MSc Mathematics Part II Revised Syllabus

Paper I: Algebra – II : Section I

Unit I. Groups (15 Lectures)

Simple groups, A_5 is simple, Solvable groups, Solvability of all groups of order less than 60, Nilpotent groups, Zassenhaus lemma, Jordan-Holder theorem, Direct and Semi-direct products,
Examples:
(i)The group of affine transformations $X \to AX + B$ as semi-direct product of the group of linear transformations acting on the group of translations.
(ii) Dihedral group D_{2n} as semi-direct product of Z_2 and Z_n.

Unit II. Representation of finite groups (15 Lectures)

Linear representations of a finite group over complex numbers, The group ring, Complete reducibility, Characters, Orthogonality, Character tables with emphasis on examples of groups of small order.

Unit III. Modules (15 Lectures)

Modules over rings, Submodules, Quotient modules, Free modules, Homomorphisms, kernels, Images, Cokernels, Noether isomorphism theorems, Matrix representations of homomorphisms between free modules, Abelian groups as modules over the ring of integers, Structure theorem for finitely generated abelian groups.

Unit IV. Modules over PID (15 Lectures)

Structure theorem for finitely generated modules over a PID, Application to Jordan canonical form and Rational Canonical form.

Reference Books:

(1) DUMMIT and FOOTE, Abstract Algebra, John Wiley and Sons, 2010.

(2) SERGE LANG, Algebra, Springer Verlag, 2004

(3) JACOBSON, Basic Algebra, Dover, 1985.

(4) M. ARTIN, Algebra, Prentice Hall of India, 1994

Section II

Unit I. Algebraic Extensions (15 Lectures)

Definition of field extensions, Algebraic elements, Algebraic extensions, Finite extensions, Degree of an algebraic element, Minimal polynomial, Degree of a field extension, Extension of a field obtained by adjoining one algebraic element, Splitting field for a set of polynomials, Composition of two sub extensions of an extension, Existence of algebraic closure.

Unit II. Normal and Separable Extensions (15 Lectures)

Separable elements, Separable extensions, In characteristic 0 all extensions are separable, Separable extensions form a distinguished class, Existence of separable closure, Normal extensions, Finite fields : existence and uniqueness.

Unit III. Galois Theory (15 Lectures)

Galois extensions, Galois groups, Subgroups, Fixed fields, Galois correspondence, Fundamental theorem of Galois theory, Galois theory - Sylow theory proof that the field of complex numbers is algebraically closed.

Unit IV. Applications (15 Lectures)

Straight edge and compass constructions, Impossibility of trisection of angle $\frac{\pi}{3}$, Solvability by radicals in terms of Galois group, Insolvability of a general quintic.

Recommended books:

(1) DUMMIT and FOOTE, Algebra: John Wiley and Sons, 2010.

(2) SERGE LANG, Algebra, 2004.

(3) N. JACOBSON, Algebra, Dover, 1985.

(3) M. ARTIN, Algebra, Prentice Hall 1994.

Paper II: Analysis-II and Fourier Analysis :

Section I

Unit I. Riemann Integration (15 Lectures)

Riemann Integration over a rectangle in $\mathbb{R}^n$, Riemann Integrable functions, Continuous functions are Riemann integrable, Measure zero sets, Lebesgue's Theorem (statement only), Fubini's Theorem

Unit II. Lebesgue Measure (15 Lectures)

Exterior measure in $\mathbb{R}^n$, Construction of Lebesgue measure in $\mathbb{R}^n$, Lebesgue measurable sets in $\mathbb{R}^n$, The sigma algebra of Lebesgue measurable sets, Borel measurable sets, Existence of non-measurable sets.

Unit III. Lebesgue Integration (15 Lectures)

Measurable functions, Simple functions, Properties of measurable functions, Lebesgue integral of complex valued measurable functions, Lebesgue integrable functions, Approximation of integrable functions by continuous functions with compact support.

Unit IV. Limit Theorems (15 Lectures)

Monotone convergence theorem, Bounded convergence theorem, Fatou's lemma, Dominated convergence theorem, Completeness of L^1.

Reference Books

(1) Stein and Shakarchi, Measure and Integration, Princeton Lectures in Analysis, Princeton University Press.

(2) Andrew Browder, Mathematical Analysis an Introduction, Springer Undergraduate Texts In Mathematics, 1999.

(3) Walter Rudin, Real and Complex Analysis, McGraw-Hill India, 1974.

Section II

Unit I. Fourier Series (15 Lectures)

Periodic Functions, Fourier series of L^1 functions, Riemann Lebesgue Lemma, Fourier series of periodic continuous functions, Uniqueness of Theorem, Dirichlet Kernel, Dirichlet Theorem on point wise convergence of Fourier series, Fejer Kernel, Fejer's Theorem, Denseness of trigonometric polynomials in $L^2[-\pi, \pi]$.

Unit II. Hilbert Spaces (15 Lectures)

Hilbert spaces, Separable Hilbert spaces, Examples, Cauchy-Schwartz inequality, Gram Schmidt, Orthonormal bases, Existence of orthonormal bases, Equivalent characterizations, Bessel's inequality, Parseval's Identity, Orthogonal decomposition.

Unit III. Riesz Fischer Theorem (15 Lectures)

The Hilbert space $L^2[-\pi, \pi]$, Orthonormal basis for $L^2[-\pi, \pi]$, Separability of $L^2[-\pi, \pi]$ Convergence of Fourier series in the L^2 norm, Best Approximation, Bessel's inequality for L^2 functions, The sequence space ℓ^2, Unitary isomorphism from $L^2[-\pi, \pi]$ onto the sequence space of square summable complex sequences.

Unit IV. Dirichlet Problem (15 Lectures)

Laplacian, Harmonic functions, Dirichlet Problem for the unit disc, The Poisson kernel, Abel summability, Abel summability of periodic continuous functions, Weierstrass Approximation Theorem as application, Solution of Dirichlet problem.

Reference Books

(1) Stein and Shakarchi, Fourier Analysis an Introduction, Princeton Lectures in Analysis: Princeton University Press, 2003.

(1) Stein and Shakarchi, Real , Measure and Integration, Princeton Lectures in Analysis: Princeton University Press, 2003.

(2) Richard Beals, Analysis an Introduction:, Cambridge University Press, 2004.

(4) R.E.Edwards, Fourier Series, A Modern Introduction (Volume I):, Springer GTM, 1982.

Paper III: Differential Geometry and Functional Analysis :

Section I

Unit I. Complete Metric Spaces (15 Lectures)

Review of complete metric spaces, Examples, Completion of a metric space, Equicontinuity, Ascoli-Arzela Theorem, Baire spaces, Baire's theorem for compact Hausdorff and complete metric spaces, Application to a sequence of continuous functions converging point wise to a limit function.

Unit II. Normed Linear Spaces (15 Lectures)

Normed Linear spaces, Banach spaces, Quotient space of a normed linear space, Examples such as the sequence spaces ℓ^p, Space of linear transformations, Function spaces L^p, Holder and Mankowski inequalities, Finite dimensional normed linear spaces, Equivalent norms, Riesz Lemma and application to infinite dimensional normed linear spaces.

Unit III. Bounded Linear Transformations (15 Lectures)

Bounded linear transformations, Equivalent characterizations, Examples, The space $B(X,Y)$, Completeness of $B(X,Y)$ when Y is complete, The dual space, Dual spaces of ℓ^1, ℓ^p and $C[a,b]$.

Unit IV. Basic Theorems (15 Lectures)

Open mapping theorem, Closed graph theorem, Uniform boundedness Principle, Hahn-Banach Theorem and applications.

Recommended Books

(1) E. Keryszig, Introductory Functional Analysis with Applications, Wiely India, 2010.

(2) G. F. Simmons, Introduction to Topology and Modern Analysis, Tata Mac Grahill , 2004.

(3) W. Rudin, Real and Complex Analysis, Mcgraw hill, India, 1966.

(4) Pedersen W. K., Analysis Now, Springer GTM, 1989.

(5) Royden, Real Analysis, Macmillian, 1968.

Section II

Unit I. Geometry of $\mathbb{R}^n$ (15 Lectures)

Hyperplanes in $\mathbb{R}^n$, Lines and planes in $\mathbb{R}^3$, Parametric equations, Inner product in $\mathbb{R}^n$, Orthonormal basis, Orthogonal transformations, Orthogonal matrices, The groups $SO(2)$, $SO(3)$, Reflections and rotations, Isometries of $\mathbb{R}^n$.

Unit II. Curves (15 Lectures)

Regular curves in $\mathbb{R}^2$ and $\mathbb{R}^3$, Arc length parametrization, Signed curvature for plane curves, Curvature and torsion for space curves, Serret-Frenet equations, Fundamental theorem for space curves.

Unit III. Regular Surfaces (15 Lectures)

Regular surfaces in $\mathbb{R}^3$, Examples, Surfaces as level sets, Surfaces as graphs, Surfaces of revolution, Tangent space to a surface at a point, Equivalent definitions, Smooth functions on a surface, Differential of a smooth function defined on a surface, Orientable surfaces.

Unit IV. Curvature (15 Lectures)

The first fundamental form, The Gauss map, The shape operator of a surface at a point, Self adjointness of the shape operator, The second fundamental form, Principle curvatures and vectors, Euler's formula, Meusnier's Theorem, Normal curvature, Gaussian curvature and mean curvature, Computation of Gaussian curvature, Isometries of surfaces, Gauss's Theorem, Covariant differentiation, Geodesics.

Reference Books

(1) M. DoCarmo, Differential geometry of curves and surfaces, Princeton University Press, 1976.

(2) S. Montiel and A. Ros, Curves and Surfaces, AMS Graduate Studies in Mathematics, 2009.

(3) A. Pressley, Elementary Differential Geometry, Springer UTM, 2009.

(3) J. Thorpe, Elementary Topics in Differential Geometry, Springer UTM, 2007.

Paper IV: Numerical Analysis :

Section I

Unit I. Basics of Numerical Analysis (15 Lectures)

Error in numerical computations, Absolute, Relative and percentage errors, Round off errors, Truncation errors, Inherent errors, Representation of numbers: Binary, Octal, Decimal, Hexadecimal.

Unit II. Solution of Algebraic & Transcendental Equations (15 Lectures)

Iteration method, Newton-Rhapson method, Muller's method, Ramanujan's method, Chebyshev method, Rate of convergence, Solution of polynomial equations, Solutions of nonlinear equations: Seidel iteration, Newton-Rhapson method.

Unit III. System of linear equations and solutions (15 Lectures)

Gaussian elimination, Gauss-Jordan method, Triangularization method: Crout's method, Cholesky method, Iteration methods: Gauss-Jacobi, Gauss-Seidel, Eigen value problem for matrices: Power method, Inverse power method, Jacobi or Given's method for real symmetric matrices, Singular value decomposition.

Unit IV. Interpolation (15 Lectures)

Difference operators, Lagrangian interpolation formula, Divided difference formula, Newton's forward and backward difference interpolation formulae, Error in interpolating polynomial, Spline interpolation, Numerical differentiation, Maxima and minima of interpolating polynomial.

Note: Pre knowledge of C or C^{++} is essential.

Section II

Unit I. Numerical Integration (15 Lectures)

Numerical Integration: Newton-Cotes quadrature formula, Trapezoidal rule, Simpson's one third and three eightth rules, Errors in trapezoidal and Simpson's rules, Romberg's method, Gaussian quadrature, Multiple integrals.

Unit II. Approximation of functions (15 Lectures)

Least squares approximation, Weighted least squares method, Gram-Schimdt orthogonalizing process, Least squares approximation by Chebyshev polynomials, Discrete Fourier transforms, Fast Fourier Transforms.

Unit III. Differential Equations (15 Lectures)

Differential equations: Solutions of linear differential equations with constant coefficients, Series solutions, Euler's modified method, Runge-Kutta methods, Predictor corrector Methods, Stability of numerical methods.

Unit IV. Numerical Solutions of partial differential Equations (15 Lectures)

Classification, Finite difference approximations to derivatives, Numerical methods of solving elliptic, Parabolic and hyperbolic equations.

Reference Books

(1) H.M.Antia , Numerical Analysis for scientists and engineers, TMH 1991.

(2) Jain, Iyengar, Numerical methods for scientific and engineering problems, New Age International, 2007.

(3) S.S.Sastry, Introductory methods of numerical analysis, Prentice-Hall India, 1977.

(3) K.E. Atkinson, An introduction to numerical analysis, John Wiley and sons, 1978.

Paper V: Graph Theory

Section I

Unit I. Connectivity (15 Lectures)

Overview of Graph theory-Definition of basic concepts such as Graph, Subgraphs, Adjacency and incidence matrix, Degree, Connected graph, Components, Isomorphism, Bipartite graphs etc., Shortest path problem-Dijkstra's algorithm, Vertex and Edge connectivity-Result $\kappa \leq \kappa^{'} \leq \delta$, Blocks, Block-cut point theorem, Construction of reliable communication network, Menger's theorem.

Unit II. Trees (15 Lectures)

Trees-Cut vertices, Cut edges, Bond, Characterizations of Trees, Spanning trees, Fundamental cycles, Vector space associated with graph, Cayley's formula, Connector problem- Kruskal's algorithm, Proof of correctness, Binary and rooted trees, Huffman coding, Searching algorithms-BFS and DFS

Unit III. Eulerian and Hamiltonian Graphs (15 Lectures)

Eulerian Graphs- Characterization of Eulerian Graph, Randomly Eulerian graphs, Chinese postman problem- Fleury's algorithm with proof of correctness. Hamiltonian graphs- Necessary condition, Dirac's theorem, Hamiltonian closure of a graph, Chvatal theorem, Degree majorisation, Maximum edges in a non-hamiltonian graph, Traveling salesman problem.

Unit IV. Matching and Ramsey Theory (15 Lectures)

Matchings-augmenting path, Berge theorem, Matching in bipartite graph, Hall's theorem, Konig's theorem, Tutte's theorem, Personal assignment problem, Independent sets and covering- $\alpha + \beta = p$, Gallai's theorem, Ramsey theorem-Existence of $r(k,l)$, Upper bounds of $r(k,l)$, Lower bound for $r(k,l) \geq 2^{\frac{m}{2}}$ where $m = \min\{k,l\}$, Generalize Ramsey numbers-$r(k_1, k_2, \ . \ . \ . \ , k_n)$, Graph Ramsey theorem, Evaluation of $r(G,H)$ when for simple graphs $G = P_3$, $H = C_4$.

Section II

Unit I. Graph Coloring (15 Lectures)

Line Graphs, Edge coloring-edge chromatic number, Vizing theorem, Timetabling problem, Vertex coloring- Vertex chromatic number, Critical graphs, Brook's theorem, Chromatic polynomial of a graph- $\pi_k(G) = \pi_k(G - e) - \pi_k(G \centerdot e)$ properties of chromatic polynomial of a graph, Existence of a triangle free graph with high vertex chromatic number, Mycieleski's construction.

Unit II. Planner Graph (15 Lectures)

Planar graph, Plane embedding of a graph, Stereographic projection, Dual of a plane graph, Euler formula, Non planarity of K_5 and $K_{3,3}$, Outer planar graph, Five color theorem, Sub-division, Kuratowski's theorem(Without Proof).

Unit III. Flow Theory (15 Lectures)

Directed graphs, Directed paths and directed cycle, Tournament, Networks, Max flow min cut theorem, Ford- Fulkerson Theorem and Algorithm.

Unit IV. Characteristic Polynomials (15 Lectures)

Spectrum of a graph, Characteristic polynomial of a graph, Coefficients of characteristic polynomial of a graph, Adjacency algebra $A(G)$ of a graph G, Dimension of $A(G) \geq diam(G) + 1$, A connected graph with diameter d has at least $d + 1$ eigen values, Circulant matrix, Determination of spectrum of graphs.

Reference Books

(1) J. A. Bondy and U. S. R. Murty, Graph Theory with Applications, The Macmillan Press, 1976.

(1) J. A. Bondy and U. S. R. Murty, Graph Theory GTM Springer, 2008.

(2) M. Behzad and G. Chartrand, Introduction to the Theory of Graphs, Allyn and Becon Inc., Boston, 1971.

(3) K. Rosen, Discrete Mathematics and its Applications, Tata-McGraw Hill, 2011.

(4) D.B.West, Introduction to Graph Theory, Prentice-Hall, India, 2009.

(5) N. Biggs, Algebraic Graph Theory, Prentice-Hall, India.