attempt any FOUR questions from the following:   Q0   Q0   Prove inverse of   Q1   Prove inverse of   Q2   Prove inverse of   Q3   Prove inverse of   Q4   Prove inverse of   Q4   Prove inverse of   Q5   Prove		(db)(db) - LCL	
Ans   $\mathbb{Z}_5^* = \{\overline{1}, \overline{2}, \overline{3}, \overline{4}\}$   Composition table   $\overline{1}$ is the identity. $\therefore$ $o(\overline{1}) = 1$   $\overline{2^2 = \overline{4}}$ , $\overline{2^3} = \overline{3}$ , $\overline{2^4 = \overline{1}}$ $\therefore$ $o(\overline{2}) = 4$   $\overline{3^2 = \overline{4}}$ , $\overline{3^3 = \overline{2}}$ , $\overline{3^4 = \overline{1}}$ $\therefore$ $o(\overline{3}) = 4$   $\overline{4^2 = \overline{1}}$ $\therefore$ $o(\overline{4}) = 2$   iii.   Let $G$ be a group and $a \in G$ . Show that $H = \{a^n   n \in \mathbb{Z} \text{ is the smallest subgroup of } G$ of Goontaining $a$ .   H $\neq \phi$ , since $e = a^0 \in H$ . Also, for $a^n, a^m \in H$ , $a^n(a^m)^{-1} = a^{n-m} \in H$ .   Therefore, $H$ is a subgroup of $G$ .   Also, if $K$ is a subgroup of $G$ containing $a$ , then $a, a^{-1} \in K$ . Now, since $K$ is a group, Also, if $K$ is a subgroup of $G$ containing $a$ , then $a, a^{-1} \in K$ , so that $H$ is the smallest subgroup of $G$ convaining $a$ .   iv.   Prove that $G = \{\begin{pmatrix} a & b \\ -b & a \end{pmatrix} + \begin{pmatrix} c & d \\ -d & c \end{pmatrix} = \begin{pmatrix} a + c & b + d \\ -b - d & a + c \end{pmatrix} \in G$   Prove Associative   Prove Identity is $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$   Prove inverse of $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ is a group under matrix addition.   Q0   Prove inverse of $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ is $\begin{pmatrix} 0 & 0 \\ -b & a \end{pmatrix}$   Q5.   Attempt any FOUR questions from the following:   Q0   Q0   Prove inverse of   Q1   Prove inverse of   Q2   Prove inverse of   Q3   Prove inverse of   Q4   Prove inverse of   Q4   Prove inverse of   Q5   Prove	1	:: G is abelian	113
Ans   $\mathbb{Z}_5^* = \{\overline{1}, \overline{2}, \overline{3}, \overline{4}\}$   Composition table   $\overline{1}$ is the identity. $\circ$ $o(\overline{1}) = 1$   $\overline{2^2 = \overline{4}}$ , $\overline{2^3} = \overline{3}$ , $\overline{2^4 = \overline{1}}$ $\circ$ $o(\overline{3}) = 4$   $\overline{2^2 = \overline{4}}$ , $\overline{3^3 = \overline{2}}$ , $\overline{3^4 = \overline{1}}$ $\circ$ $o(\overline{3}) = 4$   $\overline{4^2 = \overline{1}}$ $\circ$ $o(\overline{4}) = 2$   iii.   Let $G$ be a group and $a \in G$ . Show that $H = \{a^n   n \in \mathbb{Z} \text{ is the smallest subgroup of } G$ containing $a$ .   H $\neq \phi$ , since $e = a^0 \in H$ . Also, for $a^n, a^m \in H$ , $a^n(a^m)^{-1} = a^{n-m} \in H$ .   Therefore, $H$ is a subgroup of $G$ .   Also, if $K$ is a subgroup of $G$ containing $a$ , then $a, a^{-1} \in K$ . Now, since $K$ is a group, Also, if $K$ is a subgroup of $G$ convaining $a$ .   iv.   Prove that $G = \{\begin{pmatrix} a & b \\ -b & a \end{pmatrix} : a, b \in \mathbb{R}\}$ is a group under matrix addition.   Ans   Closure: $\begin{pmatrix} a & b \\ -b & a \end{pmatrix} + \begin{pmatrix} c & d \\ -d & c \end{pmatrix} = \begin{pmatrix} a+c & b+d \\ -b-d & a+c \end{pmatrix} \in G$   Prove Associative   Prove Identity is $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$   Prove inverse of $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ is a group under matrix addition.   Q0   Prove inverse of $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ is $\begin{pmatrix} a & b \\ -b & -a \end{pmatrix}$   Q5.   Attempt any FOUR questions from the following:   Q0   Prove that if $V_1, V_2, \dots, V_n$   Ilinearly independent then $T(v_1), T(v_2), \dots, T(v_n)$ are linearly independent elements of   Since $T$ is it. it follows that $T(c_1v_1 + \dots + c_nv_n) = o$ . As ker $T = \{o\}$ , this implies that $c_1v_1 + \dots + c_nv_n = o$ which further implies that $c_1, c_2, \dots, c_n$		Construct composition table of Z <sub>5</sub> under many	
Ans $\mathbb{Z}_5^* = \{\overline{1}, \overline{2}, \overline{3}, \overline{4}\}$ Composition table $\overline{1}$ is the identity. $\therefore o(\overline{1}) = 1$ $\overline{2^2} = \overline{4},  \overline{2^3} = \overline{3},  \overline{2^4} = \overline{1}  \therefore o(\overline{2}) = 4$ $\overline{3^2} = \overline{4},  \overline{3^3} = \overline{2},  \overline{3^4} = \overline{1}  \therefore o(\overline{3}) = 4$ $\overline{4^2} = \overline{1}  \therefore o(\overline{4}) = 2$ iii. Let $G$ be a group and $a \in G$ . Show that $H = \{a^n \mid n \in \mathbb{Z} \text{ is the smallest subgroup of } G$ containing $a$ .  Ans $H \neq \phi$ , since $e = a^0 \in H$ . Also, for $a^n, a^m \in H$ , $a^n(a^m)^{-1} = a^{n-m} \in H$ . Therefore, $H$ is a subgroup of $G$ . Also, if $K$ is a subgroup of $G$ containing $a$ , then $a, a^{-1} \in K$ . Now, since $K$ is a group, $\therefore a^n, a^{-n} = (a^{-1})^n \in K, \forall n \in \mathbb{N}$ . Whence $a^n \in K, \forall n \in \mathbb{Z}$ . $\therefore H \subseteq K$ , so that $H$ is the smallest subgroup of $G$ convaining $a$ .  iv. Prove that $G = \{\begin{pmatrix} a & b \\ -b & a \end{pmatrix} : a, b \in \mathbb{R} \}$ is a group under matrix addition.  Ans Closure: $\begin{pmatrix} a & b \\ -b & a \end{pmatrix} + \begin{pmatrix} c & d \\ -d & c \end{pmatrix} = \begin{pmatrix} a+c & b+d \\ -b-d & a+c \end{pmatrix} \in G$ Prove Associative Prove Identity is $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ Prove inverse of $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ is $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ is $\begin{pmatrix} -a & -b \\ -b & -a \end{pmatrix}$ .  Q5. Attempt any FOUR questions from the following: (20)  Ans Proof: Let $C_1 \cap C_1 \cap C_2 \cap C_2 \cap C_3 \cap C_4 \cap C_4$	1"	each element.	
Ans $\mathbb{Z}_5^* = \{\overline{1}, \overline{2}, \overline{3}, \overline{4}\}$ Composition table $\overline{1}$ is the identity. $\therefore o(\overline{1}) = 1$ $\overline{2^2} = \overline{4},  \overline{2^3} = \overline{3},  \overline{2^4} = \overline{1}  \therefore o(\overline{2}) = 4$ $\overline{3^2} = \overline{4},  \overline{3^3} = \overline{2},  \overline{3^4} = \overline{1}  \therefore o(\overline{3}) = 4$ $\overline{4^2} = \overline{1}  \therefore o(\overline{4}) = 2$ iii. Let $G$ be a group and $a \in G$ . Show that $H = \{a^n \mid n \in \mathbb{Z} \text{ is the smallest subgroup of } G$ containing $a$ .  Ans $H \neq \phi$ , since $e = a^0 \in H$ . Also, for $a^n, a^m \in H$ , $a^n(a^m)^{-1} = a^{n-m} \in H$ . Therefore, $H$ is a subgroup of $G$ . Also, if $K$ is a subgroup of $G$ containing $a$ , then $a, a^{-1} \in K$ . Now, since $K$ is a group, $\therefore a^n, a^{-n} = (a^{-1})^n \in K, \forall n \in \mathbb{N}$ . Whence $a^n \in K, \forall n \in \mathbb{Z}$ . $\therefore H \subseteq K$ , so that $H$ is the smallest subgroup of $G$ convaining $a$ .  iv. Prove that $G = \{\begin{pmatrix} a & b \\ -b & a \end{pmatrix} : a, b \in \mathbb{R} \}$ is a group under matrix addition.  Ans Closure: $\begin{pmatrix} a & b \\ -b & a \end{pmatrix} + \begin{pmatrix} c & d \\ -d & c \end{pmatrix} = \begin{pmatrix} a+c & b+d \\ -b-d & a+c \end{pmatrix} \in G$ Prove Associative Prove Identity is $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ Prove inverse of $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ is $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ is $\begin{pmatrix} -a & -b \\ -b & -a \end{pmatrix}$ .  Q5. Attempt any FOUR questions from the following: (20)  Ans Proof: Let $C_1 \cap C_1 \cap C_2 \cap C_2 \cap C_3 \cap C_4 \cap C_4$			2
Composition table  I is the identity. $\dot{o}$ o( $\bar{1}$ ) = 1 $\bar{2}^2 = 4$ , $\bar{2}^3 = \bar{3}$ , $\bar{2}^4 = \bar{1}$ $o(\bar{2}) = 4$ $\bar{3}^2 = 4$ , $\bar{3}^3 = \bar{2}$ , $\bar{3}^4 = \bar{1}$ $o(\bar{3}) = 4$ $\bar{4}^2 = \bar{1}$ $o(\bar{4}) = 2$ iii. Let $G$ be a group and $a \in G$ . Show that $H = \{a^n   n \in \mathbb{Z} \text{ is the smallest subgroup of } G$ containing $a$ .  Ans $H \neq \phi$ , since $e = a^0 \in H$ . Also, for $a^n$ , $a^m \in H$ , $a^n(a^m)^{-1} = a^{n-m} \in H$ .  Therefore, $H$ is a subgroup of $G$ .  Also, if $K$ is a subgroup of $G$ containing $a$ , then $a$ , $a^{-1} \in K$ . Now, since $K$ is a group, Also, if $K$ is a subgroup of $G$ containing $a$ .  iv. Prove that $G = \{\begin{pmatrix} a & b \\ -b & a \end{pmatrix} : a, b \in \mathbb{R} \}$ is a group under matrix addition.  Ans Closure: $\begin{pmatrix} a & b \\ -b & a \end{pmatrix} + \begin{pmatrix} c & d \\ -d & c \end{pmatrix} = \begin{pmatrix} a+c & b+d \\ -b-d & a+c \end{pmatrix} \in G$ Prove Associative  Prove Identity is $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ Prove inverse of $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ is $\begin{pmatrix} a & b \\ -b & -a \end{pmatrix}$ .  Q5. Attempt any FOUR questions from the following: (20)  Ans Proof: Let $c_1 T v_1 + c_2 T v_2 + \dots + c_n T v_n = 0$ Since $T$ is 1.t. it follows that $T$ ( $c_1 v_1 + \dots + c_n v_n = 0$ . As ker $T = \{o\}$ , this implies that $c_1 v_1 + \dots + c_n v_n = 0$ which further implies that $c_1 v_2 + \dots + c_n v_n = 0$ which further implies that $c_1 v_2 + \dots + c_n v_n = 0$ which further implies that $c_1 v_2 + \dots + c_n v_n = 0$ which further implies that $c_1 v_2 + \dots + c_n v_n = 0$ which further implies that $c_1 v_2 + \dots + c_n v_n = 0$ which further implies that $c_1 v_2 + \dots + c_n v_n = 0$ which further implies that $c_1 v_2 + \dots + c_n v_n = 0$ which further implies that $c_1 v_2 + \dots + c_n v_n = 0$ which further implies that $c_1 v_2 + \dots + c_n v_n = 0$ which further implies that $c_1 v_2 + \dots + c_n v_n = 0$ which further implies that $c_1 v_2 + \dots + c_n v_n = 0$ which further implies that $c_1 v_2 + \dots + c_n v_n = 0$ which further implies that $c_1 v_2 + \dots + c_n v_n = 0$ which further implies that $c_1 v_2 + \dots + c_n v_n = 0$ which further implies that $c_1 v_2 + \dots + c_n v_n = 0$ which further implies that $c_1 v_2$	Ans	$-\frac{1}{7/2} = \{\overline{1}, \overline{2}, \overline{3}, \overline{4}\}$	
T is the identity. $\therefore 0(1)^{-1} = 1$	Kiis	Composition table	111
Iii. Let G be a group and a of G containing a.  Ans $H \neq \phi$ , since $e = a^0 \in H$ . Also, for $a^n, a^m \in H$ , $a^n(a^m)^{-1} = a^{n-m} \in H$ . Therefore, H is a subgroup of G. Also, if K is a subgroup of G containing a, then $a, a^{-1} \in K$ . Now, since K is a group, $a = (a^{-1})^n \in K$ , $\forall n \in \mathbb{N}$ . Whence $a^n \in K$ , $\forall n \in \mathbb{Z}$ . $\therefore H \subseteq K$ , so that H is the smallest subgroup of G convaining a.  Iv. Prove that $G = \{\begin{pmatrix} a & b \\ -b & a \end{pmatrix} : a, b \in \mathbb{R} \}$ is a group under matrix addition.  Ans Closure: $\begin{pmatrix} a & b \\ -b & a \end{pmatrix} + \begin{pmatrix} c & d \\ -d & c \end{pmatrix} = \begin{pmatrix} a+c & b+d \\ -b-d & a+c \end{pmatrix} \in G$ Prove Associative Prove Identity is $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ Prove inverse of $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ is $\begin{pmatrix} -a & -b \\ b & -a \end{pmatrix}$ .  O5. Attempt any FOUR questions from the following: (20)  Ans Proof: Let $c_1 T v_1 + c_2 T v_2 + \dots + c_n T v_n = 0$ Since T is 1.T it follows that $T (c_1 v_1 + \dots + c_n v_n) = a$ . As ker $T = \{a\}$ , this implies that $c_1 v_1 + \dots + c_n v_n = a$ which further implies that $c_1 v_2 + \dots + c_n v_n = a$ .		$\bar{1}$ is the identity. $0(1) = 1$	1 1
Iii. Let G be a group and a of G containing a.  Ans $H \neq \phi$ , since $e = a^0 \in H$ . Also, for $a^n, a^m \in H$ , $a^n(a^m)^{-1} = a^{n-m} \in H$ . Therefore, H is a subgroup of G. Also, if K is a subgroup of G containing a, then $a, a^{-1} \in K$ . Now, since K is a group, $a = (a^{-1})^n \in K$ , $\forall n \in \mathbb{N}$ . Whence $a^n \in K$ , $\forall n \in \mathbb{Z}$ . $\therefore H \subseteq K$ , so that H is the smallest subgroup of G convaining a.  Iv. Prove that $G = \{\begin{pmatrix} a & b \\ -b & a \end{pmatrix} : a, b \in \mathbb{R} \}$ is a group under matrix addition.  Ans Closure: $\begin{pmatrix} a & b \\ -b & a \end{pmatrix} + \begin{pmatrix} c & d \\ -d & c \end{pmatrix} = \begin{pmatrix} a+c & b+d \\ -b-d & a+c \end{pmatrix} \in G$ Prove Associative Prove Identity is $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ Prove inverse of $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ is $\begin{pmatrix} -a & -b \\ b & -a \end{pmatrix}$ .  O5. Attempt any FOUR questions from the following: (20)  Ans Proof: Let $c_1 T v_1 + c_2 T v_2 + \dots + c_n T v_n = 0$ Since T is 1.T it follows that $T (c_1 v_1 + \dots + c_n v_n) = a$ . As ker $T = \{a\}$ , this implies that $c_1 v_1 + \dots + c_n v_n = a$ which further implies that $c_1 v_2 + \dots + c_n v_n = a$ .	1	$\frac{1}{2^2} = \overline{4}$ , $\overline{2}^3 = \overline{3}$ , $\frac{2^4}{7} = \frac{1}{7}$ $\frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} = \frac{1}{7}$	i
Iii. Let G be a group and a of G containing a.  Ans $H \neq \phi$ , since $e = a^0 \in H$ . Also, for $a^n, a^m \in H$ , $a^n(a^m)^{-1} = a^{n-m} \in H$ . Therefore, H is a subgroup of G. Also, if K is a subgroup of G containing a, then $a, a^{-1} \in K$ . Now, since K is a group, $a = (a^{-1})^n \in K$ , $\forall n \in \mathbb{N}$ . Whence $a^n \in K$ , $\forall n \in \mathbb{Z}$ . $\therefore H \subseteq K$ , so that H is the smallest subgroup of G convaining a.  Iv. Prove that $G = \{\begin{pmatrix} a & b \\ -b & a \end{pmatrix} : a, b \in \mathbb{R} \}$ is a group under matrix addition.  Ans Closure: $\begin{pmatrix} a & b \\ -b & a \end{pmatrix} + \begin{pmatrix} c & d \\ -d & c \end{pmatrix} = \begin{pmatrix} a+c & b+d \\ -b-d & a+c \end{pmatrix} \in G$ Prove Associative Prove Identity is $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ Prove inverse of $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ is $\begin{pmatrix} -a & -b \\ b & -a \end{pmatrix}$ .  O5. Attempt any FOUR questions from the following: (20)  Ans Proof: Let $c_1 T v_1 + c_2 T v_2 + \dots + c_n T v_n = 0$ Since T is 1.T it follows that $T (c_1 v_1 + \dots + c_n v_n) = a$ . As ker $T = \{a\}$ , this implies that $c_1 v_1 + \dots + c_n v_n = a$ which further implies that $c_1 v_2 + \dots + c_n v_n = a$ .		$\frac{2}{3^2} = \frac{7}{4}$ , $\frac{3}{3^3} = \frac{1}{2}$ , $3^4 = 1$ $3^4 = 1$	L
Iii. Let G be a group and a of G containing a.  Ans $H \neq \phi$ , since $e = a^0 \in H$ . Also, for $a^n, a^m \in H$ , $a^n(a^m)^{-1} = a^{n-m} \in H$ .  Therefore, H is a subgroup of G. Also, if K is a subgroup of G containing a, then $a, a^{-1} \in K$ . Now, since K is a group, $a = (a^{-1})^n \in K$ , $\forall n \in \mathbb{N}$ . Whence $a^n \in K$ , $\forall n \in \mathbb{Z}$ . $\therefore H \subseteq K$ , so that H is the smallest subgroup of G containing a.  Iv. Prove that $G = \{\begin{pmatrix} a & b \\ -b & a \end{pmatrix} : a, b \in \mathbb{R} \}$ is a group under matrix addition.  Ans Closure: $\begin{pmatrix} a & b \\ -b & a \end{pmatrix} + \begin{pmatrix} c & d \\ -d & c \end{pmatrix} = \begin{pmatrix} a+c & b+d \\ -b-d & a+c \end{pmatrix} \in G$ Prove Associative Prove Identity is $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ Prove inverse of $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ is $\begin{pmatrix} -a & -b \\ b & -a \end{pmatrix}$ .  Q5. Attempt any FOUR questions from the following: (20)  Ans Proof: Let $c_1Tv_1 + c_2Tv_2 + \dots + c_nTv_n = 0$ Since T is i.t. it follows that $T(c_1v_1 + \dots + c_nv_n) = a$ . As ker $T = \{a\}$ , this implies that $c_1v_1 + \dots + c_nv_n = a$ which further implies that $c_1v_1 + \dots + c_nv_n = a$ .		$\overline{A}^2 = \overline{1}$ : $o(\overline{A}) = 2$ $\overline{C}$ and $\overline{C}$ is the smallest subject $\overline{A}$ is the smallest subject $\overline{A}$ .	Stoab
Ans $H \neq \phi$ , since $e = a^0 \in H$ . Also, for $a^n, a^m \in H$ , $a^n(a^m)^{-1} = a^{n-m} \in H$ .  Ans $H \neq \phi$ , since $e = a^0 \in H$ . Also, for $a^n, a^m \in H$ , $a^n(a^m)^{-1} = a^{n-m} \in H$ .  Therefore, $H$ is a subgroup of $G$ .  Also, if $K$ is a subgroup of $G$ containing $a$ , then $a, a^{-1} \in K$ . Now, since $K$ is a group, Also, if $K$ is a subgroup of $G$ containing $a$ .  iv. Prove that $G = \{\begin{pmatrix} a & b \\ -b & a \end{pmatrix} : a, b \in \mathbb{R} \}$ is a group under matrix addition.  Ans Closure: $\begin{pmatrix} a & b \\ -b & a \end{pmatrix} + \begin{pmatrix} c & d \\ -d & c \end{pmatrix} = \begin{pmatrix} a+c & b+d \\ -b-d & a+c \end{pmatrix} \in G$ Prove Associative Prove Identity is $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ Prove inverse of $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ is $\begin{pmatrix} -a & -b \\ b & -a \end{pmatrix}$ .  Q5. Attempt any FOUR questions from the following:  (20)  Ans Proof: Let $c_1Tv_1 + c_2Tv_2 + \dots + c_nTv_n = 0$ Since $T$ is i.t. it follows that $T$ ( $c_1v_1 + \dots + c_nv_n = 0$ which further implies that $c_1, c_2, \dots, c_n$ implies that $c_1v_1 + \dots + c_nv_n = 0$ which further implies that $c_1, c_2, \dots, c_n$	_ <del></del>	I of C he a group and w	
Also, if $K$ is a subgroup of $G$ containing $a$ , then $a, a \in K$ . From $A$ looks if $K$ is a subgroup of $G$ containing $a$ .  iv. Prove that $G = \{\begin{pmatrix} a & b \\ -b & a \end{pmatrix} : a, b \in \mathbb{R} \}$ is a group under matrix addition.  Ans Closure: $\begin{pmatrix} a & b \\ -b & a \end{pmatrix} + \begin{pmatrix} c & d \\ -d & c \end{pmatrix} = \begin{pmatrix} a+c & b+d \\ -b-d & a+c \end{pmatrix} \in G$ Prove Associative Prove Identity is $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ Prove inverse of $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ is $\begin{pmatrix} -a & -b \\ b & -a \end{pmatrix}$ Q5. Attempt any FOUR questions from the following: (20)  Ans Proof: Let $C_1 \cap C_1 \cap C_2 \cap C_2 \cap C_3 \cap C_4 \cap C_4 \cap C_4 \cap C_5 \cap C_$	1111.	of G containing a. $\frac{1}{n} = \frac{n^{-m}}{n} \in H.$	1
Also, if $K$ is a subgroup of $G$ containing $a$ , then $a, a \in K$ . From $A$ looks if $K$ is a subgroup of $G$ containing $a$ .  iv. Prove that $G = \{\begin{pmatrix} a & b \\ -b & a \end{pmatrix} : a, b \in \mathbb{R} \}$ is a group under matrix addition.  Ans Closure: $\begin{pmatrix} a & b \\ -b & a \end{pmatrix} + \begin{pmatrix} c & d \\ -d & c \end{pmatrix} = \begin{pmatrix} a+c & b+d \\ -b-d & a+c \end{pmatrix} \in G$ Prove Associative Prove Identity is $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ Prove inverse of $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ is $\begin{pmatrix} -a & -b \\ b & -a \end{pmatrix}$ Q5. Attempt any FOUR questions from the following: (20)  Ans Proof: Let $C_1 \cap C_1 \cap C_2 \cap C_2 \cap C_3 \cap C_4 \cap C_4 \cap C_4 \cap C_5 \cap C_$		of $a$ of $a$ of $a$ $a$ $b$ $a$ $a$ $a$ $a$ $b$ $a$	3
Also, if $K$ is a subgroup of $G$ containing $a$ , then $a, a \in K$ . Find $G$ is $A$ and $A$ is an allest subgroup of $A$ containing $A$ .  In the smallest subgroup of $A$ containing $A$ containing $A$ .  In the smallest subgroup of $A$ containing $A$ containing $A$ .  In the smallest subgroup of $A$ containing $A$ containing $A$ containing $A$ .  In the smallest subgroup of $A$ containing $A$ containing $A$ containing $A$ .  In the smallest subgroup of $A$ containing $A$	Ans	$H \neq \phi$ , since $e = u$ C. If the subgroup of $G$ .	ıp,
the smallest subgroup of G convaining a.  iv. Prove that $G = \left\{ \begin{pmatrix} a & b \\ -b & a \end{pmatrix} : a, b \in \mathbb{R} \right\}$ is a group under matrix addition.  Ans Closure: $\begin{pmatrix} a & b \\ -b & a \end{pmatrix} + \begin{pmatrix} c & d \\ -d & c \end{pmatrix} = \begin{pmatrix} a+c & b+d \\ -b-d & a+c \end{pmatrix} \in G$ Prove Associative Prove Identity is $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ Prove inverse of $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ is $\begin{pmatrix} -a & -b \\ b & -a \end{pmatrix}$ Q5. Attempt any FOUR questions from the following:  (20)  Alter $T: V \to W$ be a linear transformation such that $\ker T = \{0\}$ . Prove that if $v_1, v_2, \dots, v_n$ linearly independent then $T(v_1), T(v_2), \dots, T(v_n)$ are linearly independent elements of Since $T$ is 1.t. it follows that $T(c_1v_1 + \dots + c_nv_n) = 0$ . As $\ker T = \{o\}$ , this implies that $c_1v_1 + \dots + c_nv_n = 0$ which further implies that $c_1, c_2, \dots, c_n$		Therefore, H is a subgroup of Geometrian $a$ , then $a$ , $a^{-1} \in K$ . Now, shows that H is	1
the smallest studgloup of $a$ iv. Prove that $G = \left\{ \begin{pmatrix} a & b \\ -b & a \end{pmatrix} : a, b \in \mathbb{R} \right\}$ is a group under matrix addition.  Ans Closure: $\begin{pmatrix} a & b \\ -b & a \end{pmatrix} + \begin{pmatrix} c & d \\ -d & c \end{pmatrix} = \begin{pmatrix} a+c & b+d \\ -b-d & a+c \end{pmatrix} \in G$ Prove Associative Prove Identity is $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ Prove inverse of $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ is $\begin{pmatrix} -a & -b \\ b & -a \end{pmatrix}$ Q5. Attempt any <b>FOUR</b> questions from the following: (20)  Attempt any the a linear transformation such that $\ker T = \{0\}$ . Prove that if $v_1, v_2, \ldots, v_n$ linearly independent then $T(v_1), T(v_2), \ldots, T(v_n)$ are linearly independent elements of linearly independent that $T(v_1), T(v_2), \ldots, T(v_n)$ are linearly independent elements of Since $T$ is 1.t. it follows that $T(c_1v_1 + \ldots, c_nv_n) = 0$ . As $\ker T = \{o\}$ , this implies that $c_1v_1 + \ldots, c_nv_n = 0$ which further implies that $c_1, c_2, \ldots, c_n$		Also, if K is a subgroup of $X$ in $X$ is a subgroup of $X$ in $X$ is a subgroup of $X$ in $X$ in $X$ is a subgroup of $X$ in	
the smallest studgloup of $a$ iv. Prove that $G = \left\{ \begin{pmatrix} a & b \\ -b & a \end{pmatrix} : a, b \in \mathbb{R} \right\}$ is a group under matrix addition.  Ans Closure: $\begin{pmatrix} a & b \\ -b & a \end{pmatrix} + \begin{pmatrix} c & d \\ -d & c \end{pmatrix} = \begin{pmatrix} a+c & b+d \\ -b-d & a+c \end{pmatrix} \in G$ Prove Associative Prove Identity is $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ Prove inverse of $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ is $\begin{pmatrix} -a & -b \\ b & -a \end{pmatrix}$ Q5. Attempt any <b>FOUR</b> questions from the following: (20)  Attempt any the a linear transformation such that $\ker T = \{0\}$ . Prove that if $v_1, v_2, \ldots, v_n$ linearly independent then $T(v_1), T(v_2), \ldots, T(v_n)$ are linearly independent elements of linearly independent that $T(v_1), T(v_2), \ldots, T(v_n)$ are linearly independent elements of Since $T$ is 1.t. it follows that $T(c_1v_1 + \ldots, c_nv_n) = 0$ . As $\ker T = \{o\}$ , this implies that $c_1v_1 + \ldots, c_nv_n = 0$ which further implies that $c_1, c_2, \ldots, c_n$		$a^n, a^{-n} = (a^n)$ and $a^n$ and $a^n$ are $a^n$	3
Ans Closure: $\binom{a}{-b} \binom{b}{a} + \binom{c}{-d} \binom{d}{c} = \binom{a+c}{-b-d} \binom{b+d}{a+c} \in G$ Prove Associative  Prove Identity is $\binom{0}{0} \binom{0}{0}$ Prove inverse of $\binom{a}{-b} \binom{a}{a}$ is $\binom{-a}{b-a}$ Q5. Attempt any FOUR questions from the following: (20)			
Ans Closure: $\binom{a}{-b} \binom{b}{a} + \binom{c}{-d} \binom{d}{c} = \binom{a+c}{-b-d} \binom{b+d}{a+c} \in G$ Prove Associative  Prove Identity is $\binom{0}{0} \binom{0}{0}$ Prove inverse of $\binom{a}{-b} \binom{a}{a}$ is $\binom{-a}{b-a}$ Q5. Attempt any FOUR questions from the following: (20)		$((a \ b), a \ b \in \mathbb{R})$ is a group under matrix addition.	
Ans Closure: $\binom{a}{-b} \binom{b}{a} + \binom{c}{-d} \binom{d}{c} = \binom{a+c}{-b-d} \binom{b+d}{a+c} \in G$ Prove Associative  Prove Identity is $\binom{0}{0} \binom{0}{0}$ Prove inverse of $\binom{a}{-b} \binom{a}{a}$ is $\binom{-a}{b-a}$ Q5. Attempt any FOUR questions from the following: (20)	iv.	Prove that $G = \{(-b, a), a, b \in A\}$	
Prove Associative $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ Prove Identity is $\begin{pmatrix} 0 & 0 \\ -b & a \end{pmatrix}$ is $\begin{pmatrix} -a & -b \\ b & -a \end{pmatrix}$ .  Prove inverse of $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ is $\begin{pmatrix} -a & -b \\ b & -a \end{pmatrix}$ .  Attempt any <b>FOUR</b> questions from the following:  (20)  Let $T: V \to W$ be a linear transformation such that $\ker T = \{0\}$ . Prove that if $v_1, v_2, \dots, v_n$ linearly independent then $T(v_1), T(v_2), \dots, T(v_n)$ are linearly independent elements of linearly independent elements of Since T is i.t. it follows that $T(c_1v_1 + \dots + c_nv_n) = 0$ . As $\ker T = \{0\}$ , this implies that $c_1v_1 + \dots + c_nv_n = 0$ which further implies that $c_1v_2 + \dots + c_nv_n = 0$ which further implies that $c_1v_2 + \dots + c_nv_n = 0$ which further implies that $c_1v_2 + \dots + c_nv_n = 0$ which further implies that $c_1v_2 + \dots + c_nv_n = 0$ are i.i.	Ì		1
Prove Associative $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ Prove Identity is $\begin{pmatrix} 0 & 0 \\ -b & a \end{pmatrix}$ is $\begin{pmatrix} -a & -b \\ b & -a \end{pmatrix}$ .  Prove inverse of $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ is $\begin{pmatrix} -a & -b \\ b & -a \end{pmatrix}$ .  Attempt any <b>FOUR</b> questions from the following:  (20)  Let $T: V \to W$ be a linear transformation such that $\ker T = \{0\}$ . Prove that if $v_1, v_2, \dots, v_n$ linearly independent then $T(v_1), T(v_2), \dots, T(v_n)$ are linearly independent elements of linearly independent elements of Since T is i.t. it follows that $T(c_1v_1 + \dots + c_nv_n) = 0$ . As $\ker T = \{0\}$ , this implies that $c_1v_1 + \dots + c_nv_n = 0$ which further implies that $c_1v_2 + \dots + c_nv_n = 0$ which further implies that $c_1v_2 + \dots + c_nv_n = 0$ which further implies that $c_1v_2 + \dots + c_nv_n = 0$ which further implies that $c_1v_2 + \dots + c_nv_n = 0$ are i.i.	}	$b (c d) (a+c b+d) \in G$	l
Prove Associative $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ Prove Identity is $\begin{pmatrix} 0 & 0 \\ -b & a \end{pmatrix}$ is $\begin{pmatrix} -a & -b \\ b & -a \end{pmatrix}$ .  Prove inverse of $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ is $\begin{pmatrix} -a & -b \\ b & -a \end{pmatrix}$ .  Attempt any <b>FOUR</b> questions from the following:  (20)  Let $T: V \to W$ be a linear transformation such that $\ker T = \{0\}$ . Prove that if $v_1, v_2, \dots, v_n$ linearly independent then $T(v_1), T(v_2), \dots, T(v_n)$ are linearly independent elements of linearly independent elements of Since T is i.t. it follows that $T(c_1v_1 + \dots + c_nv_n) = 0$ . As $\ker T = \{0\}$ , this implies that $c_1v_1 + \dots + c_nv_n = 0$ which further implies that $c_1v_2 + \dots + c_nv_n = 0$ which further implies that $c_1v_2 + \dots + c_nv_n = 0$ which further implies that $c_1v_2 + \dots + c_nv_n = 0$ which further implies that $c_1v_2 + \dots + c_nv_n = 0$ are i.i.	- —   Ā	This Closure: $\begin{pmatrix} a & b \\ b & a \end{pmatrix} + \begin{pmatrix} c & c \\ -d & c \end{pmatrix} = \begin{pmatrix} -b - d & a + c \end{pmatrix}$	
Prove inverse of $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ is $\begin{pmatrix} -a & -b \\ b & -a \end{pmatrix}$ Q5. Attempt any <b>FOUR</b> questions from the following:  (20)  Attempt any <b>FOUR</b> questions from the following:  (20)  Let $T: V \to W$ be a linear transformation such that $\ker T = \{0\}$ . Prove that if $v_1, v_2, \dots, v_n$ linearly independent then $T(v_1), T(v_2), \dots, T(v_n)$ are linearly independent elements of linearly independent then $T(v_1), T(v_2), \dots, T(v_n)$ are linearly independent elements of Since $T$ is i.t. it follows that $T(c_1v_1 + \dots + c_nv_n) = o$ . As $\ker T = \{o\}$ , this implies that $c_1v_1 + \dots + c_nv_n = o$ which further implies that $c_1, c_2, \dots, c_n$	1		\ '
Prove inverse of $(\underline{b}  \underline{a})^{tot}  \underline{b}  \underline{-a}^{tot}$ O5. Attempt any <b>FOUR</b> questions from the following:  (20)  (20)  (20)  (21)  (21)  (22)  (22)  (23)  (24)  (25)  (26)  (27)  (27)  (27)  (28)  (28)  (29)  (20)  (20)  (20)  (20)  (20)  (20)  (20)  (20)  (20)  (21)  (21)  (22)  (22)  (23)  (24)  (25)  (26)  (27)  (27)  (28)  (28)  (28)  (29)  (20)		Prove Associative (0 0)	
Prove inverse of $(\underline{b} - \underline{a})^{n}(\underline{b} - \underline{a})^{n}$ (20)  O5. Attempt any FOUR questions from the following:  a) Let $T: V \to W$ be a linear transformation such that $\ker T = \{0\}$ . Prove that if $v_1, v_2, \dots, v_n$ linearly independent then $T(v_1), T(v_2), \dots, T(v_n)$ are linearly independent elements of linearly independent then $T(v_1), T(v_2), \dots, T(v_n)$ are linearly independent elements of linearly independent that $C_1 V_1 + C_2 T V_2 + \dots + C_n T V_n = 0$ Ans Proof: Let $C_1 T V_1 + C_2 T V_2 + \dots + C_n V_n = 0$ which further implies that $C_1 V_1 + \dots + C_n V_n = 0$ where $C_1 V_1 + \dots + C_n V_n = 0$ where		Prove Identity is (0 0)	\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \
<ul> <li>Q5. Attempt any FOUR questions from the following: (20)</li> <li>a) Let T: V → W be a linear transformation such that kerT = {0}. Prove that if v<sub>1</sub>, v<sub>2</sub>v<sub>n</sub> linearly independent then T(v<sub>1</sub>), T(v<sub>2</sub>)T(v<sub>n</sub>) are linearly independent elements of linearly independent</li></ul>	1	$\frac{1}{a}$ inverse of $\begin{pmatrix} a & b \\ b & -a \end{pmatrix}$	
<ul> <li>Q5. Attempt any FOUR questions from the following:</li> <li>a) Let T: V → W be a linear transformation such that kerT = {0}. Prove that if v<sub>1</sub>, v<sub>2</sub>v<sub>n</sub> linearly independent then T(v<sub>1</sub>), T(v<sub>2</sub>)T(v<sub>n</sub>) are linearly independent elements of linearly independent elem</li></ul>		Prove liverse of $(-b - a)$	
<ul> <li>a) Let T: V → W be a linear transformation such that kerT = {0}. Prove that if v<sub>1</sub>, v<sub>2</sub>v<sub>n</sub> linearly independent then T(v<sub>1</sub>), T(v<sub>2</sub>)T(v<sub>n</sub>) are linearly independent elements of linearly independent elements elements of linearly indep</li></ul>		(20)	
<ul> <li>a) Let T: V → W be a linear transformation such that kerT = {0}. Prove that if v<sub>1</sub>, v<sub>2</sub>v<sub>n</sub> linearly independent then T(v<sub>1</sub>), T(v<sub>2</sub>)T(v<sub>n</sub>) are linearly independent elements of linearly independent elements elements of linearly indep</li></ul>	05.	Attempt any FOUR questions from the restaurant and	
Inearly independent then $T(v_1)$ , $T(v_2)$	1	$T_{-}(0)$ Prove that if $V_1, V_2, \dots$	$v_n$ are
Inearly independent then $T(v_1)$ , $T(v_2)$		$T \cdot V \setminus W$ be a linear transformation such that $\ker I = \{0\}$ . The variable element	ts of W
Ans Proof: Let $c_1Tv_1 + c_2Tv_2 + \dots + c_nv_n = 0$ . As ker $T = \{o\}$ , this Since T is i.t. it follows that $T(c_1v_1 + \dots + c_nv_n) = 0$ . As ker $T = \{o\}$ , this implies that $c_1v_1 + \dots + c_nv_n = 0$ which further implies that $c_1v_1 + \dots + c_nv_n = 0$ which further implies that $c_1v_1 + \dots + c_nv_n = 0$ which further implies that $c_1v_1 + \dots + c_nv_n = 0$ implies that $c_1v_1 + \dots + c_nv_n = 0$ which further implies that $c_1v_1 + \dots + c_nv_n = 0$ which further implies that $c_1v_1 + \dots + c_nv_n = 0$ which further implies that $c_1v_1 + \dots + c_nv_n = 0$ implies that $c_1v_1 + \dots + c_nv_n = 0$ which further implies that $c_1v_1 + \dots + c_nv_n = 0$ implies that $c_1v_1 + \dots + c_nv_n = 0$ which further implies that $c_1v_1 + \dots + c_nv_n = 0$ implies that $c_1v_1 + \dots + c_nv_n = 0$ which further implies that $c_1v_1 + \dots + c_nv_n = 0$ implies that $c_1v_1 + \dots + c_nv_n = 0$ implies that $c_1v_1 + \dots + c_nv_n = 0$ implies that $c_1v_1 + \dots + c_nv_n = 0$ implies that $c_1v_1 + \dots + c_nv_n = 0$ implies that $c_1v_1 + \dots + c_nv_n = 0$ implies that $c_1v_1 + \dots + c_nv_n = 0$ in the first $c_1v_1 + \dots + c_nv_n = 0$ in the first $c_1v_1 + \dots + c_nv_n = 0$ implies that $c_1v_1 + \dots + c_nv_n = 0$ in the first $c_1v_1 + \dots + c_nv_n = 0$ int	a)	Let $T: V \to V$ be a fine $T(v_n)$ are linearly independent element.	
Ans Proof: Let $c_1Tv_1 + c_2Tv_2 + \dots + c_nv_n = 0$ . As ker $T = \{o\}$ , this Since T is i.t. it follows that $T(c_1v_1 + \dots + c_nv_n) = 0$ . As ker $T = \{o\}$ , this implies that $c_1v_1 + \dots + c_nv_n = 0$ which further implies that $c_1v_1 + \dots + c_nv_n = 0$ which further implies that $c_1v_1 + \dots + c_nv_n = 0$ which further implies that $c_1v_1 + \dots + c_nv_n = 0$ implies that $c_1v_1 + \dots + c_nv_n = 0$ which further implies that $c_1v_1 + \dots + c_nv_n = 0$ which further implies that $c_1v_1 + \dots + c_nv_n = 0$ which further implies that $c_1v_1 + \dots + c_nv_n = 0$ implies that $c_1v_1 + \dots + c_nv_n = 0$ which further implies that $c_1v_1 + \dots + c_nv_n = 0$ implies that $c_1v_1 + \dots + c_nv_n = 0$ which further implies that $c_1v_1 + \dots + c_nv_n = 0$ implies that $c_1v_1 + \dots + c_nv_n = 0$ which further implies that $c_1v_1 + \dots + c_nv_n = 0$ implies that $c_1v_1 + \dots + c_nv_n = 0$ implies that $c_1v_1 + \dots + c_nv_n = 0$ implies that $c_1v_1 + \dots + c_nv_n = 0$ implies that $c_1v_1 + \dots + c_nv_n = 0$ implies that $c_1v_1 + \dots + c_nv_n = 0$ implies that $c_1v_1 + \dots + c_nv_n = 0$ in the first $c_1v_1 + \dots + c_nv_n = 0$ in the first $c_1v_1 + \dots + c_nv_n = 0$ implies that $c_1v_1 + \dots + c_nv_n = 0$ in the first $c_1v_1 + \dots + c_nv_n = 0$ int	1	linearly independent then $I(v_1), I(v_2), \dots$	
Since T is 1.t. it follows that $C_1v_1 + \dots + C_nv_n = 0$ which further implies that $c_1v_1 + \dots + c_nv_n = 0$ which further implies that $c_1v_1 + \dots + c_nv_n = 0$ which further implies that $c_1v_1 + \dots + c_nv_n = 0$ which further implies that $c_1v_1 + \dots + c_nv_n = 0$ implies that $c_1v_1 + \dots + c_nv_n = 0$ which further implies that $c_1v_1 + \dots + c_nv_n = 0$ implies that $c_1v_1 + \dots + c_nv_n = 0$ implies that $c_1v_1 + \dots + c_nv_n = 0$ implies that $c_1v_1 + \dots + c_nv_n = 0$ which further implies that $c_1v_1 + \dots + c_nv_n = 0$ in $c_1v_1 + \dots +$	<b>├</b>	Proof: Let $c_1 T v_1 + c_2 T v_2 + \dots + c_n T v_n = 0$	
implies that $c_1v_1 + \dots + c_nv_n = 0$ .  Then are l.i.	Ans	The follows that $T(c_1v_1 + \ldots + c_nv_n) = 0$ . As ker $T = \{v\}$ , thus	
implies that $c_1v_1 + \dots + c_nv_n = 0$ .  Then are l.i.	\	Since T is i.t. it to now a man $c_1$ and $c_2$ which further implies that $c_1, c_2, \ldots, c_n$	
10 at 12 170 170 170 1 170 1 1 1 1 1 1 1 1 1 1 1	1	1 · · · · · · · · · · · · · · · · · · ·	
are all zero, proving that 101, 102,	1	are all zero, proving that $Tv_1, Tv_2, \dots, Tv_n$ are i.	
$T \cdot \mathbb{P}^2 \rightarrow \mathbb{R}^2$ such that $I(3,1) \cdot (2,1)$	<del></del>	$T: \mathbb{R}^2 \to \mathbb{R}^2$ , such that $T(3, 1) = (2, -4)$ and	
b) Find the map $T: \mathbb{R}^n \to \mathbb{R}^n$ , $T(1, 1) = (0, 2)$	( b)	Find the map 1 · 22	

/		1
Ans	(x,y) = a(3,1) + b(1,1)	1
•	$\Rightarrow 3a+b=x, a+b=y \Rightarrow a=\frac{x-y}{2}, b=\frac{3y-x}{2}$	2
!	$(x,y) = \left(\frac{x-y}{2}\right)(3,1) + \left(\frac{3y-x}{2}\right)(1,1)$	1
	$T(x,y) = \left(\frac{x-y}{2}\right)T(3,1) + \left(\frac{3y-x}{2}\right)T(1,1)$	1
	$T(x,y) = \left(\frac{x-y}{2}\right)(2,3) + \left(\frac{3y-x}{2}\right)(1,4)$	
	=(x-y,5y-3x)	
(c)	Check whether the set $\{(2,0,1), (4,1,-1), (-1,0,2)\}$ is linearly dependent or independent using	5
Ans	determinants.  Columns of $A$ are linearly dependent iff $\det A = 0$	2
7,113	$\det\begin{pmatrix} 2 & 4 & -1 \\ 0 & 1 & 0 \\ 1 & -1 & 2 \end{pmatrix} = 23 \neq 0$	
	$ \begin{vmatrix} \det \begin{pmatrix} 0 & 1 & 0 \\ 1 & -1 & 2 \end{vmatrix} = 23 \neq 0 $	2
	$\{(2,0,1), (4,1,-1), (-1,0,2)\}$ is linearly independent	<u> 1</u>
d)	Find Rank of the matrix $A = \begin{pmatrix} 1 & 3 \\ 0 & -1 \\ 3 & 4 \end{pmatrix}$ . What can you say about the rank of the matrix $B$ which	h is
	obtained from A by multiplying $2^{nd}$ row of A by 3?  Number of linearly independent columns of $A = 2$	2
Ans	.Rank A=7	
	We know that rank of the matrix does not change by applying any of the row or column	2
	operations on it. $\therefore$ Rank of the matrix B which is obtained from A by multiplying $2^{nd}$ row of A by 3 is also equals 2.	1
e)	Define the order of an element a of a group G and show that order $(a^{-1}) = \operatorname{order}(a)$	
Ans	The order of an element a of a group is the smallest positive integer m such that $a^m =$	Τ-
\ \	e, where edenotes the identity element of the group.	1
	If no such mexists, $a$ is said to have infinite order. If, $a^n = e$ , then repeated multiplication by $a^{-1} - n$ times gives	'
	$(a^{-1})^n = e$ . This shows that if a is of finite order than so is $a^{-1}$ with order $(a^{-1}) \le e$ .	
	order(a). //ly, $:(a^{-1})^{-1} = a$ , therefore we see that if $a^{-1}$ is of finite order than so is $a = (a^{-1})^{-1}$ with	
	$\operatorname{order}(a) \leq \operatorname{order}(a^{-1})$ Whence, $\operatorname{order}(a^{-1}) = \operatorname{order}(a)$ , in this case.	1
	Also, if a is of infinite order then so is $a^{-1}$ (for if $a^{-1}$ is of finite order then again	2
	we can as above that $a$ is also of finite order.)  //ly, $\because (a^{-1})^{-1} = a$ , therefore if $a^{-1}$ is of infinite order then so is $a$ .	
	Thus botha and $a^{-1}$ are of infinite order or both are of finite order with	
	$order(a^{-1}) = order(a).$	_
_	This proves the result.	2
f	Show that $H = \{\overline{1}, \overline{7}, \overline{9}, \overline{23}\}$ modulo 40 is a subgroup of $U(40)$ .	

1	<i>つ</i> `
(	5

) (	)		1	
<u>/</u>	/	Since $(\overline{1}, \overline{40}) = 1, (\overline{7}, \overline{40}) = 1, (\overline{9}, \overline{40}) = 1$ and $(\overline{23}, \overline{40}) = 1$ ,	1	
		H is a subset of $U(40)$ .	2	
		Composition table	i	
1		Closure property, Associative law and Holding		\
		It itself is a group under multiplication mount		
1		Hence, His a subgroup of $U(40)$ .		
	Ĺ	110tow, Swann		