UNIVERSITY OF MUMBAI
No. UG/268 of 2017-18

CIRCULAR:-

Attention of the Principals of the affiliated Colleges in Arts is invited to this office Circular No.UG/18 of 2013-14 dated 4th May, 2013 relating to syllabus of the M.A./M.Sc. degree course. They are hereby informed that the recommendations made by the Board of Studies in Mathamatics at its meeting held on 28th April, 2017 has accepted by the Academic Council at its meeting held on 11th May, 2017 **vide** item No. 4.218 and that in accordance therewith, the revised syllabus as per the (CBCS) for(Sem. I & II) of M.A/M.Sc. degree programme in the course of Mathematics, which is available on the University's website (www.mu.ac.in) and that the same has been brought into force with effect from the academic year 2017-18 accordingly.

MUMBAI – 400 032
23rd October, 2017

(Dr.Dinesh Kamble)
I/c REGISTRAR

To,

The Head, University Department of Mathematics and the Principals of affiliated Colleges in Arts and Science.

A.C/4.218/11/05/2017

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No. UG/268-A of 2017-18 MUMBAI-400 032 23rd October, 2017

Copy forwarded with compliments for information to:-

1) The Co-Ordinator, Faculty of Arts & Humanities and Science & Technology.
2) The Chairperson, Board of Studies in Mathematics,
3) The Offg.Director, Board of Examinations and Evaluation,
4) The Director, Board of Students Development.,
5) The Professor-cum-Director, Institute of Distance and Open Learning (IDOL),
6) The Co-Ordinator, University Computerization Centre.

(Dr.Dinesh Kamble)
I/c REGISTRAR

UNIVERSITY OF MUMBAI

Proposed Syllabus for

M.A./M.Sc. Semester I & II (CBCS)

Program: M.A/M.Sc.

Course: Mathematics

with effect from the academic year 2017-2018

M.A./M.Sc. Semester I and II
Choice Based Credit System (CBCS)

Semester I

Algebra I				
Course Code	Unit	Topics	Credits	L/W
PSMT101,PAMT101	Unit I	Dual spaces	5	4
	Unit II	Determinants		
	Unit III	Characteristic polynomial		
	Unit IV	Bilinear forms		

Analysis I				
Course Code	Unit	Topics	Credits	L/W
PSMT102,PAMT102	Unit I	Euclidean space $\mathbb{R}^n$	5	4
	Unit II	Riemann integration		
	Unit III	Differential functions		
	Unit IV	Inverse function theorem , Implicit		
	Unit IV	function theorem		

Complex Analysis				
Course Code	Unit	Topics	Credits	L/W
PSMT103,PAMT103	Unit I	Holomorphic functions	5	4
	Unit II	Contour integration, Cauchy-Gursat theorem		
	Unit III	Properties of holomorphic functions		
	Unit IV	Conformal mappings, Residue Calculus		

Discrete Mathematics				
Course Code	Unit	Topics	Credits	L/W
PSMT104,PAMT104	Unit I	Number Theory	5	4
	Unit II	Advanced Counting		
	Unit III	Recurrence relations		
	Unit IV	Polya's theory of counting		

Set Theory and Logic				
Course Code	Unit	Topics	Credits	L/W
PSMT105,PAMT105	Unit I	Introduction to logic	4	4
	Unit II	Sets and functions		
	Unit III	Partial order		
	Unit IV	Lattices		

Semester II

Algebra II				
Course Code	Unit	Topics	Credits	L/W
PSMT201,PAMT201	Unit I	Groups, group homomorphisms	5	4
	Unit II	Groups acting on sets and Syllow theorems		
	Unit III	Rings, fields		
	Unit IV	Divisibility in integral domains		

Topology				
Course Code	Unit	Topics	Credits	L/W
PSMT202,PAMT202	Unit I	Topological spaces	5	4
	Unit II	Connected Topological spaces		
	Unit III	Compact Topological spaces		
	Unit IV	Compact metric spaces, Complete metric spaces		

Analysis II				
Course Code	Unit	Topics	Credits	L/W
PSMT203,PAMT203	Unit I	Measures	5	4
	Unit II	measurable functions and integration of non-negative functions		
	Unit III	Dominated convergence theorem and L^1, L^2 spaces.		
	Unit IV	Signed measures, Radon-Nykodym theorem		

Differential Equations				
Course Code	Unit	Topics	Credits	L/W
PSMT204,PAMT204	Unit I	Picard's theorem	5	4
	Unit II	Ordinary differential equations		
	Unit III	Sturm-Liouville theory		
	Unit IV	First order Partial Differential Equations		

Probability Theory				
Course Code	Unit	Topics	Credits	L/W
PSMT205,PAMT205	Unit I	Basics of Probability	4	4
	Unit II	Probability measure		
	Unit III	Random variables		
	Unit IV	Limit theorems		

Teaching Pattern for Semester I and II

1. Four lectures per week per course. Each lecture is of 60 minutes duration.
2. In addition, there shall be tutorials, seminars as necessary for each of the five courses.

SEMESTER I

All Results have to be done with proof unless otherwise stated.

PSMT101,PAMT101 ALGEBRA I

Unit I. Dual spaces (15 Lectures)

Para 1 and 2 of Unit **I** are to be reviewed without proof (no question be asked).

1. Vector spaces over a field, linear independence, basis for finite dimensional and infinite dimensional vector spaces and dimension.

2. Kernel and image, rank and nullity of a linear transformation, rank-nullity theorem (for finite dimensional vector spaces), relationship of linear transformations with matrices, invertible linear transformations. The following are equivalent for a linear map $T : V \longrightarrow V$ of a finite dimensional vector space V:

1. T is an isomorphism.
2. $\ker T = \{0\}$.
3. $\mathrm{Im}(T) = V$.

3. Linear functionals, dual spaces of a vector space, dual basis (for finite dimensional vector spaces), annihilator W° in the dual space V^* of a subspace W of a vector space V and dimension formula, a $k-$dimensional subspace of an $n-$ dimensional vector space is intersection of $n-k$ many hyperspaces. Double dual V^{**} of a Vector space V and canonical embedding of V into V^{**}. V^{**} is isomorphic to V when V is of finite dimension. (ref:[1] Hoffman K and Kunze R)

4. Transpose T^t of a linear transformation T. For finite dimensional vector spaces: rank (T^t)=rank T, range(T^t) is the annihilator of kernel (T), matrix representing T^t a rank of a matrix. (ref:[1] Hoffman K and Kunze R)

Unit II. Determinants (15 Lectures)

Determinants as alternating n-forms, existence and uniqueness , Laplace expansion of determinant, determinants of products and transposes, determinants and invertible linear transformations, determinant of a linear transformation.

Reference for Unit II: [1] Hoffman K and Kunze R, *Linear Algebra*.

Unit III. Characteristic polynomial (15 Lectures)

Eigen values and Eigen vectors of a linear transformation, Characteristic polynomial, Cayley-Hamilton theorem, Minimal polynomial, Triangulable and diagonalizable linear operators, invariant subspaces and simple matrix representation (for finite dimension). (ref: [5] N.S. Gopalkrishnan & [3] Serge Lang)

Nilpotent linear transformations on finite dimensional vector spaces, index of a Nilpotent linear transformation. Linear independence of $\{u, Nu, \cdots, N^{k-1}u\}$ where N is a nilpotent linear transformation of index $k \geq 2$ of a vector space V and $u \in V$ with $Nu \neq 0$. (Ref: [2] I.N.Herstein)

For a nilpotent linear transformation N of a finite dimensional vector space V and for any subspace W of V which is invariant under N, there exists a subspace V_1 of V such that $V = W \oplus V_1$. (Ref:[2] I.N.Herstein)

Computations of Minimum polynomials and Jordan Canonical Forms for 3×3-matrices through examples of matrices such as $\begin{pmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{pmatrix}$, $\begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 1 & 1 & 0 \end{pmatrix}$. (Ref:[6] Morris W. Hirsch and Stephen Smale)

Unit IV. Bilinear forms (15 Lectures)

Para 1 of Unit **IV** is to be reviewed without proof (no question be asked).

1. Inner product spaces, orthonormal basis.

2. Adjoint of a linear operator on an inner product space, unitary operators, self adjoint operators, normal operators, spectral theorem for a normal operator on a finite dimensional complex vector inner product space (ref:[1] Hoffman K and Kunze R).

3. Bilinear form, rank of a bilinear form, non-degenerate bilinear form and equivalent statements (ref:[1] Hoffman K and Kunze R).

4. Symmetric bilinear forms, orthogonal basis and Sylvester's Law, signature of a Symmetric bilinear form (ref:[4] Michael Artin).

Recommended Text Books

[1] Hoffman K and Kunze R: *Linear Algebra*, Prentice-Hall India.

[2] I.N.Herstein: *Topics in Algebra*, Wiley-India.

[3] Serge Lang: *Linear Algebra*, Springer-Verlag Undergraduate Text in Mathematics.

[4] Michael Artin: *Algebra*, Prentice-Hall India.

[5] N.S. Gopalkrishnan: *University Algebra*, New Age International, third edition, 2015.

[6] Morris W. Hirsch and Stephen Smale, *Differential Equations, Dynamical Systems, Linear Algebra*, Elsevier.

PSMT102/PAMT102 ANALYSIS I

Unit I. Euclidean space $\mathbb{R}^n$ (15 Lectures)

Euclidean space $\mathbb{R}^n$: inner product $\langle x, y\rangle = \sum_{j=1}^{n} x_j y_j$ of $x = (x_1, \cdot\cdot, x_n), y = (y_1, \cdot\cdot, y_n) \in \mathbb{R}^n$ and properties, norm $\|x\| = \sqrt{\sum_{j=1}^{n} x_j^2}$ of $x = (x_1, \cdot\cdot, x_n) \in \mathbb{R}^n$, Cauchy-Schwarz inequality, properties of the norm function $\|x\|$ on $\mathbb{R}^n$. ref: [4] W. Rudin or [5] M. Spivak)

Standard topology on $\mathbb{R}^n$: open subsets of $\mathbb{R}^n$, closed subsets of $\mathbb{R}^n$, interior A° and boundary ∂A of a subset A of $\mathbb{R}^n$. (ref: [5] M. Spivak)

Operator norm $\|T\|$ of a linear transformation $T : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ ($\|T\| = \sup\{\|T(v)\| : v \in \mathbb{R}^n \,\&\, \|v\| \leq 1\}$) and its properties such as: For all linear maps $S, T : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ and $R : \mathbb{R}^m \longrightarrow \mathbb{R}^k$

1. $\|S + T\| \leq \|S\| + \|T\|$,
2. $\|R \circ S\| \leq \|R\|\|S\|$, and
3. $\|cT\| = |c|\|T\|$ $(c \in \mathbb{R})$.

(Ref: [1] C.C. Pugh or [2] A. Browder)

Compactness: Open cover of a subset of $\mathbb{R}^n$, Compact subsets of $\mathbb{R}^n$ (A subset K of $\mathbb{R}^n$ is compact if every open cover of K contains a finite subcover), Heine-Borel theorem (statement only), the Cartesian product of two compact subsets of $\mathbb{R}^n$ is compact (statement only), every closed and bounded subset of $\mathbb{R}^n$ is compact. Bolzano-Weierstrass theorem: Any bounded sequence in $\mathbb{R}^n$ has a converging subsequence.

Brief review of following three topics:

1. Functions and Continuity: Notation: $A \subset \mathbb{R}^n$ arbitrary non-empty set. A function $f : A \longrightarrow \mathbb{R}^m$ and its component functions, continuity of a function (ϵ, δ definition). A function $f : A \longrightarrow \mathbb{R}^m$ is continuous if and only if for every open subset $V \subset \mathbb{R}^m$ there is an open subset U of $\mathbb{R}^n$ such that $f^{-1}(V) = A \cap U$.
2. Continuity and compactness: Let $K \subset \mathbb{R}^n$ be a compact subset and $f : K \longrightarrow \mathbb{R}^m$ be any continuous function. Then f is uniformly continuous, and $f(K)$ is a compact subset of $\mathbb{R}^m$.
3. Continuity and connectedness: Connected subsets of $\mathbb{R}$ are intervals. If $f : E \longrightarrow \mathbb{R}$ is continuous where $E \subset \mathbb{R}^n$ and E is connected , then $f(E) \subset \mathbb{R}$ is connected.

Unit II. Riemann Integration (15 Lectures)

Riemann Integration over a rectangle in $\mathbb{R}^n$, Riemann Integrable functions, Continuous functions are Riemann integrable, Measure zero sets, Lebesgues Theorem (statement only), Fubini's

Theorem and applications.
Reference for Unit II: M. Spivak, *Calculus on Manifolds*.

Unit III. Differentiable functions (15 Lectures)

Differentiable functions on $\mathbb{R}^n$, the *total derivative* $(Df)_p$ of a differentiable function $f : U \longrightarrow \mathbb{R}^m$ at $p \in U$ where U is open in $\mathbb{R}^n$, uniqueness of total derivative, differentiability implies continuity. (ref: [1] C.C. Pugh or [2] A. Browder)

Chain rule. Applications of chain rule:

1. Let γ be a differentiable curve in an open subset U of $\mathbb{R}^n$. Let $f : U \longrightarrow \mathbb{R}$ be a differentiable function and let $g(t) = f(\gamma(t))$. Then $g'(t) = \langle (\nabla f)(\gamma(t)), \gamma'(t) \rangle$.
2. Computation of total derivatives of real valued functions such as
   (a) the determinant function $\det(X)$, $(X \in M_n(\mathbb{R}))$,
   (b) the Euclidean inner product function $\langle x, y \rangle$, $((x, y) \in \mathbb{R}^n \times \mathbb{R}^n)$.

(ref: (ref: [5] M. Spivak & [4] W. Rudin &)
Results on total derivative:

1. If $f : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ is a constant function, then $(Df)_p = 0 \ \forall \ p \in \mathbb{R}^n$.
2. If $f : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ is a linear map, then $(Df)_p = f \ \forall \ p \in \mathbb{R}^n$.
3. A function $f = (f_1, f_2, \cdot\cdot, f_m) : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ is differentiable at $p \in \mathbb{R}^n$ if and only if each f_j is differentiable at $p \in \mathbb{R}^n$, and $(Df)_p = ((Df_1)_p, (Df_2)_p, \cdot\cdot, (Df_m)_p)$.

(ref: [5] M. Spivak)

Partial derivatives, directional derivative $(D_u f)(p)$ of a function f at p in the direction of the unit vector, Jacobian matrix, Jacobian determinant. Results:

1. If the total derivative of a map $f = (f_1, \cdot\cdot, f_m) : U \longrightarrow \mathbb{R}^m$ (U open subset of $\mathbb{R}^n$) exists at $p \in U$, then all the partial derivatives $\frac{\partial f_i}{\partial x_j}$ exist at p.
2. If all the partial derivatives $\frac{\partial f_i}{\partial x_j}$ of a map $f = (f_1, \cdot\cdot, f_m) : U \longrightarrow \mathbb{R}^m$ (U open subset of $\mathbb{R}^n$) exist and are continuous on U, then f is differentiable.

(ref:[4] W. Rudin)

Derivatives of higher order, $\mathscr{C}^k$-functions, $\mathscr{C}^\infty$-functions. (ref: [3] T. Apostol)

Unit IV. Inverse function theorem, Implicit function theorem (15 Lectures)

Theorem (Mean Value Inequality): Suppose $f : U \longrightarrow \mathbb{R}^m$ is differentiable on an open subset U of $\mathbb{R}^n$ and there is a real number such that $\|(Df)_x\| \leq M \ \forall \ x \in U$. If the segment $[p, q]$ is contained in U, then $\|f(q) - f(p)\| \leq M\|q - p\|$. (ref: [1] C.C. Pugh or [2] A. Browder)

Mean Value Theorem: Let $f : U \longrightarrow \mathbb{R}^m$ is differentiable on an open subset U of $\mathbb{R}^n$. Let $p, q \in U$ such that the segment $[p, q]$ is contained in U. Then for every vector $\mathbf{v} \in \mathbb{R}^n$ there is a point $x \in [p, q]$ such that $\langle \mathbf{v}, f(q) - f(p) \rangle = \langle \mathbf{v}, (Df)_x(q - p) \rangle$. (ref: [3] T. Apostol)

If $f : U \longrightarrow \mathbb{R}^m$ is differentiable on a connected open subset U of $\mathbb{R}^n$ and $(Df)_x = 0 \ \ \forall \ x \in U$, then f is a constant map.

Taylor expansion for a real valued $\mathscr{C}^m$-function defined on an open subset of $\mathbb{R}^n$, stationary points (critical points), maxima, minima, saddle points, second derivative test for extrema at a stationary point of a real valued $\mathscr{C}^2$-function defined on an open subset of $\mathbb{R}^n$. (ref: [3] T. Apostol)

Contraction mapping theorem. Inverse function theorem , Implicit function theorem. (ref: [2] A. Browder)

Recommended Text Books

[1] C.C. Pugh: *Real mathematical analysis*, Springer UTM.

[2] A. Browder: *Mathematical Analysis, An Introduction*, Springer.

[3] T. Apostol: *Mathematical Analysis*, Narosa.

[4] W. Rudin: *Principles of Mathematical Analysis*, Mcgraw-Hill India.

[5] M. Spivak: *Calculus on Manifolds*, Harper-Collins Publishers.

PSMT103/PAMT103 COMPLEX ANALYSIS

Unit I. Holomorphic functions and Mobius transformations (15 Lectures)

Note: *A complex differentiable function defined on an open subset of $\mathbb{C}$ is called a holomorphic function.*

Review: Complex Numbers, Geometry of the complex plane, Riemann sphere, Complex sequences and series, Sequences and series of functions in $\mathbb{C}$, , Weierstrass's M-test, Uniform convergence, Complex differentiable functions, Cauchy-Riemann equations (no questions be asked).

Ratio test and root test for convergence of a series of complex numbers. Complex Power series, radius of convergence of a power series, Cauchy-Hadamard formula for radius of convergence of a power series. Examples of convergent power series such as exponential series, cosine series and sine series, and the basic properties of the functions $e^z, \cos z, \sin z$. Abel's theorem: Let $\sum_{n\geq 0} a_n(z-z_0)^n$ be a power series, of radius of convergence $R > 0$. Then the function f defined by $f(z) = \sum a_n(z-z_0)^n$ is holomorphic on the open ball $|z-z_0| < R$ and $f'(z) = \sum_{n\geq 1} na_n(z-z_0)^{n-1}\ \forall\ |z-z_0| < R$. Applications of Abel's theorem such as $\exp'(z) = \exp z, \cos'(z) = -\sin z, \sin'(z) = \cos z, (z \in \mathbb{C})$.

Chain Rule. A basic result: Let Ω_1, Ω_2 be open subsets of $\mathbb{C}$. Suppose $f : \Omega_1 \longrightarrow \mathbb{C}$ is a holomorphic function with $f'(z) \neq 0\ \forall\ z \in \Omega_1$ and $g : \Omega_2 \longrightarrow \mathbb{C}$ be a continuous function such that $g(\Omega_2) \subset \Omega_1$ and $f(g(w)) = w\ \forall\ w \in \Omega_2$. Then g is a holomorphic function on Ω_2 and $g'(w) = \dfrac{1}{f'(g(w))}\ \forall\, w \in \Omega_2$. Application: The logarithm as the inverse of exponential (i.e. $\forall\, w \neq 0$ in $\mathbb{C}$, $\log(w) := \{z \in \mathbb{C} \,|\, e^z = w\}$), branches of logarithm, the principle branch $l(z)$ of the logarithmic function on $\mathbb{C} - \{z \in \mathbb{C} : z \leq 0\}$ is a holomorphic function and $l'(z) = 1/z$.

Reference for Unit I:

1. A. R. Shastri: *An introduction to complex analysis*, Macmillan.
2. Serge Lang: *Complex Analysis*.
3. L. V. Ahlfors: *Complex analysis*, McGraw Hill .
4. R. Remmert: *Theory of complex functions*, Springer.

Unit II. Contour integration, Cauchy-Gursat theorem (15 Lectures)

Contour integration, Cauchy-Goursat Theorem for a rectangular region or a triangular region. Primitives. Existence of primitives: If f is holomorphic on a disc U, then it has a primitive on U and the integral of f along any closed contour in U is 0. Local Cauchy's Formula for discs (without proof), Power series representation of holomorphic functions (without proof), Cauchy's estimates, entire functions, Liouville's theorem, Morera's theorem, the Fundamental theorem of Algebra.
Reference for unit II:

1. A. R. Shastri: *An introduction to complex analysis*, Macmillan.
2. Serge Lang: *Complex Analysis*.
3. L. V. Ahlfors: *Complex analysis*, McGraw Hill .
4. R. Remmert: *Theory of complex functions*, Springer.

Unit III. Holomorphic functions and their properties (15 Lectures)

Cauchys theorem (homotopy version or homology version) The index (winding number) of a closed curve, Cauchy integral formula.

Zeros of holomorphic functions, Identity theorem. Counting zeros; Open Mapping Theorem, Maximum modulus theorem, Schwarz's lemma. Every automorphism of unit disc with center 0 in $\mathbb{C}$ is a rotation.

Isolated singularities:removable singularities and Removable singularity theorem, poles and essential singularities. Laurent Series development. Casorati-Weierstrass's theorem.
Reference for Unit III:

1. J. B. Conway, *Functions of one Complex variable*, Springer.
2. L. V. Ahlfors: *Complex analysis*, McGraw Hill .
3. R. Remmert: *Theory of complex functions*, Springer.

Unit IV Residue calculus (15 Lectures)

Residue Theorem and evaluation of standard types of integrals by the residue calculus method.

Argument principle. Rouché 's theorem.

Conformal mappings. If $f : G \longrightarrow \mathbb{C}$ is a holomorphic function on the open subset G of $\mathbb{C}$ and $f'(z) \neq 0\ \forall\ z \in G$, then f is a conformal map. Mobius transformations (fractional linear transformation or linear transformation). Any Mobius transformation which fixes three distinct

points is necessarily the identity map. Cross ratio. A Mobius transformation preserves cross ratio. Cross ratio (z_1, z_2, z_3, z_4) is real if and only if the four points z_1, z_2, z_3, z_4 lie on a circle. A Mobius transformation takes circles onto circles. Symmetry, symmetry principle and applications to Cayley map.
Reference for Unit IV:

1. J. B. Conway, *Functions of one Complex variable*, Springer.
2. L. V. Ahlfors: *Complex analysis*, McGraw Hill .
3. R. Remmert: *Theory of complex functions*, Springer.

PSMT104/PAMT104 DISCRETE MATHEMATICS

Unit I. Number theory (15 Lectures)

Divisibility, Linear Diophantine equations, Cardano's Method, Congruences, Quadratic residues, Arithmetic functions,

Advanced counting: Types of occupancy problems, distribution of distinguishable and indistinguishable objects into distinguishable and indistinguishable boxes (with condition on distribution) Stirling numbers of second and first kind. Selections with Repetitions.

Unit II. Advanced counting (15 Lectures)

Pigeon-hole principle, generalized pigeon-hole principle and its applications, Erdos- Szekers theorem on monotone subsequences, A theorem of Ramsey. Inclusion Exclusion Principle and its applications. Derangement. Permutations with Forbidden Positions, Restricted Positions and Rook Polynomials.

Unit III.Recurrence Relations (15 Lectures)

The Fibonacci sequence, Linear homogeneous recurrence relations with constant coefficient. Proof of the solution in case of distinct roots and statement of the theorem giving a general solution (in case of repeated roots), Iteration and Induction. Ordinary generating Functions, Exponential Generating Functions, algebraic manipulations with power series, generating functions for counting combinations with and without repetitions, exponential generating function for bell numbers, applications to counting, use of generating functions for solving recurrence relations.

Unit IV. Polyas Theory of counting (15 Lectures)

Equivalence relations and orbits under a permutation group action. Orbit stabiliser theorem, Burnside Lemma and its applications, Cycle index, Polyas Formula, Applications of Polyas Formula.

Recommended Text Books

1. D. M. Burton, *Introduction to Number Theory*, McGraw-Hill.

2. Nadkarni and Telang, Introduction to Number Theory
3. V. Krishnamurthy:*Combinatorics: Theory and applications*, Affiliated East-West Press.
4. Richard A. Brualdi: *Introductory Combinatorics*, Pearson.
5. A. Tucker: *Applied Combinatorics*, John Wiley & Sons.
6. Norman L. Biggs: *Discrete Mathematics*, Oxford University Press.
7. Kenneth Rosen: *Discrete Mathematics and its applications*, Tata McGraw Hills.
8. Sharad S. Sane, *Combinatorial Techniques*, Hindustan Book Agency, 2013.

PSMT105/PAMT105 SET THEORY AND LOGIC

Unit I. Introduction to logic (15 Lectures)

Statements, Propositions and Theorems, Truth value, Logical connectives and Truth tables, Conditional statements, Logical inferences, Methods of proof, examples.

Basic Set theory: Union, intersection and complement, indexed sets, the algebra of sets, power set, Cartesian product, relations, equivalence relations, partitions, discussion of the example congruence modulo $-m$ relation on the set of integers.

Unit II. Sets and functions (15 Lectures)

Functions, composition of functions, surjections, injections, bijections, inverse functions, Cardinality Finite and infinite sets, Comparing sets, Cardinality, $|A| < |P(A)|$, Schroeder-Bernstein theorem (with Proof) , Countable sets, Uncountable sets, Cardinalities of $\mathbb{N}$, $\mathbb{N} \times \mathbb{N}$, $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{R} \times \mathbb{R}$.

Unit III. Partial order (15 Lectures)

Order relations, order types, partial order, total order, Well ordered sets, Principle of Mathematical Induction, Russells paradox, Statements of the Axiom of Choice, the Well Ordering Principle, Zorns lemma, applications of Zorns lemma to maximal ideals and to bases of vector spaces.

Unit IV. Lattices (15 Lectures)

Mobius inversion formula on a partially ordered set, Hasse Diagrams of a partially ordered set, Lattices, Distributive and Modular Lattices, complements, Boolean Algebra, Boolean expressions, Only elementary Applications.

Recommended Text Books

1. Robert R. Stoll: *Set theory and logic*, Freeman & Co.

2. James Munkres: *Topology*, Prentice-Hall India;

3. J. F. Simmons, *Introduction to Topology and real analysis.*

4. Richard A. Brualdi: *Introductory Combinatorics*, Pearson.

5. Kenneth Rosen: *Discrete Mathematics and its applications*, Tata McGraw Hills.

6. Larry J. Gerstein: *Introduction to mathematical structures and proofs*, Springer.

7. Joel L. Mott, Abraham Kandel, Theodore P. Baker: *Discrete mathematics for Computer scientists and mathematicians*, Prentice-Hall India.

8. Robert Wolf: *Proof, logic and conjecture, the mathematicians toolbox*, W. H. Freemon.

SEMESTER II

All Results have to be done with proof unless otherwise stated.

PSMT201/PAMT201 ALGEBRA II

Unit I. Groups, group Homomorphisms (15 lectures)

Review: Groups, subgroups, normal subgroups, center $Z(G)$ of a group. The kernel of a homomorphism is a normal subgroup. Cyclic groups. Lagrange's theorem. The product set $HK := \{hk \,|\, h \in H \;\&\; k \in K\}$ of two subgroups of a group G. Examples of groups such as Permutation groups, Dihedral groups, Matrix groups, U_n−the group of units of $\mathbb{Z}_n$ (no questions be asked).

Quotient groups. First Isomorphism Theorem and the following two applications (reference: *Algebra* by Michael Artin)

1. Let $\mathbb{C}^*$ be the multiplicative group of non-zero complex numbers and $\mathbb{R}^{>0}$ be the multiplicative group of positive real numbers. Then the quotient group $\mathbb{C}^*/U$ is isomorphic to $\mathbb{R}^{>0}$.
2. The quotient group $GL_n(\mathbb{R})/SL_n(\mathbb{R})$ is isomorphic to the multiplicative group of non-zero real numbers $\mathbb{R}^*$.

Second and third isomorphism theorems for groups, applications.

Product of groups. The group $\mathbb{Z}_m \times \mathbb{Z}_n$ is isomorphic to $\mathbb{Z}_{mn}$ if and only if the g.c.d. of m and n is 1. *Internal direct product* (A group G is an internal direct product of two normal subgroups H, K if $G = HK$ and every $g \in G$ can be written as $g = hk$ where $h \in H, k \in K$ in a unique way). If H, K are two finite subgroups of a group, then $|HK| = \frac{|H|\,|K|}{|H \cap K|}$. If H, K are two normal subgroups of a group G such that $H \cap K = \{e\}$ and $HK = G$, then G is internal direct product of H and K. If a group G is an internal direct product of two normal subgroups H and K, then G is isomorphic to $H \times K$. (Ref: *Algebra* by MICHAEL ARTIN)

Automorphisms of a group. If G is a group, then $\mathscr{A}(G)$, the set of all automorphisms of G, is a group under composition. If G is a finite cyclic group of order r, then $\mathscr{A}(G)$ is isomorphic to U_r, the groups of all units of $\mathbb{Z}_r$ under multiplication modulo r. For the infinite cyclic group $\mathbb{Z}$, $\mathscr{A}(\mathbb{Z})$ is isomorphic to $\mathbb{Z}_2$. Inner automorphisms of a group. *Topics in Algebra* by I.N.HERSTEIN).

Structure theorem of Abelian groups(statement only) and applications (ref: *A first Course in Abstract Algebra* by J. B. FRALEIGH,)

Unit II. Groups acting on sets, Syllow theorems

Center of a group, centralizer or normalizer $N(a)$ of an element $a \in G$, *conjugacy class* $C(a)$ of a in G. In finite group G, $|C(a)| = o(G)/o(N(a))$ and $o(G) = \sum \frac{o(G)}{o(N(a))}$ where the summation is over one element in each conjugacy class, applications such as: (1) If G is a group of order p^n where p is a prime number, then $Z(G) \neq \{e\}$. (2) Any group of order p^2, where p is a prime number, is Abelian (Reference: *Topics in Algebra* by I.N.HERSTEIN).

Groups acting on sets, Class equation, Cauchy's theorem: If p is a prime number and $p|o(G)$ where G is finite group,then G has an element of order p. (Reference: *Topics in Algebra* by I.N.HERSTEIN).

p-groups, Syllow's theorems and applications:

1. There are exactly two isomorphism classes of groups of order 6.
2. Any group of order 15 is cyclic

(Reference for Syllow's theorems and applications: *Algebra* by MICHAEL ARTIN).

Unit III. Rings, Fields (15 lectures)

Review: Rings (with unity), ideals, quotient rings, prime ideals, maximal ideals, ring homomorphisms, characteristic of a ring, first and second Isomorphism theorems for rings, correspondence theorem for rings (If $f : R \longrightarrow R'$ is a surjective ring homomorphism, then there is a $1-1$ correspondence between the ideals of R containing the ker$f$ and the ideals of R'). Integral domains, construction of the quotient field of an integral domain. (no questions be asked).

For a commutative ring R with unity:

1. An ideal M of R is a maximal ideal if and only if the quotient ring R/M is a field.
2. An ideal N of R is a prime ideal if and only if the quotient ring R/M is an integral domain.
3. Every maximal ideal is a prime ideal.

Definition of field, characteristic of a field, subfield of a field. A field contains a subfield isomorphic to Z_p or $\mathbb{Q}$.

Polynomial ring $F[X]$ over a field, irreducible polynomials over a field. Prime ideals, and maximal ideals of a Polynomial ring $F[X]$ over a field F. A non-constant polynomial $p(X)$ is irreducible in a polynomial ring $F[X]$ over a field F if and only if the ideal $(p(X))$ is a maximal ideal of $F[X]$. Unique Factorization Theorem for polynomials over a field (statement only).

Definition of field extension, algebraic elements, minimal polynomial of an algebraic element, extension of a field obtained by adjoining one algebraic element. Kronecker's theorem: Let F be any field and let $f(X) \in F[X]$ be such that $f(X)$ has no root in F. Then there exists a field E containing F as a subfield such that f has a root in E. Application of Kronecker's theorem: Let F be any field and let $f(X) \in F[X]$. Then there exists a field E containing F as a subfield such that $f(X)$ factorises completely into linear factors in $E[X]$. (reference: *Rings, fields and Groups, An Introduction to Abstract Algebra* by R.B.J.T. ALLENBY).

Finite fields: A finite field of characteristic p contains exactly p^n elements for some $n \in \mathbb{N}$. Existence result for finite fields: For every prime number p and positive integer n, there exists a filed with exactly p^n elements (reference: *Rings, fields and Groups, An Introduction to Abstract Algebra* by R.B.J.T. ALLENBY).

Unit IV. Divisibility in integral domains (15 lectures)

Prime elements, irreducible elements, Unique Factorization Domains, Principle Ideal Domains, Gauss's lemma, $\mathbb{Z}[X]$ is a UFD, irreducibility criterion, Eisenstein's criterion, Euclidean domains. $\mathbb{Z}[\sqrt{-5}]$ is not a UFD.
Reference for Unit IV: MICHAEL ARTIN: *Algebra*, Prentice-Hall India.

Recommended Text Books

1. MICHAEL ARTIN: *Algebra*, Prentice-Hall India.
2. I.N.HERSTEIN: *Topics in Algebra*, Wiley-India.
3. R.B.J.T. ALLENBY: *Rings, fields and Groups, An Introduction to Abstract Algebra*, Elsevier (Indian edition).
4. J. B. FRALEIGH, *A first Course in Abstract Algebra*, Narosa.
5. DAVID DUMMIT, RICHARD FOOT: *Abstract Algebra*, Wiley-India.

PSMT202/PAMT202 TOPOLOGY

Unit I. Topological spaces (15 Lectures)

Topological spaces, basis, sub-basis, product topology (finite factors only), subspace topology, closure, interior, continuous functions, T_1, T_2 spaces, quotient spaces.

Unit II. Connected topological spaces (15 Lectures)

Connected topological spaces, path-connected topological spaces, continuity and connectedness, Connected components of a topological space, Path components of a topological space. Countability Axioms, Separation Axioms, Separable spaces, Lindeloff spaces, Second countable spaces.

Unit III. Compact topological spaces (15 Lectures)

Compact spaces, limit point compact spaces, continuity and compactness, tube lemma, compactness and product topology (finite factors only), local compactness, one point compactification. A compact T_2 space is regular and normal space.

Unit IV. Compact metric spaces, Complete metric spaces (15 Lectures)

Complete metric spaces, Completion of a metric space, total boundedness , compactness in Metric spaces, sequentially compact metric spaces, uniform continuity, Lebesgue covering lemma.

Recommended Text Books

1. JAMES MUNKRES: *Topology*, Pearson.
1. GEORGE SIMMONS: *Topology and Modern Analysis*, Tata Mcgraw-Hill.
2. M.A.ARMSTRONG: *Basic Topology*, Springer UTM.

PSMT203/PAMT203 ANALYSIS II

Unit I. Measures (15 lectures)

Outer measure μ^* on a set X, μ^*-measurable subsets of X (A subset E of a set X with outer measure μ^* is said to be μ^*-measurable if $\mu^*(A) = \mu^*(A \cap E) + \mu^*(A \cap (X \setminus E)\ \forall A \subseteq X$ (definition due to Carathéodory)), the collection Σ of all μ^*-measurable subsets of X form a σ-algebra, measure space (X, Σ, μ).

Volume $\lambda(I)$ of any rectangle in $\mathbb{R}^d$ (for the interval $I = \Pi_{i=1}^{d}(a_i, b_i)$ of $\mathbb{R}^d$,
$\lambda(I) = \Pi_{i=1}^{d}(b_i - a_i)$), Lebesgue's Outer measure m^* in R^d and results:

1. Lebesgue's Outer measure m^* is translation invariant.
2. Let A, B be any two subsets of $\mathbb{R}^d$ with $d(A, B) > 0$. Then $m^*(A \cup B) = m^*(A) + m^*(B)$.
3. For any bounded interval $I = (a, b)$ of $\mathbb{R}$, $m^*(I) = b - a$.
4. For any interval I of $\mathbb{R}^d$, $m^*(I) = \lambda(I)$.

The σ-algebra $\mathscr{M}$ of all Lebesgue measurable subsets of $\mathbb{R}^d$, existence of a non-measurable set.

Unit II. Measurable functions and integration of non-negative functions (15 lectures)

Measurable functions on (X, Σ, μ), simple functions, properties of measurable functions. If f is a non-negative measurable function, then there exists a monotone increasing sequence (s_n) of non-negative simple measurable functions converging to pointwise to the function f.

Integral of a non-negative simple measurable function with respect to a measure μ and properties, integral of a non-negative measurable function, Monotone convergence theorem. If $f \geq 0$ and $g \geq 0$ are measurable functions, then $\int (f + g)\, d\mu = \int f\, d\mu + \int g\, d\mu$.

Unit III. Dominated convergence theorem and $L^1(\mu)$, $L^2(\mu)$ spaces. (15 lectures) integrable functions (A measurable function f is integrable (or summable) if $\int f^+ \, d\mu < \infty$ and $\int f^- \, d\mu < \infty$) and linearity properties.

Fatous lemma, Dominated convergence theorem, Completeness of $L^1(\mu), L^2(\mu)$.

Lebesgue and Riemann integrals: A bounded real valued function on $[a, b]$ is Riemann integrable if and only if it is continuous at almost every point of $[a, b]$; in this case, its Riemann integral and Lebesgue integral coincide.

Unit IV. Signed measures, Radon-Nykodym theorem (15 lectures)

Borel σ- algebra of $\mathbb{R}^d$. Any closed subset and any open subset of $\mathbb{R}^d$ is Lebesgue measurable. Every Borel set in $\mathbb{R}^d$ is Lebesgue measurable. For any bounded Lebesgue measurable subset E of $\mathbb{R}^d$, and any $\epsilon > 0$, there exist a compact set K and open set U in $\mathbb{R}^n$ such that $K \subseteq E \subseteq U$ such that $m(U \setminus K) < \epsilon$. For any Lebesgue measurable subset E of $\mathbb{R}^d$, there exist Borel sets F, G in $\mathbb{R}^n$ such that $F \subseteq E \subseteq G$ & $m(E \setminus F) = 0 = m(G \setminus E)$.

Complex valued Lebesgue measurable functions on $\mathbb{R}^d$, Lebesgue integral of complex valued measurable functions, Approximation of Lebesgue integrable functions by continuous functions with compact support.

Signed measures, positive measurable sets and negatives measurable sets for a signed measure. Notion of Absolutely continuity $\nu << \mu$ of a signed measure ν with respect to a positive measure μ, Hahn Decomposition theorem, Jordan Decomposition of a signed measure, examples.

Radon-Nykodym theorem and Radon-Nykodym derivative.
Reference for unit IV:

1. H.L. Royden, *Real Analysis* by , PHI.
2. Andrew Browder, *Mathematical Analysis, An Introduction*, Springer Undergraduate Texts in Mathematics.

See also *Measure, Integral and probability* by M. Capinski and E. Kopp, Springer (SUMS) for a proof of the Radon-Nykodym theorem.

Recommended Text Books:

1. Andrew Browder, *Mathematical Analysis, An Introduction*, Springer Undergraduate Texts in Mathematics.
2. H.L. Royden, *Real Analysis*, PHI.
3. Walter Rudin, *Real and Complex Analysis*, McGraw-Hill India, 1974.

PSMT204/PAMT204 DIFFERENTIAL EQUATIONS

Unit I. Picards Theorem (15 Lectures)

Existence and Uniqueness of solutions to initial value problem of first order ODE- both autonomous, non-autonomous (Picard's Theorem), Picard's scheme of successive Approximations, system of first order linear ODE with constant coefficients and variable coefficients, reduction of an n-th order linear ODE to a system of first order ODE.

Unit II. Ordinary Differential Equations (15 Lectures)

Existence and uniqueness results for an n-th order linear ODE with constant coefficients and variable coefficients, linear dependence and independence of solutions of a homogeneous n-th order linear ODE, Wronskian matrix, Lagrange's Method (variation of parameters), algebraic properties of the space of solutions of a non-homogeneous n-th order linear ODE.

Unit III. Sturm-Liouville theory (15 Lectures)

Solutions in the form of power series for second order linear equations of Legendre and Bessel, Legendre polynomials, Bessel functions.

Sturm- Liouville Theory: Sturm-Liouville Separation and comparison Theorems, Oscillation properties of solutions, Eigenvalues and eigenfunctions of Sturm-Liouville Boundary Value Problem, the vibrating string.

Unit IV. First Order Partial Differential Equation (15 Lectures)

First order quasi-linear PDE in two variables: Integral surfaces, Characteristic curves, Cauchys method of characteristics for solving First order quasi-linear PDE in two variables.

First order non-linear PDE in two variables, Characteristic equations, Characteristic strip, Cauchy problem and its solution for first order non linear PDE in two variables.

Note : ODE stands for Ordinary Differential Equations and PDE stands for Partial Differential Equations.

Recommended Text Books:

1. For Units I and II:
   (a) E.A. Codington, N. Levinson: *Theory of ordinary differential Equations*, Tata McGraw-Hill, India.
   (b) Morris W. Hirsch and Stephen Smale, *Differential Equations, Dynamical Systems, Linear Algebra*, Elsevier.
2. For Unit III: G.F. Simmons: *Differential equations with applications and historical notes*, McGraw-Hill international edition.
3. For Unit IV:
   (a) Fritz John: *Partial Differential Equations*, Springer.
   (b) Hurewicz W.: *Lectures on ordinary differential equations*, M.I.T. Press.

PSMT205/PAMT205 PROBABILITY THEORY

Unit I. Probability basics (15 Lectures)

Modelling Random Experiments: Introduction to probability, probability space, events.

Classical probability spaces: uniform probability measure, fields, finite fields, finitely additive probability, Inclusion-exclusion principle, σ-fields, σ-fields generated by a family of sets, σ-field of Borel sets, Limit superior and limit inferior for a sequence of events.

Unit II. Probability measure (15 Lectures)

Probability measure, Continuity of probabilities, First Borel-Cantelli lemma, Discussion of Lebesgue measure on σ-field of Borel subsets of assuming its existence, Discussion of Lebesgue integral for non-negative Borel functions assuming its construction.

Discrete and absolutely continuous probability measures, conditional probability, total probability formula, Bayes formula, Independent events.

Unit III. Random variables (15 Lectures)

Random variables, simple random variables, discrete and absolutely continuous random variables, distribution of a random variable, distribution function of a random variable, Bernoulli, Binomial, Poisson and Normal distributions, Independent random variables, Expectation and variance of random variables both discrete and absolutely continuous.

Unit IV. Limit Theorems (15 Lectures)

Conditional expectations and their properties, characteristic functions, examples, Higher moments examples, Chebyshev inequality, Weak law of large numbers, Convergence of random variables, Kolmogorov strong law of large numbers (statement only), Central limit theorem (statement only).

Recommended Text Books:

1. M. Capinski, Tomasz Zastawniak: Probability Through Problems.
2. J. F. Rosenthal: *A First Look at Rigorous Probability Theory*, World Scientific.
3. Kai Lai Chung, Farid AitSahlia: *Elementary Probability Theory*, Springer Verlag.

Scheme of Examination

The scheme of examination for the syllabus of Semesters I & II of M.A./M.Sc. Programme (CBCS) in the subject of Mathematics will be as follows. There shall be a Semester-end External Theory examination with 100 marks to be conducted by the University.

(i) Duration:- Examination shall be of 3 Hours duration.
(ii) Theory Question Paper Pattern:-

1. There shall be five questions each of 20 marks.
2. On each unit there will be one question and the fifth one will be based on entire syllabus.

3. All questions shall be compulsory with internal choice within each question.
4. Each question may be subdivided into sub-questions a, b, c, ·· and the allocation of marks depend on the weightage of the topic.
5. Each question will be of 30 marks when marks of all the sub-questions are added (including the options) in that question.

Questions		Marks
Q1	Based on Unit I	20
Q2	Based on Unit II	20
Q3	Based on Unit III	20
Q4	Based on Unit IV	20
Q5	Based on Units I,II,III& IV	20
	Total Marks	100

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