

**UNIVERSITY OF MUMBAI**

**No. UG/97 of 2015-16**

**CIRCULAR:-**

A reference is invited to the Syllabi relating to the B.Sc. degree course , **vide** this office Circular No. UG/05 of 2012, dated 15<sup>th</sup> February, 2012 and the Principals of affiliated Colleges in Science are hereby informed that the recommendation made by the Faculty of Science at its meeting held on 11<sup>th</sup> August, 2015 has been accepted by the Academic Council at its meeting held on 31<sup>st</sup> August, 2015 **vide** item No. 4.6 and that in accordance therewith, the revised syllabus as per Credit Based Semester and Grading System for the T.Y. B.A./B.Sc programme in Mathematics (Sem.V & Sem. VI), which is available on the University's web site ([www.mu.ac.in](http://www.mu.ac.in)) that the same has been brought into force with effect from the academic year 2016-17.

MUMBAI – 400 032  
13<sup>th</sup> October, 2015

REGISTRAR

To,

The Principals of affiliated Colleges in Science and the Heads of the recognized Science Institution concerned.

**A.C/4.6/31/08/2015**

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No. UG/97 -A of 2015-16

MUMBAI-400 032

13<sup>th</sup> October, 2015

Copy forwarded with compliments for information to :-

- 1) The Dean, Faculty of Science,
- 2) The Director, Board of Colleges and University Development,
- 3) The Professor-cum-Director, Institute of Distance and Open Learning(IDOL),
- 4) The Controller of Examinations,
- 5) The Co-Ordinator, University Computerization Centre.

  
REGISTRAR

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# UNIVERSITY OF MUMBAI



**Syllabus for: T. Y. B. Sc. /T. Y. B.A.  
Program: B.Sc. /B.A.**

**Course: Mathematics**

(Credit Based Semester and Grading  
System with effect from the Academic year  
(2016–2017))

**Revised Syllabus in Mathematics**  
**Credit Based Semester and Grading System**  
**Third Year B. Sc. / B. A. 2016-2017**

**SEMESTER V**

| <b>Integral Calculus</b>                                 |      |   |         |         |
|--|------|---|---------|---------|
| Course Code  | UNIT | TOPICS  | Credits | L/ Week |
| USMT501<br>UAMT501                                       | I    | Multiple Integrals  | 2.5     | 3       |
|  | II   | Line Integrals  |         |         |
|  | III  | Surface Integrals   |         |         |
| <b>Linear Algebra</b>                                    |      |   |         |         |
| USMT502<br>UAMT502                                       | I    | Quotient Spaces and Orthogonal Linear Transformations     | 2.5     | 3       |
|  | II   | Eigenvalues and Eigen vectors                             |         |         |
|  | III  | Diagonalisation   |         |         |
| <b>Topology of Metric Spaces</b>                         |      |   |         |         |
| USMT503<br>UAMT503                                       | I    | Metric spaces   | 2.5     | 3       |
|  | II   | Closed sets, Limit Points and Sequences                   |         |         |
|  | III  | Compact sets  |         |         |
| <b>Numerical Analysis I (Elective A)</b>                 |      |   |         |         |
| USMT5A4<br>UAMT5A4                                       | I    | Errors Analysis and Transcendental & Polynomial Equations | 2.5     | 3       |
|  | II   | Transcendental and Polynomial Equations                   |         |         |
|  | III  | Linear System of Equations                                |         |         |
| <b>Number Theory and its applications I (Elective B)</b> |      |   |         |         |
| USMT5B4<br>UAMT5B4                                       | I    | Congruences and Factorization                             | 2.5     | 3       |
|  | II   | Diophantine equations and their solutions                 |         |         |
|  | III  | Primitive Roots and Cryptography                          |         |         |
| <b>Graph Theory (Elective C)</b>                         |      |   |         |         |
| USMT5C4<br>UAMT5C4                                       | I    | Basics of Graphs  | 2.5     | 3       |
|  | II   | Trees   |         |         |
|  | III  | Eulerian and Hamiltonian graphs                           |         |         |

| <b>Basic Concepts of Probability and Random Variables (Elective D)</b> |   |  |                |               |
|--|---|--|----------------|---------------|
| USMT5D4<br>UAMT5D4   | I   | Basic Concepts of Probability and Random Variables           | 2.5            | 3             |
|  | II  | Properties of Distribution function , Joint Density function |                |               |
|  | III   | Weak Law of Large Numbers                                    |                |               |
| <b>Course</b>  | <b>Practicals</b>   |  | <b>Credits</b> | <b>L/Week</b> |
| USMTP05<br>UAMTP05   | Practicals based on USMT501/UAMT501 and USMT502/UAMT502   |  | 3              | 6             |
| USMTP06<br>UAMTP06   | Practicals based on USMT503/UAMT503 and USMT5A4/UAMT5A4 OR SMT5B4/UAMT5B4 USMT5C4/UAMT5C4 OR SMT5D4/UAMT5D4 |  | 3              | 6             |

**Revised Syllabus in Mathematics**  
**Credit Based Semester and Grading System**  
**Third Year B. Sc. / B. A. 2016-2017**

**SEMESTER VI**

| <b>Real and Complex Analysis</b>                          |      |  |         |         |
|---|------|--|---------|---------|
| Course Code   | UNIT | TOPICS   | Credits | L/ Week |
| USMT601<br>UAMT601  | I    | Sequence and series of functions                         | 2.5     | 3       |
|   | II   | Introduction to Complex Analysis                         |         |         |
|   | III  | Complex power series                                     |         |         |
| <b>Algebra</b>  |      |  |         |         |
| USMT602<br>UAMT602  | I    | Group Theory   | 2.5     | 3       |
|   | II   | Ring Theory  |         |         |
|   | III  | Polynomial Rings and Field theory                        |         |         |
| <b>Metric Topology</b>                                    |      |  |         |         |
| USMT603<br>UAMT603  | I    | Complete metric spaces                                   | 2.5     | 3       |
|   | II   | Continuous functions on metric spaces                    |         |         |
|   | III  | Connected sets   |         |         |
| <b>Numerical Analysis II (Elective A)</b>                 |      |  |         |         |
| USMT6A4<br>UAMT6A4  | I    | Interpolation  | 2.5     | 3       |
|   | II   | Polynomial Approximations and Numerical Differentiation  |         |         |
|   | III  | Numerical Integration                                    |         |         |
| <b>Number Theory and Its Applications II (Elective B)</b> |      |  |         |         |
| USMT6B4<br>UAMT6B4  | I    | Quadratic Reciprocity                                    | 2.5     | 3       |
|   | II   | Continued Fractions                                      |         |         |
|   | III  | Pell's equation, Arithmetic function and Special numbers |         |         |
| <b>Graph Theory and Combinatorics (Elective C)</b>        |      |  |         |         |
| USMT6C4<br>UAMT6C4  | I    | Colorings of graph                                       | 2.5     | 3       |
|   | II   | Planar graph   |         |         |
|   | III  | Combinatorics  |         |         |
| <b>Operations Research</b>                                |      |  |         |         |

|                    |  |                       |         |        |
|--------------------|--|-----------------------|---------|--------|
| USMT6D4<br>UAMT6D4 | I  | Linear Programming I  | 2.5     | 3      |
|                    | II   | Linear programming II |         |        |
|                    | III  | Queuing Systems       |         |        |
| COURSE             | <b>Practicals</b>  |                       | Credits | L/Week |
| USMTP07<br>UAMTP07 | Practicals based on USMT601/UAMT601 and USMT602/UAMT602  |                       | 3       | 6      |
| USMTP08<br>UAMTP08 | Practicals based on USMT603/UAMT603 and USMT6A4/UAMT6A4 OR UT6B4/UAMT6B4 USMT6C4/UAMT6C4 OR SMT6D4/UAMT6D4 |                       | 3       | 6      |

- Note:**
1. USMT501/UAMT501, USMT502/UAMT502, USMT503/UAMT503 are compulsory courses for Semester V.
  2. Candidate has to opt one Elective Course from USMT5A4/ UAMT5A4, USMT5B4/ UAMT5B4, USMT5C4/ UAMT5C4 and USMT5D4/ UAMT5D4 for Semester V.
  3. USMT601/UAMT601, USMT602/UAMT602, USMT603/UAMT603 are compulsory courses for Semester VI.
  4. Candidate has to opt one Elective Course from USMT6A4/ UAMT6A4, USMT6B4/ UAMT6B4, USMT6C4/ UAMT6C4 and USMT6D4/ UAMT6D4 for Semester VI.
  5. Passing in theory and practical shall be separate.

**Teaching Pattern for B. Sc /B.A.:**

1. Three lectures per week per course (1 lecture/period is of 48 minutes duration).
2. One practical of three periods per week per course (1 lecture/period is of 48 minutes duration).

**SEMESTER V**  
**Course: Integral Calculus**  
**Course Code: USMT501 / UAMT501**

**ALL Results have to be done with proof unless otherwise stated.**

**Unit I. Multiple Integrals (15L)**

Definition of double (respectively: triple) integral of a function bounded on a rectangle (respectively: box), Geometric interpretation as area and volume. Fubini's Theorem over rectangles and any closed bounded sets, Iterated Integrals. Basic properties of double and triple integrals proved using the Fubini's theorem such as; Integrability of the sums, scalar multiples, products, and (under suitable conditions) quotients of integrable functions, Formulae for the integrals of sums and scalar multiples of integrable functions, Integrability of continuous functions. More generally, integrability of bounded functions having finite number of points of discontinuity, Domain additivity of the integral. Integrability and the integral over arbitrary bounded domains. Change of variables formula (Statement only), Polar, cylindrical and spherical coordinates and integration using these coordinates. Differentiation under the integral sign. Applications to finding the center of gravity and moments of inertia.

**References for Unit I:**

1. Apostol, Calculus, Vol. 2, Second Ed., John Wiley, New York, 1969 Section 1.1 to 11.8
2. James Stewart , Calculus with early transcendental Functions - Section 15
3. J. E. Marsden and A.J. Tromba, Vector Calculus, Fourth Ed., W.H. Freeman and Co., New York, 1996. Section 5.2 to 5.6.

**Unit II. Line Integrals (15L)**

Review of Scalar and Vector fields on  $\mathbb{R}$ , Vector Differential Operators.

Gradient Paths (parametrized curves) in  $\mathbb{R}^3$  (emphasis on  $\mathbb{R}^2$  and  $\mathbb{R}^3$ ), Smooth and piecewise smooth paths, Closed paths, Equivalence and orientation preserving equivalence of paths. Definition of the line integral of a vector field over a piecewise smooth path, Basic properties of line integrals including linearity, path-additivity and behavior under a change of parameters, Examples.

Line integrals of the gradient vector field, Fundamental Theorem of Calculus for Line Integrals, Necessary and sufficient conditions for a vector field to be

conservative, Green's Theorem (proof in the case of rectangular domains).  
Applications to evaluation of line integrals.

### **References for Unit II:**

1. Lawrence Corwin and Robert Szczarba, Multivariable Calculus, Chapter 12.
2. Apostol, Calculus, Vol. 2, Second Ed., John Wiley, New York, 1969 Section 10.1 to 10.5, 10.10 to 10.18
3. James Stewart, Calculus with early transcendental Functions - Section 16.1 to 16.4.
4. J. E. Marsden and A. J. Tromba, Vector Calculus, Fourth Ed., W.H. Freeman and Co., New York, 1996. Section 6.1, 7.1.7.4.

### **Unit III. Surface Integrals (15L)**

Parameterized surfaces. Smoothly equivalent parameterizations, Area of such surfaces. Definition of surface integrals of scalar-valued functions as well as of vector fields defined on a surface. Curl and divergence of a vector field, Elementary identities involving gradient, curl and divergence. Stoke's Theorem (proof assuming the general form of Green's Theorem), Examples. Gauss' Divergence Theorem (proof only in the case of cubical domains), Examples.

### **References for Unit III:**

1. Apostol, Calculus, Vol. 2, Second Ed., John Wiley, New York, 1969 Section 1.1 to 11.8
2. James Stewart, Calculus with early transcendental Functions - Section 16.5 to 16.9
3. J. E. Marsden and A. J. Tromba, Vector Calculus, Fourth Ed., W.H. Freeman and Co., New York, 1996 Section 6.2 to 6.4.

### **Additional Reference Books.**

1. T Apostol, Mathematical Analysis, Second Ed., Narosa, New Delhi. 1947.
2. R. Courant and F. John, Introduction to Calculus and Analysis, Vol.2, Springer Verlag, New York, 1989.
3. W. Fleming, Functions of Several Variables, Second Ed., Springer-Verlag, New York, 1977.
4. M. H. Protter and C. B. Morrey, Jr., Intermediate Calculus, Second Ed., Springer-Verlag, New York, 1995.
5. G. B. Thomas and R. L. Finney, Calculus and Analytic Geometry, Ninth Ed. (ISE Reprint), Addison- Wesley, Reading Mass, 1998.
6. D. V. Widder Advanced Calculus, Second Ed., Dover Pub., New York. 1989.



7. Sudhir R. Ghorpade and Balmohan Limaye, A course in Multivariable Calculus and Analysis, Springer International Edition.

**Course: Linear Algebra**  
**Course Code: USMT502 / UAMT502**

**Unit I. Quotient Spaces and Orthogonal Linear Transformations (15L)**

Review of vector spaces over  $\mathbb{R}$ , sub spaces and linear transformation.  
Quotient Spaces: For a real vector space  $V$  and a subspace  $W$ , the cosets  $v + W$  and the quotient space  $V/W$ , First Isomorphism theorem of real vector spaces (fundamental theorem of homomorphism of vector spaces), Dimension and basis of the quotient space  $V/W$ , when  $V$  is finite dimensional.

Orthogonal transformations: Isometries of a real finite dimensional inner product space, Translations and Reflections with respect to a hyperplane, Orthogonal matrices over  $\mathbb{R}$ , Equivalence of orthogonal transformations and isometries fixing origin on a finite dimensional inner product space, Orthogonal transformation of  $\mathbb{R}^n$ , Any orthogonal transformation in  $\mathbb{R}^n$  is a reflection or a rotation, Characterization of isometries as composites of orthogonal transformations and

translation. Characteristic polynomial of an  $n \times n$

real matrix. Cayley Hamilton

Theorem and its Applications (Proof assuming the result (1)) =

for an  $n \times n$

matrix over the polynomial ring  $\mathbb{R}[x]$ .

**Unit II. Eigenvalues and eigen vectors (15L)**

Eigen values and eigen vectors of a linear transformation  $T: V \rightarrow V$ ,

where  $V$  is a finite dimensional real vector space and examples, Eigen values and Eigen vectors of  $n \times n$  real matrices, The linear independence of eigenvectors corresponding to distinct eigenvalues of a linear transformation and a Matrix. The characteristic polynomial of a  $n \times n$  real matrix and a linear transformation of a finite dimensional real vector space to itself, characteristic roots, Similar matrices, Relation with change of basis, Invariance of the characteristic polynomial and (hence of the) eigen values of similar matrices, Every square matrix is similar to an upper triangular matrix. Minimal Polynomial of a matrix, Examples like minimal polynomial of scalar matrix, diagonal matrix, similar matrix, Invariant subspaces

### Unit III: Diagonalisation (15L)

Geometric multiplicity and Algebraic multiplicity of eigen values of an  $n \times n$  real matrix, An  $n \times n$  matrix is diagonalizable if and only if has a basis of eigenvectors of if and only if the sum of dimension of eigen spaces of is if and only if the algebraic and geometric multiplicities of eigen values of coincide, Examples of

non diagonalizable matrices, Diagonalisation of a linear transformation :  $\rightarrow$ ,

where is a finite dimensional real vector space and examples. Orthogonal diagonalisation and Quadratic Forms. Diagonalisation of real Symmetric matrices, Examples, Applications to real Quadratic forms, Rank and Signature of a Real Quadratic form, Classification of conics in  $\mathbb{R}$  and quadric surfaces in  $\mathbb{R}$ . Positive definite and semi definite matrices, Characterization of positive definite matrices in terms of principal minors.

#### Recommended Books.

1. S. Kumaresan, Linear Algebra: A Geometric Approach.
2. Ramachandra Rao and P. Bhimasankaram, Tata McGraw Hill Publishing Company.

#### Additional Reference Books

1. T. Banchoff and J. Wermer, Linear Algebra through Geometry, Springer.
2. L. Smith, Linear Algebra, Springer.
3. M. R. Adhikari and Avishek Adhikari, Introduction to linear Algebra, Asian Books Private Ltd.
4. K Hoffman and Kunze, Linear Algebra, Prentice Hall of India, New Delhi.
5. Inder K Rana, Introduction to Linear Algebra, Ane Books Pvt. Ltd.

### Course: Topology of Metric Spaces Course Code: USMT503 /UAMT503

All concepts have to be taught with plenty of examples and worked out in special case of Euclidean space, Complex plane and other metric spaces mentioned in Unit I.

### Unit I. Metric spaces (15L)

Definition, examples of metric spaces  $\mathbb{R}$ ,  $\mathbb{C}$ , Euclidean space  $\mathbb{R}^n$  sup and sum metric,  $\mathbb{C}$  (complex numbers), the spaces  $l^p$  and  $l^\infty$  the space  $C(X)$ , of real valued continuous functions on  $X$ .

with its Euclidean of sequences and , Discrete metric

space. Distance metric induced by the norm, translation invariance of the metric induced by the norm. Metric subspaces, Product of two metric spaces. Open balls and open set in a metric space, examples of open sets in various metric spaces, Hausdorff property, Interior of a set, Properties of open sets, Structure of an open set in  $\mathbb{R}$ , Equivalent metrics. Distance of a point from a set, between sets, diameter of a set in a metric space and bounded sets.

## **Unit II. Closed sets, Limit Points and Sequences (15L)**

Closed ball in a metric space, Closed sets- definition, examples. Limit point of a set, Isolated point, A closed set contains all its limit points, Closure of a set and boundary, Sequences in a metric space, Convergent sequence in a metric space, Cauchy sequence in a metric space, subsequences, examples of convergent and Cauchy sequence in finite metric spaces,  $\mathbb{R}$  with different metrics and other metric spaces. Characterization of limit points and closure points in terms of sequences. Definition and examples of relative openness/closeness in subspaces, Dense subsets in a metric space and Separability

## **Unit III. Compact sets (15L)**

Definition of compact metric space using open cover, examples of compact sets in different metric spaces  $\mathbb{R}$ ,  $\mathbb{R}^n$ ,  $\mathbb{R}$  and other metric spaces. Properties of compact sets—compact set is closed and bounded, every infinite bounded subset of a compact metric space has a limit point, Heine Borel theorem-every subset of Euclidean metric space  $\mathbb{R}^n$  is compact if and only if it is closed and bounded. Equivalent statements for compact sets in  $\mathbb{R}^n$ ; Heine-Borel property, Closed and boundedness property, Bolzano-Weierstrass property, Sequentially compactness property.

### **Recommended Books.**

1. S. Kumaresan, Topology of Metric spaces.
2. E. T. Copson. Metric Spaces. Universal Book Stall, New Delhi, 1996.

### **Additional Reference Books.**

1. W. Rudin, Principles of Mathematical Analysis.
2. T. Apostol. Mathematical Analysis, Second edition, Narosa, New Delhi, 1974
3. E. T. Copson. Metric Spaces. Universal Book Stall, New Delhi, 1996.
4. R. R. Goldberg Methods of Real Analysis, Oxford and IBH Pub. Co., New Delhi 1970.

5. P. K. Jain. K. Ahmed. Metric Spaces. Narosa, New Delhi, 1996.
6. W. Rudin. Principles of Mathematical Analysis, Third Ed, McGraw-Hill, Auckland, 1976.
7. D. Somasundaram, B. Choudhary. A first Course in Mathematical Analysis. Narosa, New Delhi
8. G.F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill, New York, 1963.
9. Sutherland, Topology.

**Course: Numerical Analysis I (Elective A)**  
**Course Code: USMT5A4 / UAMT5A4**

**N.B.** Derivations and geometrical interpretation of all numerical methods have to be covered.

**Unit I. Errors Analysis and Transcendental & Polynomial Equations (15L)**

Measures of Errors: Relative, absolute and percentage errors. Types of errors: Inherent error, Round-off error and Truncation error. Taylor's series example. Significant digits and numerical stability. Concept of simple and multiple roots. Iterative methods, error tolerance, use of intermediate value theorem. Iteration methods based on first degree equation: Newton-Raphson method, Secant method, Regula-Falsi method, Iteration Method. Condition of convergence and Rate of convergence of all above methods

**Unit II. Transcendental and Polynomial Equations (15L)**

Iteration methods based on second degree equation: Muller method, Chebyshev method, Multipoint iteration method. Iterative methods for polynomial equations; Descart's rule of signs, Birge-Vieta method, Bairstrow method. Methods for multiple roots. Newton-Raphson method. System of non-linear equations by Newton-Raphson method. Methods for complex roots. Condition of convergence and Rate of convergence of all above methods.

**Unit III. Linear System of Equations (15L)**

Matrix representation of linear system of equations. Direct methods: Gauss elimination method. Pivot element, Partial and complete pivoting, Forward and backward substitution method, Triangularization methods-Doolittle and Crouts method, Cholesky's method. Error analysis of direct methods. Iteration methods: Jacobi iteration method, Gauss-Siedal method. Convergence analysis of iterative method. Eigen value problem, Jacobi's method for symmetric matrices  
 Power method to determine largest eigenvalue and eigenvector.

## Recommended Books

1. Kendall E. and Atkinson, An Introduction to Numerical Analysis, Wiley.
2. M. K. Jain, S. R. K. Iyengar and R. K. Jain, Numerical Methods for Scientific and Engineering Computation, New Age International Publications.
3. S.D. Comte and Carl de Boor, Elementary Numerical Analysis, An algorithmic approach, McGraw Hill International Book Company.
4. S. Sastry, Introductory methods of Numerical Analysis, PHI Learning.
5. Hildebrand F.B., Introduction to Numerical Analysis, Dover Publication, NY.
6. Scarborough James B., Numerical Mathematical Analysis, Oxford University Press, New Delhi.

## Course: Number Theory and its applications I (Elective B) Course Code: USMT5B4 / UAMT5B4

### Unit I. Congruences and Factorization (15L)

Review of Divisibility, Primes and The fundamental theorem of Arithmetic.

Congruences : Definition and elementary properties, Complete residue system modulo  $m$ , Reduced residue system modulo  $m$ , Euler's function and its properties, Fermat's little Theorem, Euler's generalization of Fermat's little Theorem, Wilson's theorem, Linear congruence, The Chinese remainder Theorem, Congruences of higher degree, The Fermat-Kraitchik Factorization Method.

### Unit II. Diophantine equations and their solutions (15L)

The linear equations  $ax + by = c$ . The equations  $x^2 + y^2 = p$ , where  $p$  is a prime. The equation  $x^4 + y^4 = z^4$ , Pythagorean triples, primitive solutions, The equations  $x^4 + y^4 = z^4$  and  $x^4 + y^4 = z^4$  have no solutions  $(x; y; z)$  with  $xyz \neq 0$ . Every positive integer  $n$  can be expressed as sum of squares of four integers, Universal quadratic forms  $x^2 + y^2 + z^2 + t^2$ . Assorted examples –section 5.4 of Number theory by Niven-Zuckermann-Montgomery.

### Unit III. Primitive Roots and Cryptography (15L)

Order of an integer and Primitive Roots. Basic notions such as encryption (enciphering) and decryption (deciphering), Cryptosystems, symmetric key cryptography, Simple examples such as shift cipher, Affine cipher, Hill's cipher, Vigenere cipher. Concept of Public Key Cryptosystem; RSA Algorithm. An application of Primitive Roots to Cryptography.

**Reference for Unit III:** Elementary number theory, David M. Burton, Chapter 8 sections 8.1, 8.2 and 8.3, Chapter 10, sections 10.1, 10.2 and 10.3

### **Recommended Books**

1. Niven, H. Zuckerman and H. Montgomery, An Introduction to the Theory of Numbers, John Wiley & Sons. Inc.
2. David M. Burton, An Introduction to the Theory of Numbers. Tata McGraw Hill Edition.
3. G. H. Hardy and E.M. Wright. An Introduction to the Theory of Numbers. Low priced edition. The English Language Book Society and Oxford University Press, 1981.
4. Neville Robins. Beginning Number Theory. Narosa Publications.
5. S.D. Adhikari. An introduction to Commutative Algebra and Number Theory. Narosa Publishing House.
6. N. Koblitz. A course in Number theory and Cryptography, Springer.
7. M. Artin, Algebra. Prentice Hall.
8. K. Ireland, M. Rosen. A classical introduction to Modern Number Theory. Second edition, Springer Verlag.
9. William Stalling. Cryptology and network security.

**Course: Graph Theory (Elective C)**  
**Course Code: USMT5C4 / UAMT5C4**

### **Unit I. Basics of Graphs (15L)**

Definition of general graph, Directed and undirected graph, Simple and multiple graph, Types of graphs- Complete graph, Null graph, Complementary graphs, Regular graphs Sub graph of a graph, Vertex and Edge induced sub graphs, Spanning sub graphs. Basic terminology- degree of a vertex, Minimum and maximum degree, Walk, Trail, Circuit, Path, Cycle. Handshaking theorem and its applications, Isomorphism between the graphs and consequences of isomorphism between the graphs, Self complementary graphs, Connected graphs, Connected components. Matrices associated with the graphs – Adjacency and Incidence matrix of a graph- properties, Bipartite graphs and characterization in terms of cycle lengths. Degree sequence and Havel-Hakimi theorem, Distance in a graph- shortest path problems, Dijkstra's algorithm.

## **Unit II. Trees (15L)**

Cut edges and cut vertices and relevant results, Characterization of cut edge, Definition of a tree and its characterizations, Spanning tree, Recurrence relation of spanning trees and Cayley formula for spanning trees of  $n$ , Algorithms for spanning tree-BFS and DFS, Binary and m-ary tree, Prefix codes and Huffman coding, Weighted graphs and minimal spanning trees - Kruskal's algorithm for minimal spanning trees.

## **Unit III. Eulerian and Hamiltonian graphs (15L)**

Eulerian graph and its characterization- Fleury's Algorithm-(Chinese postman problem), Hamiltonian graph, Necessary condition for Hamiltonian graphs using G-S where S is a proper subset of  $V(G)$ , Sufficient condition for Hamiltonian graphs- Ore's theorem and Dirac's theorem, Hamiltonian closure of a graph, Cube graphs and properties like regular, bipartite, Connected and Hamiltonian nature of cube graph, Line graph of graph and simple results.

### **Recommended Books.**

1. Bondy and Murty Graph, Theory with Applications.
2. Balkrishnan and Ranganathan, Graph theory and applications.
3. West D G. , Graph theory.

### **Additional Reference Book.**

1. Behzad and Chartrand Graph theory.
2. Choudam S. A., Introductory Graph theory.

## **Course: Basic Concepts of Probability and Random Variables (Elective D) Course Code: USMT5D4 / UAMT5D4**

### **Unit I. Basic Concepts of Probability and Random Variables.**

Basic Concepts: Algebra of events including countable unions and intersections, Sigma field  $\mathcal{F}$ , Probability measure  $P$  on  $\mathcal{F}$ , Probability Space as a triple  $(\Omega, \mathcal{F}, P)$ , Properties of  $P$  including Subadditivity. Discrete Probability Space, Independence and Conditional Probability, Theorem of Total Probability. Random Variable on  $(\Omega, \mathcal{F}, P)$  – Definition as a measurable function, Classification of random variables - Discrete Random variable, Probability function, Distribution function, Density function and Probability measure on Borel subsets of  $\mathbb{R}$ , Absolutely continuous random variable. Function of a random variable; Result on a random variable  $R$  with distribution function  $F$  to be absolutely continuous, Assume  $F$  is continuous everywhere and has a

continuous derivative at all points except possibly at finite number of points, Result on density function of where  $=g(h^{-1}(x))$ ,  $h$  is inverse of  $g$  over a 'suitable' subinterval  $(a, b) \subset \mathbb{R}$  under suitable conditions.

**Reference for Unit 1**, Sections 1.1-1.6, 2.1-2.5 of Basic Probability theory by Robert Ash, Dover Publication, 2008.

### Unit II. Properties of Distribution function, Joint Density function

Properties of distribution function  $F$ ,  $F$  is non-decreasing,  $\lim_{x \rightarrow -\infty} F(x) = 0$ ,  $\lim_{x \rightarrow \infty} F(x) = 1$ , Right continuity of  $F$ ,  $F(x) = \int_{-\infty}^x f(t) dt$ , Joint distribution, Joint Density, Results on Relationship between Joint and Individual densities, Related result for Independent random variables. Examples of distributions like Binomial, Poisson and Normal distribution. Expectation and  $k^{\text{th}}$  moments of a random variable with properties.

**Reference for Unit II**, Sections 2.5-2.7, 2.9, 3.2-3.3, 3.6 of Basic Probability theory by Robert Ash, Dover Publication, 2008.

### Unit III. Weak Law of Large Numbers

Joint Moments, Joint Central Moments, Schwarz Inequality, Bounds on Correlation Coefficient, Result on  $\text{Cov}(X, Y) \leq \sqrt{\text{Var}(X)\text{Var}(Y)}$  as a measure of linear dependence,  $\text{Var}(\sum a_i X_i) = \sum a_i^2 \text{Var}(X_i) + 2 \sum_{i < j} a_i a_j \text{Cov}(X_i, X_j)$ . Method of Indicators to find expectation of a random variable, Chebyshev's Inequality, Weak law of Large numbers.

**Reference for Unit III**, Sections 3.4, 3.5, 3.7, 4.1-4.4 of Basic Probability theory by Robert Ash, Dover Publication, 2008

### Additional Reference Books.

1. Marek Capinski, Probability through Problems, Springer

**Course: Practicals (Based on USMT501 / UAMT501 and USMT502 / UAMT502) Course Code: USMTP05 / UAMTP05**

### Suggested Practicals (Based on USMT501 / UAMT501)

1. Evaluation of double and triple integrals.
2. Change of variables in double and triple integrals and applications
3. Line integrals of scalar and vector fields
4. Green's theorem, conservative field and applications
5. Evaluation of surface integrals



6. Stoke's and Gauss divergence theorem
7. Miscellaneous theory questions on units 1, 2 and 3.

### **Suggested Practicals (Based on USMT502 / UAMT502**

1. Quotient Spaces, Orthogonal Transformations.
2. Cayley Hamilton Theorem and Applications
3. Eigen Values & Eigen Vectors of a linear Transformation/ Square Matrices
4. Similar Matrices, Minimal Polynomial, Invariant Subspaces
5. Diagonalisation of a matrix
6. Orthogonal Diagonalisation and Quadratic Forms.
7. Miscellaneous Theory Questions

### **Course: Practicals (Based on USMT503 / UAMT503 and USMT5A4 / UAMT5A4 OR USMT5B4 / UAMT5B4 OR USMT5C4 / UAMT5C4 OR USMT5D4 / UAMT5D4)**

**Course Code: USMTP06 / UAMTP06**

### **Suggested Practicals USMT503 / UAMT503:**

1. Examples of Metric Spaces
2. Open balls and Open sets in Metric / Normed Linear spaces, Interior Points
3. Subspaces, Closed Sets and Closure, Equivalent Metrics and Norms
4. Sequences, Convergent and Cauchy Sequences in a Metric Space
5. Limit Points, Diameter of a set, Dense Sets and Separability
6. Examples of Compact Sets
7. Miscellaneous Theory Questions

### **Suggested Practicals on USMT5A4 / UAMT5A4**

The Practicals should be performed using non-programmable scientific calculator. (The use of programming language like C or Mathematical Software like Mathematica, Matlab, MuPad, and Maple may be encouraged).

1. Newton-Raphson method, Secant method, Regula-Falsi method, Iteration Method
2. Muller method, Chebyshev method, Multipoint iteration method
3. Descart's rule of signs, Birge-Vieta method, Bairstrow method
4. Gauss elimination method, Forward and backward substitution method,
5. Triangularization methods-Doolittle's and Crouts method, Cholesky's method
6. Jacobi iteration method, Gauss-Siedal method
7. Eigen value problem: Jacobi's method for symmetric matrices and Power method to determine largest eigenvalue and eigenvector

### **Suggested Practicals based on USMT5B4 / UAMT5B4**

1. Congruences.

2. Linear congruences and congruences of higher degree.
3. Linear diophantine equation.
4. Pythagorean triples and sum of squares.
5. Cryptosystems (Private Key).
6. Cryptosystems (Public Key) and primitive roots.
7. Miscellaneous theoretical questions based on full USMT5B4 / UAMT5B4.

#### **Suggested Practicals based on USMT5C4 / UAMT5C4**

1. Handshaking Lemma and Isomorphism.
2. Degree sequence and Dijkstra's algorithm
3. Trees, Cayley Formula
4. Applications of Trees
5. Eulerian Graphs.
6. Hamiltonian Graphs.
7. Miscellaneous Problems.

#### **Suggested Practicals based on USMT5D4 / UAMT5D4**

1. Basic concepts of Probability (Algebra of events, Probability space, Probability measure, combinatorial problems)
2. Conditional Probability, Random variable (Independence of events. Definition, Classification and function of a random variable)
3. Distribution function, Joint Density function
4. Expectation of a random variable, Normal distribution
5. Method of Indicators, Weak law of large numbers
6. Conditional density, Conditional expectation
7. Miscellaneous Theoretical questions based on full paper

## SEMESTER VI

**Course: Real and Complex Analysis**  
**Course Code: USMT601 / UAMT601**

### Unit I. Sequence and series of functions (15L)

Sequence of functions - pointwise and uniform convergence of sequences of real-valued functions, examples. Uniform convergence implies pointwise convergence, example to show converse not true, series of functions, convergence of series of functions, Weierstrass M-test. Examples. Properties of uniform convergence: Continuity of the uniform limit of a sequence of continuous function, conditions under which integral and the derivative of sequence of functions converge to the integral and derivative of uniform limit on a closed and bounded interval. Examples. Consequences of these properties for series of functions, term by term differentiation and integration. Power series in  $\mathbb{R}$  centered at origin and at some point in  $\mathbb{R}$ , radius of convergence, region (interval) of convergence, uniform convergence, term by-term differentiation and integration of power series, Examples. Uniqueness of series representation, functions represented by power series, classical functions defined by power series such as exponential, cosine and sine functions, the basic properties of these functions.

#### Reference for Unit I:

1. R.R. Goldberg, Methods of Real Analysis, Oxford and International Book House (IBH) Publishers, New Delhi.
2. Ajit Kumar, S. Kumaresan, Introduction to Real Analysis

### Unit II. Introduction to Complex Analysis (15L)

Review of complex numbers: Complex plane, polar coordinates, exponential map, powers and roots of complex numbers, De Moivre's formula,  $\mathbb{C}$  as a metric space, bounded and unbounded sets, point at infinity-extended complex plane, sketching of set in complex plane (No question be asked).

Limit at a point, theorems on limits, convergence of sequences of complex numbers and results using properties of real sequences. Functions :  $\mathbb{C} \rightarrow \mathbb{C}$  real and imaginary part of functions, continuity at a point and algebra of continuous functions. Derivative of :  $\mathbb{C} \rightarrow \mathbb{C}$ , comparison between differentiability in real and complex sense, Cauchy-Riemann equations, sufficient conditions for differentiability, analytic function, , analytic then + , - , and / are analytic, chain rule.

Theorem: If  $f(z) = 0$  everywhere in a domain  $D$ , then  $f(z)$  must be constant throughout  $D$ , Harmonic functions and harmonic conjugate.

### Reference for Unit II:

1. J. W. Brown and R. V. Churchill, Complex variables and applications: Sections 18, 19, 20, 21, 23, 24, 25

### Unit III. Complex power series (15L)

Explain how to evaluate the line integral  $\int_C f(z) dz$  over  $|z-z_0|=r$  and prove the Cauchy integral formula: If  $f(z)$  is analytic in  $B(z_0, r)$  then for any  $z$  in  $B(z_0, r)$  we have

$f(z) = \frac{1}{2\pi i} \int_C \frac{f(\zeta) d\zeta}{\zeta - z}$ , over  $|z-z_0|=r$ . Taylor's theorem for analytic function. Mobius transformations --definition and examples. Exponential function, its properties, trigonometric function, hyperbolic functions, Power series of complex numbers and related results following from Unit I, radius of convergences, disc of convergence, uniqueness of series representation, examples. Definition of Laurent series, Definition of isolated singularity, statement (without proof) of existence of Laurent series expansion in neighbourhood of an isolated singularity, type of isolated singularities viz. removable, pole and essential defined using Laurent series expansion, statement of residue theorem and calculation of residue.

### Reference for Unit III:

2. J. W. Brown and R.V. Churchill, Complex analysis and Applications: sections 28, 33, 34, 47, 48, 53, 54, 55, Chapter 5, page 231 section 65, define residue of a function at a pole using Theorem in section 66 page 234, Statement of Cauchy's residue theorem on page 225, section 71 and 72 from chapter 7.

### Additional Reference Boos.

3. T. Apostol, Mathematical Analysis, Second Ed., Narosa, New Delhi. 1947.
4. R. Courant and F. John, Introduction to Calculus and Analysis, Vol.2, Springer Verlag, New York, 1989.
5. W. Fleming, Functions of Several Variables, Second Ed., Springer-Verlag, New York, 1977.
6. M. H. Protter and C. B. Morrey Jr., Intermediate Calculus, Second Ed., Springer-Verlag, New York, 1995.
7. G.B. Thomas and R.L. Finney, Calculus and Analytic Geometry, Ninth Ed. (ISE Reprint), Addison- Wesley, Reading Mass, 1998.
8. D. V. Widder Advanced Calculus, Second Ed., Dover Pub., New York. 1989.

9. Sudhir R. Ghorpade and Balmohan Limaye, A course in Multivariable Calculus and Analysis. Springer International Edition.
10. Robert E. Greene and Steven G. Krantz, Function theory of one complex variable
11. T. W. Gamelin, Complex analysis

**Course: Algebra**  
**Course Code: USMT602 / UAMT602**

**Unit I. Group Theory (15L)**

Review of Groups, Subgroups, Abelian groups, Order of a group, Finite and infinite groups, Cyclic groups, The Center  $Z(G)$  of a group  $G$ , Cosets, Lagrange's theorem, Group homomorphisms, isomorphisms, automorphisms, inner automorphisms (No question be asked)

Normal subgroups: Normal subgroups of a group, definition and examples including center of a group, Quotient group, Alternating group  $A_n$ , Cycles. Listing normal subgroups of  $A_4$ ,  $S_3$ . First Isomorphism theorem (or Fundamental Theorem of homomorphisms of groups), Second Isomorphism theorem, third Isomorphism theorem, Cayley's theorem, External direct product of a group, Properties of external direct products, Order of an element in a direct product, criterion for direct product to be cyclic, Classification of groups of order  $\leq 7$ .

**Unit II. Ring Theory (15L)**

Motivation: Integers & Polynomials.

Definitions of a ring (The definition should include the existence of a unity element),

zero divisor, unit, the multiplicative group of units of a ring. Basic Properties & examples of rings, including  $\mathbb{Z}$ ,  $\mathbb{Q}$ ,  $\mathbb{R}$ ,  $\mathbb{C}$ ,  $M_n(\mathbb{R})$ ,  $\mathbb{Q}[X]$ ,  $\mathbb{R}[\overline{X}]$ ,  $\mathbb{C}[X]$ ,  $\mathbb{Z}[i]$ ,  $\mathbb{Z}[\sqrt{2}]$ ,  $\mathbb{Z}[\sqrt{-5}]$ ,  $\mathbb{Z}_n$ . Definitions of Commutative ring, integral domain (ID), Division ring, examples. Theorem such as: A commutative ring  $R$  is an integral domain if and only if for  $a, b, c \in R$  with  $a \neq 0$  the relation  $ab = ac$  implies that  $b = c$ . Definitions of Subring, examples. Ring homomorphisms, Properties of ring homomorphisms, Kernel of ring homomorphism, Ideals, Operations on ideals and Quotient rings, examples. Factor theorem and First and second Isomorphism theorems for rings, Correspondence Theorem for rings: ( If  $f: R \rightarrow S$  is a surjective ring homomorphism, then there is a 1-1 correspondence between the ideals of  $R$  containing the  $\ker f$  and the ideals of  $S$  )

. Definitions of characteristic of a ring, Characteristic of an ID.

### **Unit III. Polynomial Rings and Field theory (15L)**

Principal ideal, maximal ideal, prime ideal, the characterization of the prime and maximal ideals in terms of quotient rings. Polynomial rings,  $R[X]$  when  $R$  is an integral domain/ Field. Divisibility in Integral Domain, Definitions of associates, irreducible and primes. Prime (irreducible) elements in  $\mathbb{R}[X]$ ,  $\mathbb{Q}[X]$ ,  $\mathbb{Z}_p[X]$ . Eisenstein's criterion for irreducibility of a polynomial over  $\mathbb{Z}$ . Prime and maximal ideals in polynomial rings. Definition of field, subfield and examples, characteristic of fields. Any field is an ID and a finite ID is a field. Characterization of fields in terms of maximal ideals, irreducible polynomials. Construction of quotient field of an integral domain (Emphasis on  $\mathbb{Z}$ ,  $\mathbb{Q}$ ). A field contains a subfield isomorphic to  $\mathbb{Z}_p$  or  $\mathbb{Q}$ .

#### **Recommended Books**

1. P. B. Bhattacharya, S. K. Jain, and S. R. Nagpaul, Abstract Algebra, Second edition, Foundation Books, New Delhi, 1995.
2. N. S. Gopalakrishnan, University Algebra, Wiley Eastern Limited.
3. N. Herstein. Topics in Algebra, Wiley Eastern Limited, Second edition.
4. M. Artin, Algebra, Prentice Hall of India, New Delhi.
5. J. B. Fraleigh, A First course in Abstract Algebra, Third edition, Narosa, New Delhi.
6. J. Gallian, Contemporary Abstract Algebra, Narosa, New Delhi.

#### **Additional Reference Books**

1. S. Adhikari, An Introduction to Commutative Algebra and Number theory, Narosa Publishing House.
2. T.W. Hungerford. Algebra, Springer.
3. D. Dummit, R. Foote. Abstract Algebra, John Wiley & Sons, Inc.
4. I.S. Luthar, I.B.S. Passi. Algebra, Vol. I and II.
5. U. M. Swamy, A. V. S. N. Murthy Algebra Abstract and Modern, Pearson.
6. Charles Lanski, Concepts Abstract Algebra, American Mathematical Society
7. Sen, Ghosh and Mukhopadhyay, Topics in Abstract Algebra, Universities press

**Course: Metric Topology**  
**Course Code: USMT603/ UAMT603**

All concepts have to be taught with plenty of examples and worked out in special case of Euclidean space, Complex plane and other metric spaces.

## Unit I. Complete metric spaces (15L)

Definition of complete metric spaces, Examples of complete metric spaces. Completeness property in subspaces. Nested Interval theorem in  $\mathbb{R}$ . Cantor's Intersection Theorem. Applications of Cantor's Intersection Theorem: The set of real numbers is uncountable, Density of rational numbers (Between any two real numbers there exists a rational number), Bolzano Weierstrass Theorem: Every bounded sequence of real numbers has a convergent subsequence, Intermediate

Value theorem: Let  $f: [a, b] \rightarrow \mathbb{R}$  be continuous, and assume that  $f(a) < c < f(b)$  and  $f(b) > c > f(a)$  are

of different signs say,  $f(a) < 0 < f(b)$  and  $f(b) > 0 > f(a)$ . Then there exists  $\xi \in (a, b)$  such that

$f(\xi) = c$ . Heine Borel theorem: Let  $C \subseteq \mathbb{R}$  be a closed and bounded interval and

let  $\mathcal{C} = \{I_\alpha\}_{\alpha \in A}$  be a family of open intervals such that  $C \subseteq \bigcup_{\alpha \in A} I_\alpha$ . Then there exists a

finite subset  $C' \subseteq \mathcal{C}$  such that  $C \subseteq \bigcup_{\alpha \in C'} I_\alpha$ . that is,  $C$  is contained in the union of a finite

number of open intervals of the given family. Finite intersection property of closed sets for compact metric space, hence every compact metric space is complete.

## Reference for Unit I: Expository articles of MTTTS programme

## Unit II. Continuous functions on metric spaces (15L)

Epsilon-delta definition of continuity at a point of a function from one metric space to another, Characterization of continuity at a point in terms of sequences, open sets and closed sets and examples. Algebra of continuous real valued functions on a metric space, Continuity of composite of continuous functions, Continuous image of compact set is compact. Uniform continuity in a metric space, definition and examples (emphasis on  $\mathbb{R}$ ). Contraction mapping and fixed point theorem, Applications

## Unit III. Connected sets (15L)

Separated sets- definition and examples, disconnected sets, disconnected and connected metric spaces, Connected subsets of a metric space. Connected subsets of  $\mathbb{R}$ , A subset of  $\mathbb{R}$  is connected if and only if it is an interval. A continuous image of a connected set is connected, Characterization of a connected space, viz. a metric space is connected if and only if every continuous function from it to  $\{1, -1\}$  is a constant function. Path connectedness in  $\mathbb{R}$ , definition and examples, A path connected subset of  $\mathbb{R}$  is connected, convex sets are path connected, Connected components, An example of a connected subset of  $\mathbb{R}$  which is not path connected.

**Reference Books:**

1. S. Kumaresan, Topology of Metric spaces.
2. E. T. Copson. Metric Spaces. Universal Book Stall, New Delhi, 1996.

**Additional Reference Books.**

1. W. Rudin, Principles of Mathematical Analysis.
2. T. Apostol. Mathematical Analysis, Second edition, Narosa, New Delhi, 1974
3. E. T. Copson. Metric Spaces. Universal Book Stall, New Delhi, 1996.
4. R. R. Goldberg Methods of Real Analysis, Oxford and IBH Pub. Co., New Delhi 1970.
5. P. K. Jain. K. Ahmed. Metric Spaces. Narosa, New Delhi, 1996.
6. W. Rudin. Principles of Mathematical Analysis, Third Ed, McGraw-Hill, Auckland, 1976.
7. D. Somasundaram, B. Choudhary. A first Course in Mathematical Analysis. Narosa, New Delhi
8. G.F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill, New York, 1963.
9. Sutherland. Topology.

**Course: Numerical Analysis II (Elective A)****Course Code: USMT6A4 / UAMT6A4**

**N.B.** Derivations and geometrical interpretation of all numerical methods with theorem mentioned have to be covered.

**Unit I. Interpolation (15L)**

Interpolating polynomials, Uniqueness of interpolating polynomials. Linear, Quadratic and higher order interpolation. Lagrange's Interpolation. Finite difference operators: Shift operator, forward, backward and central difference operator, Average operator and relation between them. Difference table, Relation between difference and derivatives. Interpolating polynomials using finite differences Gregory-Newton forward difference interpolation, Gregory-Newton backward difference interpolation, Stirling's Interpolation. Results on interpolation error.

**Unit II. Polynomial Approximations and Numerical Differentiation (15L)**

Piecewise Interpolation: Linear, Quadratic and Cubic. Bivariate Interpolation: Lagrange's Bivariate Interpolation, Newton's Bivariate Interpolation. Numerical differentiation: Numerical differentiation based on Interpolation, Numerical



differentiation based on finite differences (forward, backward and central), Numerical Partial differentiation.

### **Unit III. Numerical Integration (15L)**

Numerical Integration based on Interpolation. Newton-Cotes Methods, Trapezoidal rule, Simpson's  $1/3^{\text{rd}}$  rule, Simpson's  $3/8^{\text{th}}$  rule. Determination of error term for all above methods. Convergence of numerical integration: Necessary and sufficient condition (with proof). Composite integration methods; Trapezoidal rule, Simpson's rule.

### **Recommended Books**

1. Kendall E, Atkinson, An Introduction to Numerical Analysis, Wiley.
2. M. K. Jain, S. R. K. Iyengar and R. K. Jain, Numerical Methods for Scientific and Engineering Computation, New Age International Publications.
3. S.D. Comte and Carl de Boor, Elementary Numerical Analysis, An algorithmic approach, McGraw Hill International Book Company.
4. S. Sastry, Introductory methods of Numerical Analysis, PHI Learning.
5. Hildebrand F.B, Introduction to Numerical Analysis, Dover Publication, NY.
6. Scarborough James B., Numerical Mathematical Analysis, Oxford University Press, New Delhi.

## **Course: Number Theory and its applications II (Elective B)**

**Course Code: USMT6B4 / UAMT6B4**

### **Unit I. Quadratic Reciprocity (15 L)**

Quadratic residues and Legendre Symbol, Gauss's Lemma, Theorem on Legendre Symbol (–), the result: If  $p$  is an odd prime and 'a' is an odd integer  $(a, p) = 1$  then  $\left(\frac{a}{p}\right) = (-1)^t$  where  $t = \sum_{i=1}^{a-1} \frac{i}{p}$ , Quadratic Reciprocity law. Theorem on Legendre Symbol (–). The Jacobi Symbol and law of reciprocity for Jacobi Symbol. Quadratic Congruences with Composite moduli.

### **Unit II. Continued Fractions (15 L)**

Finite continued fractions. Infinite continued fractions and representation of an irrational number by an infinite simple continued fraction, Rational approximations to

irrational numbers and order of convergence, Best possible approximations. Periodic continued fractions.

### **Unit III. Pell's equation, Arithmetic function and Special numbers (15 L)**

Pell's equation  $x^2 - dy^2 = n$ , where  $d$  is not a square of an integer. Solutions of Pell's equation (The proofs of convergence theorems to be omitted). Arithmetic functions of number theory:  $d(n)$  (or  $T(n)$ ),  $\sigma(n)$ ,  $\sigma_k(n)$ ,  $w(n)$  and their properties,  $\mu(n)$  and the Möbius inversion formula. Special numbers: Fermat numbers, Mersenne numbers, Perfect numbers, Amicable numbers, Pseudo primes, Carmichael numbers

#### **Recommended Books**

1. Niven, H. Zuckerman and H. Montgomery. An Introduction to the Theory of Numbers. John Wiley & Sons. Inc.
2. David M. Burton. An Introduction to the Theory of Numbers. Tata McGraw Hill Edition.
3. G. H. Hardy and E.M. Wright. An Introduction to the Theory of Numbers. Low priced edition. The English Language Book Society and Oxford University Press, 1981.
4. Neville Robins. Beginning Number Theory. Narosa Publications.
5. S. D. Adhikari. An introduction to Commutative Algebra and Number Theory. Narosa Publishing House
6. .N. Koblitz. A course in Number theory and Cryptography. Springer.
7. M. Artin. Algebra. Prentice Hall.
8. K. Ireland, M. Rosen. A classical introduction to Modern Number Theory. Second edition, Springer Verlag.
9. William Stallng. Cryptology and network security.

### **Course: Graph Theory and Combinatorics (Elective C) Course Code: USMT6C4 /UAMT6C4**

#### **Unit I. Colorings of graph (15L)**

Vertex coloring- evaluation of vertex chromatic number of some standard graphs, critical graph. Upper and lower bounds of Vertex chromatic Number- Statement of Brooks theorem. Edge coloring- Evaluation of edge chromatic number of standard graphs such as complete graph, complete bipartite graph, cycle. Statement of Vizing

Theorem. Chromatic polynomial of graphs- Recurrence Relation and properties of Chromatic polynomials. Vertex and Edge cuts vertex and edge connectivity and the relation between vertex and edge connectivity. Equality of vertex and edge connectivity of cubic graphs. Whitney's theorem on 2-vertex connected graphs.

### **Unit II. Planar graph (15L)**

Definition of planar graph. Euler formula and its consequences. Non planarity of  $K_5$  ;  $K(3; 3)$ . Dual of a graph. Polyhedron in  $\mathbb{R}^3$  and existence of exactly five regular polyhedra- (Platonic solids) Colorability of planar graphs- 5 color theorem for planar graphs, statement of 4 color theorem. Networks and flow and cut in a network- value of a flow and the capacity of cut in a network, relation between flow and cut. Maximal flow and minimal cut in a network and Ford-Fulkerson theorem.

### **Unit III. Combinatorics (15L)**

Applications of Inclusion Exclusion Principle- Rook polynomial, Forbidden position problems Introduction to partial fractions and using Newton's binomial theorem for real power find series, expansion of some standard functions. Forming recurrence relation and getting a generating function. Solving a recurrence relation using ordinary generating functions. System of Distinct Representatives and Hall's theorem of SDR. Introduction to matching, M alternating and M augmenting path, Berge theorem. Bipartite graphs.

### **Recommended Books.**

- 1 Bondy and Murty Graph, Theory with Applications.
- 2 Balkrishnan and Ranganathan, Graph theory and applications.
- 3 West D G. , Graph theory.
- 4 Richard Brualdi, Introduction to Combinatorics.

### **Additional Reference Book.**

- 1 Behzad and Chartrand Graph theory.
- 2 Choudam S. A., Introductory Graph theory.
- 3 Cohen, Combinatorics.

**Course: Operations Research Elective D)**

**Course Code: USMT6D4 / UAMT6D4**

### **Unit I. Linear Programming-I (15L)**

Prerequisites: Vector Space, Linear independence and dependence, Basis, Convex sets, Dimension of polyhedron, Faces.

Formation of LPP, Graphical Method. Theory of the Simplex Method- Standard form of LPP, Feasible solution to basic feasible solution, Improving BFS, Optimality Condition, Unbounded solution, Alternative optima, Correspondence between BFS and extreme points. Simplex Method – Simplex Algorithm, Simplex Tableau.

1. G. Hadley, Linear Programming, Narosa Publishing, (Chapter 3).

### **Unit II. Linear programming-II (15L)**

Simplex Method – Case of Degeneracy, Big-M Method, Infeasible solution, Alternate solution, Solution of LPP for unrestricted variable. Transportation Problem: Formation of TP, Concepts of solution, feasible solution, Finding Initial Basic Feasible Solution by North West Corner Method, Matrix Minima Method, Vogel's Approximation Method. Optimal Solution by MODI method, Unbalanced and maximization type of TP.

2. G. Hadley, Linear Programming, Narosa Publishing, (Chapter 4 and 9).
3. J. K. Sharma, Operations Research, Theory and Applications, (Chapter 4, 9).

### **Unit III. Queuing Systems (15L)**

Elements of Queuing Model, Role of Exponential Distribution. Pure Birth and Death Models; Generalized Poisson Queuing Mode. Specialized Poisson Queues: Steady-state Measures of Performance, Single Server Models, Multiple Server Models, Self-service Model, Machine-servicing Model.

#### **Reference for Unit III:**

1. J. K. Sharma, Operations Research, Theory and Applications.
2. H. A. Taha, Operations Research, Prentice Hall of India.

#### **Additional Reference Books:**

1. Hillier and Lieberman, Introduction to Operations Research.
2. Richard Bronson, Schaum Series Book in Operations Research, Tata McGraw Hill Publishing Company Ltd.

**Course: Practicals (Based on USMT601 / UAMT601 and USMT602 / UAMT602) Course Code: USMTP07 / UAMTP07**

#### **Suggested Practicals (Based on USMT601 / UAMT601)**

1. Pointwise and uniform convergence of sequence functions, properties
2. Point wise and uniform convergence of series of functions and properties
3. Limit continuity and derivatives of functions of complex variables,
4. Analytic function, finding harmonic conjugate, Mobius transformations

5. Cauchy integral formula, Taylor series, power series
6. Finding isolated singularities- removable, pole and essential, Laurent series, Calculation of residue.
7. Miscellaneous theory questions.

**Suggested Practicals (Based on USMT602 / UAMT602)**

1. Normal Subgroups and quotient groups.
2. Cayley's Theorem and external direct product of groups.
3. Rings, Subrings, Ideals, Ring Homomorphism and Isomorphism
4. Prime Ideals and Maximal Ideals
5. Polynomial Rings
6. Fields.
7. Miscellaneous Theoretical questions on Unit 1, 2 and 3.

**Course: Practicals (Based on USMT603 / UAMT603 and USMT6A4 / UAMT6A4 OR USMT6B4 / UAMT6B4 OR USMT6C4 / UAMT6C4 OR USMT6D4 / UAMT6D4)  
Course Code: USMTP08 / UAMTP08**

**Suggested practicals Based on USMT603 / UAMT603:**

1. Examples of complete metric spaces
2. Cantor's Theorem and Applications
3. Continuous functions on metric spaces
4. Uniform continuity, fixed point theorem
5. Examples of connected sets and connected metric spaces
6. Path connectedness, convex sets, equivalent condition for connected set using continuous function.
7. Miscellaneous theory questions.

**Suggested Practicals based on USMT6A4 / UAMT6A4**

The Practicals should be performed using non-programmable scientific calculator. (The use of programming language like C or Mathematical Software like Mathematica, Matlab, MuPad, and Maple may be encouraged).

1. Linear, Quadratic and higher order interpolation, Interpolating polynomial by Lagrange's Interpolation
2. Interpolating polynomial by Gregory-Newton forward and backward difference Interpolation and Stirling Interpolation.
3. Bivariate Interpolation: Lagrange's Interpolation and Newton's Interpolation
4. Numerical differentiation: Finite differences (forward, backward and central), Numerical Partial differentiation

5. Numerical differentiation and Integration based on Interpolation
6. Numerical Integration: Trapezoidal rule, Simpson's  $1/3^{\text{rd}}$  rule, Simpson's  $3/8^{\text{th}}$  rule
7. Composite integration methods: Trapezoidal rule, Simpson's rule

#### **Suggested Practicals based on USMT6B4 / UAMT6B4**

1. Legendre Symbol.
2. Jacobi Symbol and Quadratic congruences with composite moduli.
3. Finite continued fractions.
4. Infinite continued fractions.
5. Pell's equations and Arithmetic functions of number theory.
6. Special Numbers.
7. Miscellaneous Theoretical questions based on full USMT6B4 / UAMT6B4.

#### **Suggested Practicals based on USMT6C4 / UAMT6C4**

- 1) Coloring of Graphs
- 2) Chromatic polynomials and connectivity.
- 3) Planar graphs
- 4) Flow theory.
- 5) Inclusion Exclusion Principle and Recurrence relation.
- 6) SDR and Matching.
- 7) Miscellaneous Problem

#### **Suggested Practicals based on USMT6D4 / UAMT6D4**

All practicals to be done manually as well as using software TORA / EXCEL solver.

1. LPP formation, graphical method and simple problems on theory of simplex method
2. LPP Simplex Method
3. Big-M method, special cases of solutions.
4. Transportation Problem
5. Queuing Theory; single server models
6. Queuing Theory; multiple server models
7. Miscellaneous Theory Questions.

#### **Scheme of Examination for Semester V&VI**

The performance of the learners shall be evaluated into two parts. The learner's performance shall be assessed by Internal Assessment with 25% marks in the first part by conducting the Semester End Examinations with 75% marks in the second part. The allocation of marks for the Internal Assessment and Semester End Examinations are as shown below:-

**(a) Internal Assessment 25 %**

Courses with tutorials (Mathematics)

| Sr. No | Evaluation type   | Marks |
|--------|---|-------|
| 1      | One class Test  | 20    |
| 2      | Active participation in routine class instructional deliveries/Tutorials. Overall conduct as a responsible student, mannerism and articulation and exhibit of leadership qualities in organizing related academic activities. | 05    |

**(b) External Theory examination 75 %.**

1. Duration – The examinations shall be of 2.5 Hours duration.
2. Theory Question Paper Pattern:-There shall be four questions. Question number 1, 2 and 3 will be of 20 marks each (with internal options), while Question 4 will be of 15 marks (with internal options).
3. All questions shall be compulsory with internal choice within the questions.
4. Questions may be sub divided into sub questions as a, b, c, d & e, etc & the allocation of marks depends on the weightage of the topic.

**B. Sc / B. A. Mathematics Paper pattern.**

**External Examination:**

1. There shall be 4 questions, first three questions shall be of 20 marks on each unit and fourth question will be of 15 marks based on Unit I, II and Unit III.
2. All questions shall be compulsory with internal choice within the questions.

| Questions   | Subquestions  | Maximum marks |
|-------------|---|---------------|
| Q1          | <b>Part A:</b><br>Two theory subquestions each one is of 8 marks and attempt any one.<br><b>Part B:</b><br>Six subquestions (essentially examples), each one is of 4 marks and attempt any three. | 20x3          |
| Q2          |   |               |
| Q3          |   |               |
| Q4          | There shall be 6 subquestions each one is of 5 marks (2 subquestions on each unit) and attempt any 3.   | 15            |
| Total Marks |   | 75            |

### Internal Examination.

1. There shall be 2 questions.

2. All questions shall be compulsory with internal choice within the questions.

| Questions   | Subquestions  | Maximum marks |
|-------------|---|---------------|
| Q1          | Seven objective subquestions each one is of 1 mark and attempt any five.            | 05            |
| Q2          | Four subquestions (essentially examples), attempt any three each one is of 5 marks. | 15            |
| Total Marks |   | 20            |

### S. Y. B.Sc., Paper III (Sem III&IV):

**Teaching Pattern:** 1. Three lectures per week. 2. One practical per week per batch (The batches to be formed as prescribed by the University).

### Internal Assessment 25%.

There will be no class test for Paper III, instead there shall be a separate practical examination (conducted by the college) at the end of each semester (III&IV) of 50 marks and marks be converted out of 25.

### Paper pattern for 50 marks:

|          |           |
|----------|-----------|
| Q 1. :   | 20 Marks  |
| Q 2. :   | 20 Marks  |
| Journal: | 05 Marks  |
| Viva:    | 05 Marks  |
| Total:   | 50 Marks. |

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