M.Sc. (Mathematics) Part – I
Algebra – I (Rev)
Paper - I
April: - 2016

QP Code : 33145

Revised] (3 Hours) **[Total Marks:80**

Instructions:

- Attempt **any two** questions from **each section**
- **All questions** carry **equal marks**
- **Answers** to **Section I** and **Section II** should be written in the **same answer book**.

SECTION I (Attempt **any two** questions)

1. (a) Let $T : V \to W$ be a linear transformation. Show that T is an isomorphism if and only if dim V =dim W.

   (b) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined by :
   (i) $T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$, (ii) $T(x, y, z) = (x + y + z, 2x + y - z, x + z)$.
   Find a basis and the dimension of Im T and Ker T.

2. (a) Let $k_1, k_2, \ldots, k_n$ and $j_1, j_2, \ldots, j_n$ be positive integers. For $A \in M_n(k)$ define $D(A) = a_{j_1}k_1 a_{j_2}k_2 \ldots a_{j_n}k_n$. Show that D is n-linear if and only if $j_1, j_2, \ldots, j_n$ are distinct.

   (b) (i) Find the rank of the matrix $\begin{pmatrix} 1 & 1 & -1 & 2 \\ 2 & -2 & 0 & 2 \\ 2 & -8 & 3 & -1 \end{pmatrix}$

   (ii) Using Cramer's rule solve the equation:

$$4x - y + 3z = 2$$
$$x + 5y - 2z = 3$$
$$3x + 2y + 4z = 6$$

3. (a) Let T be a linear operator on a finite dimensional vector space V. If f is the characteristic polynomial for T, than f(T)=0.

   (b) Show that similar matrices have the same minimal polynomial.

4. (a) Let V be an inner product space, then for any vectors $u, v \in V$ and scalar α prove the following:
   (i) $\|\alpha u\| = |\alpha|\|u\|$, (ii) $|\langle u, v\rangle| \leq \|u\|\|v\|$

   (b) Determine canonical form of the real non-degenerate symmetric bilinear form $\begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 2 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 1 & 4 & 1 & 5 \end{pmatrix}$

SECTION II (Attempt **any two** questions)

5. (a) (i) A cyclic group is abelian.

   (ii) List all abelian groups (upto isomorphism) of order 180.

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(b) Prove that every group is isomorphic to a group of permutations.

6. (a) Let G be a group and A be a non-empty subset of G. Define centralizer $C_G(A)$ and normalizer $N_G(A)$. Prove that both are subgroups of G. If G is an abelian group what can we say about its centralizer? Justify your answer.

   (b) Let G be a finite group and p be a prime that divides order of G. Prove that G has an element of order p.

7. (a) (i) Let R, S be commutative rings with unity. Let $\varphi : R \to S$ be a ring homomorphism. Prove that Ker φ is an ideal of R.

   (ii) Let $R = \left\{ \begin{pmatrix} a & b \\ b & a \end{pmatrix} \mid a, b \in \mathbb{Z} \right\}$. Define $\varphi : R \to \mathbb{Z}$ by $\varphi\left(\begin{pmatrix} a & b \\ b & a \end{pmatrix}\right) = a - b$. Verify that φ is a ring homomorphism and determine Ker φ.

   (b) Let I, J be ideals of a commutative ring R with unity. Suppose $J \subseteq I$. Show that I/J is an ideal of R/J and $(R/J)/(I/J)$ is isomorphic to R/I.

8. (a) Prove that every principal ideal domain (PID) is unique factorization domain (UFD).

   (b) Explain whether (i) $\mathbb{Z}$ is a PID, (ii) $\mathbb{Z}[\sqrt{-5}]$ is a UFD. Justify your answer.

M.Sc. (Mathematics) Part – I
Analysis & Topology (Rev)
Paper - II
April: - 2016

QP Code : 33148

Revised] (3 Hours) [Total Marks:80

Instructions:
- Attempt **any two** questions from **each section**
- **All questions** carry **equal marks**
- **Answers** to **Section I** and **Section II** should be written in the **same answer book**.

SECTION I (Attempt **any two** questions)

1. (a) Prove that for $x = (x_1, x_2); y = (y_1, y_2) \in \mathbb{R}^2$, $d_1(x,y) = \max\{|x_1 - y_1|, |x_2 - y_2|\}$ is a metric on $\mathbb{R}^2$. Let $d_2(x,y)$ be the metric on $\mathbb{R}^2$ given by $d_2(x,y) = \sqrt{(x_1-y_1)^2 + (x_2-y_2)^2}$. Prove that d_1 and d_2 are equivalent.

   (b) Define n cell on $\mathbb{R}^n$. Prove that an n-cell I of $\mathbb{R}^n$ is compact.

2. (a) Define connected set in a metric space (X, d). Prove that a metric space (X, d) is connected if and only if every continuous characteristic function is a constant function.

   (b) Define uniform continuity of a function f on a metric space (X, d). Give an example of a function f on a suitable metric space (X, d) which is continuous but not uniformly continuous.

3. (a) Define differentiability of a function $f : E \subset \mathbb{R}^n \to \mathbb{R}^m$ at $a \in E$, where E is an open subset of $\mathbb{R}^n$. Prove that if f is differentiable, then the total derivative of f is unique.

   (b) Give with correct justification, an example of a function such that all directional derivatives at a point exist, but f is not differentiable.

4. (a) State (without proof) inverse function theorem. Show that the function $f : \mathbb{R}^2 \to \mathbb{R}^2$ given by $f(x,y) = (2xy, x^2 - y^2)$ is not invertible on $\mathbb{R}^2$, but locally invertible at every point of $E = \{(x,y) \in \mathbb{R}^2 \mid x > 0\}$. Also find the inverse function at one such point.

   (b) Determine with correct justification if $f(x,y) = x^3 + y^3 - 2xy$ can be expressed by an explicit function $y = g(x)$ in a neighbourhood of the point $(1, 1)$.

SECTION II (Attempt **any two** questions)

5. (a) Define topological space (X, τ). Define base of a topological space. Prove that a family $\mathcal{B}$ of subsets of X is a base to the topological space X if

   (a) $X = \cup_{\{B \in \mathcal{B}\}} B$

   (b) If $B_1, B_2 \in \mathcal{B}$ and if $x \in B_1 \cap B_2$, then there exists $B \in \mathcal{B}$ such that $x \in B \subset B_1 \cap B_2$.

   (b) Define closed set in a topological space (X, τ). Prove that a set A in a topological space (X, τ) is closed if and only if $X \setminus A$ is an open set in (X, τ).

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6. (a) Let (X, τ) be a topological space. When is (X, τ) called a T_0 space? Prove that being T_0 is a topological property (i.e., it is preserved under homeomorphisms.)
   (b) Define Lindelöff topological space. Prove that if (X, d) is a Lindelöff metric space, then (X, d) is a separable metric space.
7. (a) Define compact set in a topological space (X, τ). Let Y be a subspace of X. Prove that Y is compact if and only if every open covering of Y by sets open in X contains a finite subcollection covering of Y.
   (b) State and prove tube lemma.
8. (a) Define sequential compactness in a metric space (X, d). Prove that if (X, d) is sequentially compact, then for every $\varepsilon > 0$, there exists a finite covering of X by ε balls.
   (b) State and prove Lebesgue covering lemma.

M.Sc. (Mathematics) Part – I
Complex Analysis (Rev)
Paper - IV
April: - 2016

QP Code : 33154

External(New Syllabus)] (3 Hours) **[Total Marks:80**

Instructions:
- Attempt **any two** questions from **each section**
- **All questions** carry **equal marks**
- **Answers** to **Section I** and **Section II** should be written in the **same answer book**.

SECTION I (Attempt **any two** questions)

1. (a) Suppose $z_n = x_n + iy_n$ and $z_0 = x_0 + iy_0$ then prove that $\lim_{n\to\infty} z_n = z_0$ if and only if $\lim_{n\to\infty} x_n = x_0$ and $\lim_{n\to\infty} y_n = y_0$.

   (b) Find the domain of region of convergence of the following power series $\sum_{n=1}^{\infty}\left(\frac{iz-1}{3+4i}\right)^n$

2. (a) Prove that Bilinear Transformation preserves Cross Ratio. Hence or otherwise find the fixed points of $w = \frac{1+3iz}{i+z}$.

   (b) Prove that the circle $|z-2| = 3$ is mapped onto a circle $\left|w + \frac{2}{5}\right| = \frac{9}{25}$ under the transformation $w = \frac{1}{z}$.

3. (a) Prove: Let u and v be real valued functions defined on the domain $G \subset \mathbb{C}$ and suppose that u and v have continuous partial derivatives then $f : G \to \mathbb{C}$ defined by $f(z) = u(z) + iv(z)$ is analytic if and only if u and v satisfy Cauchy Riemann equation.

   (b) If $v = e^x \sin y$, prove that v is a harmonic function. Also find the corresponding harmonic conjugate and analytic function.

4. (a) State and prove the Cauchy-Goursat theorem.

   (b) Evaluate $\int_0^{1+i} x^2 + iy \, dz$ along:
   (i) The line $y = x$
   (ii) Along the parabola $y = x^2$. Is the integral independent of path?

SECTION II (Attempt **any two** questions)

5. (a) State Cauchy Integral Formula. Hence or otherwise evaluate $\int_c \frac{z+3}{2z^2+3z-2} dz$.

   (b) State and prove Moreras theorem.

6. (a) State and prove Schwarz Lemma.

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MT-Con. 3677-16.

(b) Suppose f is non-constant and analytic in a domain of G. If $|f|$ attains minimum in G at α , then prove that $f(\alpha) = 0$.

7. (a) State and prove Casorti Weiestrass theorem.

(b) Find all the possible Laurent Series expansions of $f(z) = \dfrac{1}{(z-1)(z-2)}$.

8. (a) State and prove Rouche's Theorem.

(b) Use the contour integration to evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{x^2+x+2}{x^4+10x^2+9} dx$

M.Sc. (Mathematics) Part – I
Discrete Mathematics (Rev)
April: - 2016

QP Code : 33151

External(Revised)] (3 Hours) [Total Marks:80

Instructions:
- Attempt **any two** questions from **each section**
- **All questions** carry **equal marks**
- **Answers** to **Section I** and **Section II** should be written in the **same answer book**.

SECTION I (Attempt **any two** questions)

1. (a) (i) Find the remainder when 2^{50} and 41^{65} are divided by 7.
   (ii) Define Mobius function μ. Prove that μ is a multiplicative function.

   (b) Using Cardano's method solve the cubic equation $x^3 - 2x^2 - 5x + 6 = 0$

2. (a) In how many ways can m distinguishable balls be put into n indistinguishable boxes?
   (b) Define Stirling's number of first kind $s(n, k)$.
   Show that (i) $s(n, 1) = (n-1)!$, (ii) $\sum_{k=0}^{n} s(n, k) = n!$

3. (a) State Pigeonhole Principle. Let T be an equilateral triangle with side length 1 unit. Show that if 5 points are chosen in T then two of them will be at most 1/2 unit apart.
   (b) (i) Let m and n be relative prime positive integer. Prove that the system $x \equiv a(\text{mod } m)$ and $x \equiv b(\text{mod } n)$ has a solution.
   (ii) Given any six integers in $1, 2, \ldots, 10$ prove that there exist atleast two that add upto 11.

4. (a) (i) A tree has two vertices of degree 2, one vertex of degree 3 and three vertices of degree 4. How many vertices of degree 1 does it have?
   (ii) Show that an edge e of graph G is not a cut edge if and only if e belongs to a cycle in graph G
   (b) Show that the following are equivalent in a Boolean algebra: (i) $a + b = b$, (ii) $a * b = a$, (iii) $a' + b = 1$, (iv) $a * b' = 0$

SECTION II (Attempt **any two** questions)

5. (a) (i) For a matrix $A = [a_{ij}]$ of order $n \times n$ define e^A and prove that it is a well defined matrix of the same order.
   (ii) Obtain with justification the solution of the initial value problem:

$$\frac{dx}{dt} = A(x), \quad x(t_0) = x_0 \in \mathbb{R}$$

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(b) Solve :

$$\frac{dx}{dt} = 4x + 3y, \quad x(2) = 2$$
$$\frac{dy}{dt} = -3x + 4y, \quad y(2) = 4$$

6. (a) (i) Explain the method of reducing an intial value problem of order $n(\geq 2)$ to a system of first order oridinary differential equations with initial conditions.

(ii) State and prove a condition guaranteeing a unique solution of the order $n(\geq 2)$ initial value problem.

(b) Reduce the following initial value problem to a first order system with initial conditions and solve it:

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} = 2x + 3t$$
$$x(0) = 1$$
$$\frac{dx}{dt}(0) = 2$$

7. (a) For the vibrating string problem :
$\frac{\partial^2 X}{\partial t^2} = c^2\frac{\partial^2 X}{\partial x^2}, t \geq 0, 0 \leq x \leq L$, c, L being constant real numbers, describe the method of separation of variables.

(b) Find all the eigenvalues of the boundary value problem:
$\frac{d^2X}{dt^2} + 10X = 0, X(0) = 0 = \frac{dX}{dt}(2)$

8. (a) Describe the Cauchy problem for the non-linear partial differential equation $F(x, y, u, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}) = 0$ in a function $u = u(x, y)$ and obtain the five characteristic equations for it.

(b) Solve the quasilinear Cauchy problem:

$$x\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = y$$
$$u(x, 0) = x^2$$

M.Sc. (Mathematics) Part – I

Soft Skills, Logic & Elementary

(Rev) April: - 2016

QP Code : 33157

Revised] (3 Hours) [Total Marks:80

Instructions:

- Attempt **any two** questions from **each section**
- **All questions** carry **equal marks**
- **Answers** to **Section I** and **Section II** should be written in the **same answer book.**

SECTION I (Attempt **any two** questions)

1. (a) Prepare the truth table of the following:
   (i)$[(p \wedge q) \vee (-p \wedge r)] \vee (q \wedge r)$
   (ii)$[p \Rightarrow (-q \vee r)] \wedge [-q \vee (p \Leftrightarrow -r)]$
   (b) Show that the relation of congruence modulo p has p distinct equivalence classes.

2. (a) State and prove Schroder - Bernstein theorem.
   (b) Check given relations are Equivalence or not (i)$S = \Re \times \Re$ and relation is $(x_1, y_1)R(x_2, y_2)$ iff $x_1 + x_2 = y_1 + y_2$ (ii) $S = Z$ and relation R is defined as,for $x, y \in Z \in, xRy$ iff $2x + y$ is divisible by 3.

3. (a) Let S be an arbitrary poset with a subset X. Show by an example that X may have just one maximal but no supremum
   (b) Prove using principle of Mathematical Induction $8^n - 3^n$ is divisble for $n \in \mathbb{N}$.

4. (a) If α and β are disjoint cycles then prove that disjoint cycles are commutative i.e. $\alpha\beta = \beta\alpha$. Also prove that every permutation in S_n is a product of disjoint cycles.
   (b) Compute σ^{1057} for the given permutations:
   (i) $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 1 & 4 & 5 & 3 & 7 & 6 \end{pmatrix}$, (ii) $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 6 & 1 & 7 & 4 & 2 & 5 \end{pmatrix}$

SECTION II (Attempt **any two** questions)

5. (a) (i) From a standard deck of 52 cards, we draw 13 cards. What is the probability that we have 5 spades and three diamonds cards in our hands?
   (ii) Let $\mathcal{F}$ be a family of subsets of a non-empty set Ω. What are the conditions which makes $\mathcal{F}$ a field? For $\Omega = \{1, 2, 3\}$ find smallest field on Ω containing $\{1\}, \{2\}$.
   (b) (i) Let P be a finitely additive probability measure on a filed $\mathcal{F}$. Show that $P(A \Delta B) = P(A) + P(B) - 2P(A \cap B)$ for $A, B \in F$.
   (ii) Define σ-field. Is the family $\mathcal{F}$ consisting of all finite subsets of a non-empty set Ω and their complements always a σ-field? Justify your answer.

6. (a) (i) Define countably additive probability measure. Let $\mathcal{F}$ be the family of all subsets of $\mathbb{N}$. Put $P(\{i\}) = \alpha_i, i = 1, 2, \ldots$ and extend P to a countably additive probability measure on $\mathcal{F}$ under suitable conditions on the numbers α_i.

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(ii) Elderly friend of yours has two children, one of whom is a girl, but you do not know the sex of the other child. How likely is it that it is a boy? Justify your answer.

(b) (i) Let Ω be an n-element set with uniform probability and let $A, B \subset \Omega$ be independent. Show that if A has i elements, then B must have $j = k\dfrac{n}{\gcd(i,n)}$ for some $k \in \{0, 1, \ldots, \gcd(i,n)\}$.

(ii) Let $(\Omega, \mathcal{F}, P)$ be a probability space. Define $P(A|B)$, a conditional probability of A given B, for $A, B \in \mathcal{F}$. Suppose $A, B \in \mathcal{F}$ such that $0 \neq P(B) \neq 1$. Show that $P(A) = P(A|B)$ if and only if $P(A) = P(A|(\Omega \setminus B))$.

7. (a) (i) Define a random variable on Ω, given a σ-field $\mathcal{F}$ on a non-empty set Ω. Suppose a random variable X has the normal distribution $N(0,1)$. Then compute the normal distribution of $a + bX$, where $a, b \in \mathbb{R}$.

(ii) When is a random variable X said to have the binomial distribution $B(n,p)$ for $n \in \mathbb{N}$ and $p \in [0,1]$? Let X_n be a sequence of random variables with binomial distribution $B(n, \lambda/n)$ for some $\lambda > 0$. Show that $\lim_{n\to\infty} P(\{X_n = k\}) = e^{-\lambda}\dfrac{\lambda^k}{k!}$.

(b) (i) Define a simple random variable X and define expectation $E(X)$. For independent simple random variables X and Y, show that $E(XY) = E(X)E(Y)$.

(ii) Define variance of a discrete random variable. Compute the variance of a random variable with normal distribution.

8. (a) (i) Define the characteristic function of a random variable. If X is a random variable such that, $P(\{X \in \mathbb{Z}\}) = 1$ then show that characteristic function of X is a periodic function with period 2π.

(ii) How many independent tosses of a fair coin are required for the probability that the average number of Heads differ from 0.5 by less than 2% to be atleast 0.99? (Given that area under normal curve between $x = 0$ and $x = 2.59$ is 0.4950.)

(b) (i) State Chebyshev inequality. A survey of is taken by using a random sample of n families, having a single child. Let S_n denote the number of families in the sample, having a daughter as the single child. Use Chebyshev inequality to find a lower bound of the probability that $\dfrac{S_n}{n}$, differs from 1/2, by less than 1% when $n = 100$.

(ii) State Central limit theorem. What is the probability that number of heads in 10000 tosses will be greater than 5100? (Given that area under normal curve between $x = 0$ and $x = 2$ is 0.477).

M.Sc. (Mathematics) Part – I

Algebra – I (Old)

Paper - I

April: - 2016

QP Code : 25193

(3 Hours) [Total Marks:100

Scheme B (Internal)] (2 Hours) [Total Marks: 40

Instructions:

- **Mention** on the top of the **answer book** the **scheme** under which you are appearing
- **Scheme A** students should **attempt any five** questions
- **Scheme B** students should **attempt any three** questions
- All **questions** carry **equal marks**

1. (a) Show that a finite semi-group G in which cancellation laws hold is a group.

   (b) Show that the number of generators of an infinite cyclic group is two.

2. (a) Show that the order of a subgroup of a finite group divides the order of the group.

   (b) Prove that every group of prime order is abelian.

3. (a) Prove that a group of order 36 is not simple.

   (b) Show that every quotient group of a cyclic group is cyclic, give an example to show that the converse need not to be true.

4. (a) Show that the intersection of two subrings of a ring R is a subring of R. Give one example to show that the union of two subrings of R need not to be a subring of R.

   (b) If R is commutative ring with unity whose only ideals are $\{0\}$ and R, then show that R is a field.

5. (a) Let R be a commutative ring with unit element in which every ideal is a prime ideal. Prove that R is a field.

   (b) If p and q are prime elements in an integral domain R with unity such that $p|q$ then show that p and q are associates.

6. (a) If $U(F)$ and $V(F)$ are two vector spaces and T is a linear transformation from U into V then show that the null space of T is sub space of U.

   (b) Show that there exists a basis for each finite dimensional vector space.

7. (a) Let U, V be vector spaces over the field F and let T be a linear transformation from U into V. If T is one-one and onto, then show that the inverse function T^{-1} is a linear transformation from V into U.

   (b) In $V_3(\mathbb{R})$, examine each of the following sets of vectors for linear dependence

      1. {(-1,2,1), (3,0,-1), (-5,4,3)}
      2. {(1,3,2), (1,-7,-8), (2,1,-1)}

8. (a) Suppose that α and β are vectors in an inner product space. If $|(\alpha, \beta)| = \|\alpha\|\|\beta\|$. Then show that α and β are linearly dependent.

   (b) Find all (complex) characteristic values and characteristic vectors of $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

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M.Sc. (Mathematics) Part – I
Analysis (Old)
Paper - II
April: - 2016

QP Code : 25202

Scheme A (External)]	(3 Hours)	**[Total Marks:100**
Scheme B (Internal)]	(2 Hours)	**[Total Marks: 40**

Instructions:

- **Mention** on the top of the **answer book** the **scheme** under which you are appearing
- **Scheme A** students should **attempt any five** questions
- **Scheme B** students should **attempt any three** questions
- All **questions** carry **equal marks**

1. (a) State and prove Heine-Borel theorem.

   (b) Prove that every bounded sequence in $\mathbb{R}$ has a convergent subsequence.

2. (a) If $f : \mathbb{R}^n \to \mathbb{R}^m$ is given by $f = \{f_1, f_2, \ldots, f_m\}$, show that f is continuous at $a \in \mathbb{R}^n$ if and only if each f_i is continuous at a for $i = 1, 2, \ldots, m$.

   (b) Discuss the continuity and differentiability of f at $(0, 0)$ if $f(x, y) = \sqrt{xy}, \forall (x, y) \in \mathbb{R}^2$

3. (a) Define the pointwise and uniform convergence of a sequence $\{f_n(x)\}$ of real-valued functions on S, where S is a non-empty subset of $\mathbb{R}$. Show that if $\{f_n(x)\}$ is uniformly convergent then it is convergent.

   (b) Show that the sequence $\left\{\dfrac{e^{-nx}}{3n}\right\}$ converges uniformly on $\mathbb{R}$.

4. (a) State and prove Weirstrass test for uniform convergence of a series $\sum f_n(x)$.

   (b) State Ratio test for convergence of a positive term series. Hence discuss the convergence of $\sum \dfrac{x^n}{(2n)!}, x \in \mathbb{R}$

5. (a) When do you say that a function $f : \mathbb{R}^n \to \mathbb{R}$ is differentiable at $a \in \mathbb{R}^n$? Find the total derivative of $f(x, y, z) = x^2y - yz^3$ at $(1, 1, -1)$.

   (b) State and prove Mean value theorem for a differentiable function $f : \mathbb{R}^n \to \mathbb{R}$.

6. (a) State Taylor's theorem and use it to expand the function $f(x, y) = \tan x . \tan y$ near $(\pi, -\pi)$ upto and including second degree terms.

   (b) Find the jacobians of f, g and $f \circ g$ at $(1, 1, 2)$ given that $f(x, y, z) = (xy, yz, zx)$ and $g(x, y, z) = (y - 3z, x + y, z^2 - x)$

7. (a) State and prove Fubini's theorem for a double integral over a rectangle in xy-plane.

   (b) Evaluate the double integral of $f(x, y) = 3x$ over the bounded region between the lines $x + y = 1, x + y = 2, 2x - y = 2$ and $2x - y = 3$.

8. (a) Show that the improper integral $\int_a^b \frac{dx}{(x-a)^p}$ $(p > 0)$ converges if and only if $p < 1$. Hence discuss the convergence of $\int_0^1 \frac{dx}{\sqrt{x}(x+1)}$

   (b) Define the convergence of $\int_a^\infty f(x)dx$. Hence show that $\int_a^\infty \frac{dx}{1+x^2}$ converges but $\int_a^\infty x^2 dx$ does not.

MT-Con. 3672-16.

M.Sc. (Mathematics) Part – I
Topology (Old)
Paper – III April: - 2016

Scheme A (External)]	(3 Hours)	[Total Marks:100
Scheme B (Internal)]	(2 Hours)	[Total Marks: 40

Instructions:
- **Scheme A** students should attempt **any five** questions.
- **Scheme B** students should attempt **any three** questions.
- **All** questions carry **equal marks.**
- **Mention** clearly the **Scheme** under which you are appearing.

1. (a) Let $f : X \to Y$. Prove that f is injective iff $f(A \cap B) = f(A) \cap f(B)$ for all subsets A, B of X.

   (b) Let $n \in \mathbb{N}$ and P be a set with $p_0 \in P$. Prove that there exists a bijective correspondence f on the set P with the set $\{1, 2, \ldots, n+1\}$ iff there exists a bijective correspondence g on the set $P - p_0$ with the set $\{1, 2, \ldots, n\}$.

2. (a) Let X be a topological space and $A \subseteq X$. Prove that the following statements are equivalent: (i) A is closed (ii) $A = \overline{A}$ (iii) A contains all its limit points.

   (b) Define a connected topological space. Let X be a topological space and A be a subset of X. Show that if A is connected then its closure $\overline{A}$ is also connected.

3. (a) Let X, Y be topological spaces. Show that if A is closed in X and B is closed in Y then $A \times B$ is closed in $X \times Y$.

   (b) Show that every open subset of $\mathbb{R}$ is the union of disjoint sequence of open intervals.

4. (a) Let $p : X \to Y$ be a quotient map. Suppose $f : X \to Z, g : Y \to Z$ are maps with f being continuous and $g \circ p = f$. Prove that g is a continuous map.

   (b) Let X, Y be topological spaces. Define product topology on $X \times Y$. Let $\pi_1 : X \times Y \to X$ be defined as $\pi_1(x, y) = x, \forall (x, y) \in X \times Y$. Is π_1 an open map? Is π_1 a closed map? Justify your answer.

5. (a) Prove that every compact subset of a Hausdroff space is closed.

   (b) Let X, Y be topological spaces. If Y is compact then prove that the projection map $\pi_1 : X \times Y \to X$ is a closed map.

6. (a) Let f and g be two paths in a topological space with same initial point and same end point. Define the path homotopy relation $\simeq_p$ and show that it is an equivalence relation.

   (b) Prove that the image of a compact space under a continuous map is compact.

7. (a) Prove that any connected open subset of $\mathbb{R}$ is a countable union of disjoint open intervals.

   (b) State and prove the path lifting lemma.

8. (a) Prove or disprove: "The set of all irrational numbers in $\mathbb{R}$ is a Baire space".

   (b) Prove that the fundamental group $\pi_1(S^1, (1, 0))$ is isomorphic to the group $\mathbb{Z}$ of integers under addition.

M.Sc. (Mathematics) Part – I
Combinatorics
Paper V April: - 2016

QP Code : 25229

	(3 Hours)	Total marks: 100
Scheme B(Internal/External)	(2 Hours)	Total marks: 40

N.B: 1) **Scheme A** students answer **any five** questions.
2) **Scheme B** students answer **any three** questions.
3) All **questions** carry **equal marks.**
4) Write on the **top** of your **answer book** the **scheme** under which you are appearing.

1. (a) Determine 12-combinations of multiset S = {4.a, 3.b, 4.c ,5.d}
(b) In how many ways can 9 gentleman and 5 ladies be seated at round table if no two ladies are to sit in alternate seats?
2. (a) Define Stirling numbers of second kind S(n,k) and prove the identity:
$x^n = \sum_{k=1}^{n} S(n,k)[x]_k$ where $[x]_k$ denotes the falling factorial.
(b) Show that number of surjective functions from an n-set to m-set where $m \leq n$ is $m!\,S(n,m)$.
3. (a) Give one application of Pigeon hole principle by stating strong form of Pigeon hole principle.
(b) Write a note on derangement of n objects D_n. Derive formula for D_n.
4. (a) State and prove Mobius inversion formula.
(b) Solve the recurrence relation $a_n - 3a_{n-1} = 2 - 2n^2$ given that $a_0 = 3$.
5. (a) How many 6 digits numbers can be formed using digits 1, 2, 3, 4, 5 such that any digit that appears in the number appears at least twice?
(b) Find sum of all coefficients in $(3x - 5y + z)^3$.
6. (a) You have two coins, a fair one with probability of tails ½ and unfair on with probability of tails 1/3, but otherwise identical. A coin is selected at random, falling tails up. How likely is that it is a fair coin?
(b) In a sample 2% of the population have a certain blood disease in a serious form. 20% have it in a mild form; 88% don't have it at all. A new blood test is developed; the probability of testing positive is 9/10 if the subject (patient) has the serious form, 6/10 if the subject has the mild form and 1/10 if the subject doesn't have the disease. A person X just tested positive. What is the probability that X has the serious form of disease?
7. (a) State and prove Baye's theorem.
(b) If X and Y are independent random variables then show that $\phi_{X+Y}(t) = \phi_X(t).\phi_Y(t)$. Hence show that if X and Y are two independently normally distributed random variables then X + Y is normally distributed random variable.
8. (a) Let X and Y be independent random variables with binomial distributions $B(m, p)$ and $B(h, p)$ respectively. Prove that $X + Y$ is also binomial distribution with $B(m+n, p)$.
(b) Define variance of a discrete random variable. Compute the variance of random variable with normal distribution.

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